THE

THEORY AND PRACTICE

OF FINDING THE

LONGITUDE

AT SEA OR LAND.

VOL. I.

B. Tickner

3052

THE

THEORY AND PRACTICE

OF FINDING THE

LONGITUDE

AT SEA OR LAND:

TO WHICH ARE ADDED,

VARIOUS METHODS OF DETERMINING

THE LATITUDE OF A PLACE,

AND VARIATION OF THE COMPASS;

WITH

NEW TABLES.

BY

ANDREW MACKAY, *LL. D. F. R. S. EDIN.*

HONORARY MEMBER OF THE LITERARY AND PHILOSOPHICAL SOCIETY OF NEWCASTLE-UPON-TYNE, AND MATHEMATICAL EXAMINER TO THE HONOURABLE THE CORPORATION OF TRINITY-HOUSE, THE HONOURABLE THE EAST INDIA COMPANY, CHRIST'S HOSPITAL, ETC.

IN TWO VOLUMES.

THE THIRD EDITION, IMPROVED AND ENLARGED.

VOL. I.

Cœli enarrant Gloriam Dei.

LONDON:

PRINTED FOR AND SOLD BY THE AUTHOR,

No. 2, George Street, Trinity Square, Tower Hill;

AND BY THE PRINCIPAL BOOKSELLERS IN THE UNITED KINGDOM,

BY R. WILKS, 89, CHANCERY LANE.

1809.

ENTERED AT STATIONERS' HALL,
ACCORDING TO ACT OF PARLIAMENT.

TO THE

RIGHT HONOURABLE AND HONOURABLE

THE MASTER,

DEPUTY MASTER,

WARDEN, AND ASSISTANTS,

OF THE

CORPORATION OF TRINITY HOUSE,

THIS TREATISE

IS DEDICATED,

WITH THE UTMOST DEFERENCE

AND RESPECT,

BY THEIR

MUCH OBLIGED,

MOST OBEDIENT,

AND FAITHFUL SERVANT,

THE AUTHOR.

PREFACE.

In every commercial state, any work that has for its object the improvement of the art of Navigation, will always be favourably received; and should it not, in all respects, answer the public expectation, the author may still have some claim to indulgence from his good intentions, and having exercised his best abilities for the good of his country. With this view, the author of the following Treatise has bestowed much time and labour to render it as complete, and as generally useful, as possible; and, though he has not the vanity to imagine it to be by any means a perfect work, he has yet been flattered, that it may be of service to Navigation, and, therefore, not altogether unworthy of the attention of the public. He ventures, therefore, to submit it, with much diffidence, to their judgement; and if it shall be found any ways deserving of such a character, he will esteem their approbation his highest and best reward. Should this be the case, he may be encouraged, at a future period, to offer some opinions on the mechanical construction of ships, an ancient art, but perhaps less improved by long experience than any which we now cultivate with so much success.

It is well known, that, although the course and distance between any two places, whose latitude and longitude are accurately settled, may be ascertained with the greatest precision, yet a ship at sea is so liable to be put out of the proper course, by storms, contrary winds, currents, &c. that, after all reasonable allowance is made for the errors arising from these causes, the place of the ship by account is very seldom found to agree with its real place, or that deduced from observation, the difference frequently amounting to several degrees,

degrees. It is, therefore, obvious, that the ship may be exposed to the greatest hazard when the seamen think themselves most secure; and from this it appears that there is an absolute necessity of taking observations, whenever an opportunity offers, for obtaining, with accuracy, the true place of the ship.

At sea, the latitude of a ship is easily deduced either from the meridian altitude of the Sun, or from that of a star or planet, or from double, treble, &c. altitudes of any of these objects; but with regard to the longitude, a more difficult operation becomes necessary. At land, indeed, the longitude of any place may be found with very little trouble by several methods, particularly by the eclipses of Jupiter's satellites; and the longitude at sea might be found by this method with the same facility, provided a telescope possessed of a sufficient magnifying power could be employed for that purpose. But this is found to be extremely difficult, by reason of the perpetual agitation of the sea. However, attempts to render a telescope manageable at sea have been made by Dr. Irwin and others, and an instrument for this purpose is described in Rutledge's "*Theorie Astronomique* *." But a telescope, with a small magnifying power, may be used at sea in moderate weather; and, since, even with such an instrument, solar eclipses and occultations may often be observed with tolerable accuracy, and consequently the longitude may be ascertained from such observations, the opportunities of making them ought never to be neglected when the ship is either out of sight of land, or near an unknown coast. The calculations, indeed, which necessarily attend these methods, seem to be considered as too laborious and difficult for the generality of seamen; and, therefore, the method by observing the distance between the Moon and the Sun, or a fixed star, is now commonly preferred, both on account of its superior facility, and because it can be practised at sea almost as often as necessary. Yet, as even this method is attended with a calculation that is, by many navigators, thought troublesome, various ways have been proposed, to shorten, as much as possible, the

* This method is mentioned in the *Cosmolabe de Jacques Besson*, printed at Paris in 1807.

operations

operations for reducing the apparent to the true distance; and it is now accordingly reduced to a tolerably simple computation. By this method the longitude may, in most cases, be determined within half a degree, which at sea is esteemed no very great error; and if the calculations are accurately performed, from the mean of several sets of good observations, taken at short intervals of time, the error will, probably, be greatly diminished.

If a chronometer, or time-keeper, could be constructed, so as to go uniformly when placed in every different position, and under different degrees of heat, then would this method of finding the longitude at sea be a most valuable acquisition to the navigator. Indeed some of our ingenious countrymen have brought this art to a degree of perfection formerly unknown; and every person acquainted with the principles of watch-making must highly admire the accuracy of Harrison's time-keeper. Those made by Messrs. Arnold, Kendal, Earnshaw, &c. are also excellent; but it is to be hoped, that instruments of this kind may be still farther improved, and may be afforded much cheaper than at present: for the high price alone is a very great objection to them, and very much prevents their being more generally used. It is well known, that every chronometer hitherto contrived is subject to irregularities; the smallest shock is found to affect them, and the rate of going is found to be altered by changes in the atmosphere, even though provided with thermometer pieces. Upon this and other accounts, therefore, their accuracy is very much to be suspected; so that at present they are chiefly used for experiments, or to connect observations, for which purpose they certainly make a valuable appendage to a set of nautical instruments.

The following work, which contains the method of finding the longitude already mentioned, and also those from which any advantage can be derived for the solution of this important problem, is divided into two volumes; and the first volume is divided into six books.

The first book contains the general principles necessary for a proper knowledge of the subject.

Book second contains the description, rectification, and use of the Quadrant, Sextant, and Circular Instrument, in their present improved state; also, an account of the corrections to be applied to

the observed altitude of any celestial object, in order to reduce it to the true altitude.

In book third is contained a complete system of lunar observations, with an introductory account of this method of finding the longitude at sea. It also contains a new method of finding the longitude and latitude of a ship at sea, together with the apparent time, from the same set of observations; for which the author had the honour of receiving the thanks of the Boards of Longitude of England and of France.

Book fourth contains various methods of finding the Longitude of a place; some of which, though scarcely practicable at sea, are yet, perhaps, the very best that can be used for determining the longitude of any place at land. These are, by the Moon's transit over the meridian, by the meridian altitude of the Moon, and by the observed altitude of the Moon when not on the meridian; by lunar eclipses, solar eclipses, occultations of fixed stars by the Moon, eclipses of the satellites of Jupiter, by a chronometer, and by the Variation Chart.

Every treatise must be considered as defective, wherein rules are given without demonstrations: and this is particularly the case with most books on the subject of navigation, probably because sea-faring people in general are very indifferent as to the *rationale* of any rule given for making an observation, provided it holds good in practice. On the other hand, it not unfrequently happens, that to the persons not very fond of mathematical investigations, the rule and demonstration following each other in regular order, may be disagreeable. For these reasons, and to gratify the scientific seaman in the present highly-improved state of nautical knowledge, the demonstrations of the *rules* and *formulæ* are separately inserted at the end of the work, in the fifth book. And, in order to render this work still more generally useful, there is farther added to what was originally proposed, book sixth, containing various methods of finding a ship's latitude, and the variation of the compass.

Volume second contains the tables, together with the necessary explanations of them.

This

This work, therefore, it is hoped, will be found to comprehend a collection of the best methods of making and reducing all sorts of observations necessary to be made at sea, for ascertaining a ship's place; or for reducing the true course between any two places to the course per compass; or for correcting the course steered. The author, however, flatters himself, that it will not be considered merely as a compilation from the works of others, but that the intelligent reader will discover, in various parts of it, some things that are at least new and of his own invention, however little other merit they may be thought to possess; and in every part of it, a suitable plainness and perspicuity of style, and a warm zeal for the improvement of one of the noblest and most useful of human arts. In a word, he has exerted his best endeavours to render his work worthy the notice of the navigator and astronomer, as well as, in some respects, of the practical geographer and surveyor.

The favourable reception of the first edition of this work, has induced the author to revise the whole; in doing which he has made many additions to it, which he flatters himself will be found useful. The size of the page has been also enlarged, in order to prevent that irregularity, or confusion, which would otherwise happen in the arrangement of the calculations, when the page had been too much contracted. And, with a view to render the computations more easy, and to make the work complete, some of the tables in the second volume of the first edition have been altered and extended, and new tables added; which has been a work of considerable labour.

To this third edition, still further improvements and additions have been made, as will be evident by comparing it with the former editions, throughout the whole of the work. In the second volume, the Table of Natural Versed Sines is extended to an arch of 180°, with a view to facilitate the operation of reducing the apparent distance to the true distance, by Method First, page 150: and also for the purpose of employing various other methods of effecting the same thing; see page 157, &c. A table of Logarithmic Tangents, to every ten seconds, with the proportional parts to each second, similar to that of the Logarithmic Sines, is also added. The various other additions, alterations, and corrections, will be obvious; and no expense whatever has been spared, with respect to print, paper, &c. in order to render it worthy of the attention of the public.

The author is extremely sorry to have occasion to observe, that some late authors, or he may rather say, pretended authors, have copied largely from the former editions of this work, without mentioning from whence they obtained their information. Many of the tables they have copied; and, strange to tell! have asserted they were either constructed or altered to the present form by themselves*: many other of the tables they have altered, and to which they have given new titles, with a view to disguise them as much as possible. They have copied several of the various methods, rules, &c. by which they have entailed upon themselves the most opprobrious of epithets, namely, that of plagiarists. This, however, is not the case with foreigners, particularly the French, Dutch, Americans, &c. who have candidly acknowledged from whence they derived their information.

The author cannot conclude without mentioning, that many favours have been conferred upon him by his worthy and valuable friend, Mr. BARON MASERES: to whom the public is, in a great measure, indebted for the present edition of this work. His superior knowledge raises him high in the literary and scientific world; and he has, without the least degree of ostentation, an ardent desire to promote true knowledge, and also the happiness of those who are similarly inclined.

The Author most respectfully informs the Public, that his signature, as below, is placed to every copy of this work, and which can only be had by application to himself, or by his appointment.

George Street, Trinity Square,
January 16, 1809.

A. Mackay

* For the truth of this, the reader is requested to compare a Collection of Mathematical Tables, printed in London in 1803, and an Epitome of Navigation, of which these Tables make a part, printed in 1805, with the second volume of the author's Treatise on the Longitude, printed in 1801.

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CHAP.

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BOOK V.

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BOOK VI.

Containing various Methods of finding the Latitude of a Place, and the Variation of the Compass.

EXPLA-

EXPLANATION OF CHARACTERS.

The planetary characters, see page 28, are also used to represent the days of the week.

Thus, ☉ denotes Sunday, ☽,—Monday, ♂,—Tuesday,
☿ Wednesday, ♃,—Thursday, ♀,—Friday.
♄ - Saturday,

For the characters of the zodiacal constellations, see page 11.

Also, ☊ the ascending node of a planet, and ☋ the descending node; A. M. (*ante meridiem*) put after any hour, signifies that the time is between midnight and noon; and P. M. (*post meridiem*) that the given time is between noon and midnight.

Aspects *of the* Planets

The aspect of a planet is its situation with respect to the Sun or to another planet. There are usually reckoned five aspects, viz.

☌	denotes	Conjunction, or planets having the same longitude.
✱	——	Sextile, the difference of longitude of the planets being 2 signs, or 60°.
□	——	Quartile, the difference of longitude of the planets being 3 signs, or 90°.
△	——	Trine, the difference of longitude being 4 signs, or 120°.
☍	——	Opposition, or planets situated in opposite longitudes, or differing 6 signs from each other.

Algebraic Characters.

Each of these characters, except the radical, is supposed to be placed between two quantities, to indicate whether the *sum*, *difference*, &c. of these quantities is to be taken.

The sign $+$, (*plus*) signifies,		that the second quantity is to be added to the first.
$-$, (*minus*)	-	the second quantity is to be subtracted from the first.
$\times$,	-	the two quantities are to be multiplied.
$\div$,	- -	the first to be divided by the second.
$\backsim$	-	the diff. of the two quantities is to be taken.
$\sqsubset$, or $\angle$	-	the first is greater than the second.
$\sqsupset$, or $\measuredangle$	-	the first is less than the second.

It may be observed, that the less quantity is placed at the open part of the character, and the greater at the close part.

$\surd$	- - -	the square root of the quant. within it.

When

When the sum or difference of two quantities is to be multiplied by a third; then these quantities, connected with their proper sign, are placed either within a parenthesis, or have a line, called a *Vinculum*, drawn above them. Thus, if the sum of a and b is to be multiplied by x, the product will be $(a+b)\times x$, or $\overline{a+b}\times x$.

THE

THE

THEORY AND PRACTICE

OF FINDING THE

LONGITUDE AT SEA OR LAND.

BOOK I.

CONTAINING,

The Principles of the Astronomical Methods of finding the LONGITUDE *at* SEA *or* LAND.

CHAP. I.

Of the FIGURE *and* MAGNITUDE *of the* EARTH.

INTRODUCTION.

WITHOUT a previous knowledge of the figure and magnitude of the Earth, the places of the heavenly bodies could not be accurately settled, from observations made on its surface; and therefore, computations made from observations of these bodies, could not be depended on for ascertaining the position of places on the Earth; hence, the necessity of knowing both the Figure and magnitude of the Earth is apparently obvious.

Of the FIGURE *of the* EARTH.

The opinions of the ancients concerning the figure of the Earth were various. It was supposed by many to be a plane indefinitely extended; some imagined it to be of a cylindric form, and a few supposed it spherical; but the discovery of its real figure was left to the illus-

trious Sir Isaac Newton. The following are a few of the arguments commonly used to prove, that the figure of the Earth is either spherical, or nearly so.

I.

The Earth has been circumnavigated by many persons, at different periods. The first who attempted this circumnavigation was Ferdinand Magellan, a Portuguese. He sailed from Seville, in Spain, August 10, 1519, in the ship called the Victory, accompanied by four other vessels. In April 1521, Magellan was killed, at the Island of *Sebu* or *Zebu*,* one of the Philippines, in a skirmish with the natives; and one of his vessels arrived at St. Lucar, near Seville, Sept. 7, 1522. The next who circumnavigated the Earth was Sir Francis Drake. He sailed from Plymouth, December 13, 1577, with five vessels,† and arrived at the same place, Sept. 26, 1580. Since that time, the circumnavigation of the Earth has been performed by Sir Thomas Cavendish, Messrs. Cordes, Noort, Scharten, Heremites, Dampier, Woodes, Rogers, Schovten, Roggewein, Lord Anson, Byron, Carteret, Wallis, Bougainville, Cook, King, Clerk, Vancouver, and many others. These navigators, by sailing in a westerly direction, allowance being made for promontories, &c. arrived at the country they sailed from. Hence, the Earth must be either of a cylindric, or globular figure; but it cannot be in the form of a cylinder, because then the difference of longitude and meridian distance between any two places would be equal, which is contrary to observation; the figure of the Earth is, therefore, spherical.

II.

The upper parts only of distant ships are visible, when viewed with a telescope, the lower parts being hid by the interposed water, and more or less become visible, according to the distance. In making the land, the most elevated parts are first seen, and the lower parts become visible as the land is approached. The sun is observed sooner at rising, and later at setting, by a person at the mast-head of a ship, than by one on deck. These phenomena evidently arise from the spherical figure of the Earth; and, therefore, serve to prove the Earth to be of that figure.

III.

The continual appearance of the sun above the horizon, during the space of several months, in the neighbourhood of one pole, while at a place equally distant from the other pole, the sun is as long absent, is another proof that the Earth is spherical.

* This island is also sometimes called Matan or Mactan; its metropolis is in latitude 10° 33′ N and longitude 123° 48″ E.

† The largest of these vessels was only 100 tons; the others were 80, 50, 30 and 15 tons respectively.

IV.

IV.

LEMMATA.

1st, The distance of the nearest fixed star, when compared with the magnitude of the Earth, is so immense, that rays flowing therefrom, to any two points on the surface of the Earth, are physically parallel.

2d, If in a curve, the arches are proportional to the correspondent angles, that curve is a circle.

Now, if the Earth was an extended plane, the meridian zenith distance of any given fixed star would be the same in all places of the Earth, by lemma 1st; but it is found to be variable, and in such a manner, that the difference of the meridian altitudes of the same star is proportional to the intercepted arch of a meridian; hence, by lemma 2d, that meridian is circular: and since this is found to be the case in every part of the Earth, its figure is, therefore, spherical.

V.

LEMMA.

If the shadow of any body, when turned in every position with respect to the luminous body, be circular, when projected on a plane perpendicular to the line joining the centres of both, the body itself is a sphere.

Now, since a lunar eclipse arises from the passage of the moon through the shadow of the Earth; and as that portion of the Earth's shadow, which is projected on the lunar disk, is observed to be always circular, in every different position of the Earth—the figure of the Earth must, therefore, be that of a sphere.

VI.

If a lunar eclipse be observed at two places, differing in longitude, the time of the beginning, or end, will be observed to be later at the eastern than at the western place; and the difference of time will be found to be propotional to the difference of longitude. And if the eclipse be observed at two places in the same parallel of latitude, and also at two other places, under the same meridian as the former, but in a different parallel of latitude; then, although the same interval of time will be observed in each of the parallels, yet the meridian distances between the places will be different; and, upon calcalation, will be found to answer to a spherical figure, or nearly so. From observations of this kind the magnitude of the earth might also be determined.

VII.

The planets hitherto discovered are observed to be globular; but the Earth is also a planet, subject to the same laws, and revolving round the sun in the same manner as the other planets; therefore, by analogy, the Earth is globular.

Although it appears from the preceding proofs, that the Earth is of a spherical figure, yet it is not a perfect sphere, but an Oblate

SPHEROID,* which is a solid generated by the rotation of a semi-ellipse about its shortest axis. The circumstance that gave rise to the knowledge of its spheroidal figure is briefly as follows: Soon after the establishment of the Royal Academy of Sciences of Paris, in 1666,† it was suggested, that, granting the rotation of the Earth, a weight ought to descend with less force at the equator than at the poles; or that a pendulum vibrating seconds in France, ought to be shortened, in order to vibrate seconds at or near the equator. To settle this point, therefore, along with others, the Academy of Sciences sent Mr. Richer to the island of Cayenne,‡ in South America. Richer accordingly left Paris in October 1671, and the following year he found that his pendulum, which vibrated seconds in Paris, lost two minutes twenty-eight seconds daily at Cayenne; and therefore, in order that the peudulum should keep time at that place, it ought to be shortened 1¼ lines.‖ This difference, however, being greater than what should have arisen from the excess of the centrifugal force§ at Cayenne above that at Paris, excited the curiosity of Newton and Huygens, who, enquiring minutely into the cause of this phenomenon, shewed that it resulted from the rotation of the Earth combined with its spheroidal figure; and Newton, supposing the Earth to assume the same figure that a homogeneous fluid would take under like circumstances, computed the ratio of the equatoreal diameter to the axis of the earth to be as 230 : 229. ¶

This hypothesis was objected to by M. Cassini, who affirmed, the figure of the Earth was that of an oblong spheroid; and the contest was carried on with much ingenuity by their followers. To settle this dispute, however, by actual experiment, the French King ordered the length of a degree to be measured at the equator, and at the polar circle; so that, by comparing the length of one of these degrees with that measured in France, it would be known whether the figure of the Earth was that of an *oblate* or *oblong* spheroid.

For this purpose, therefore, two companies of mathematicians were appointed; the one company to measure the length of a degree at Peru, in South America, and the other in Swedish Lapland. The company for South America, consisting of Messrs. Bouguer, de la Condamine, and Godin, sailed from Rochelle, in France, May, 16, 1735; and were joined at Carthagena by Don J. Juan, and Don Antonio de Ulloa, and their operations were not finished until 1744. The company for the north, consisting of Messrs. de Maupertuis, Clairault, Camus, Le Monnier, and Outhier, left Paris, April 20, 1736; and were joined by M. Celsus, Professor of Astronomy at Upsal. Their measures being completed, they returned to Paris, August 19, 1737.

* From observations made in different parts of the Earth, it would appear, that its figure is not that of a regular spheroid.

† The first meeting of the Society was December 22, 1666.

‡ Latitude of Cayenne, 4° 56′ N. longitude, 52° 16′ W.

‖ A line is a twefth part of a French inch.

§ Centrifugal force, is that force by which any body moving in a curve, endeavours to fly of in a tangent to that curve—and centripetal force is, that by which any revolving body is impelled towards some point as a center.

¶ Newton's Principia, Book iii. Prop. xix. Prob. iii.

The

The lengths of a degree, as deduced from the mensurations of each of these companies, being compared with themselves, and also with that formerly obtained from mensurations taken in France, confirmed Newton's theory. For, by comparing the length of a degree at the polar circle with that measured in France, the Earth will be found to be more flattened at the poles than as assigned by Newton; but, according to Newton's theory, the Earth is flatter at the poles than that deduced from a comparison of the lengths of a degree at the equator, and in France—whereas, by comparing the first and last of these measures, the figure of the Earth deduced therefrom will nearly agree with that assigned by Newton.*

Various opinions have been formed with respect to the internal structure of the Earth. According to some, the Earth was supposed to consist of a shell or crust, the interior part being either a cavity, or filled with water. And Dr. Halley, in order to explain the phenomena of the magnetic needle, supposed an internal nucleus.† Reason, however, was against these hypotheses, and experiment has confuted them. For according to Dr. Hutton's very elaborate calculations,‡ made from the mensurations taken at the hill of Schehallien, in Perthshire, in the years 1774, 1775 and 1776, by the direction, and partly under the inspection, of Dr. Maskelyne, Astronomer Royal, the density of the Earth is to that of the hill as 9 to 5 nearly; and the density of the matter in the hill being supposed to be to that of rain water as $2\frac{1}{2}$ to 1:‖ hence, these two ratios being compounded, there results the ratio of $4\frac{1}{2}$ to 1, for the ratio of the densities of the Earth and rain water.

"Sir Isaac Newton," says Dr. Hutton, "thought it probable, that the mean density of the Earth might be five or six times as great as the density of water; and we have now found, by experiment, that it is very little less than what he thought it to be. So much justness was even in the surmises of this wonderful man! Since then, the mean density of the whole Earth is about double that of the general matter near the surface, and within our reach, it follows, that there must be somewhere within the Earth, toward the more central parts, great quantities of metals, or such like dense matter, to counter-

* Length of a degree at the polar circle, corrected,	-	57422 toises
- - in France,	-	57069
- - at the equator, according to Bouguer,	-	56753

Hence a degree at the Polar circle exceeds that in France by 353 toises, and that at the equator by 669 toises. If the Earth is a regular spheroid, its figure may be determined by any two of these measures See, the Figure of the Earth determined, &c. by M de Maupertuis, page 164.—M. de la Lande's Astronomie, tom iii p. 28.—Emerson's Geography, p. 28.—Jones' Solution in Gardner's Log. Tables —Letherland's Solution in Robertson's Navigation, vol, ii. p. 205.—Martin's New Principles, &c.

† Miscellanea Curiosa, vol. I. page 48.

‡ Philosophical Transactions, vol. lxviii.

‖ Doctor Hutton, in a letter dated Woolwich, July 5th, 1800, says, "The density of the hill not being got, we were forced to make use of an arbitrary or assumed number for it, viz. the gravity of common stone ($2\frac{1}{2}$); which we have reason to believe, is much below that of the hill in question—and that consequently, our conclusion thence deduced is much below the truth."

balance

balance the lighter materials, and produce such a considerable mean density. If we suppose, for instance, the density of metal to be 10, which is about a mean among the various kinds of it, the density of water being 1, it would require sixteen parts out of twenty-seven, or a little more than one-half of the matter in the whole Earth, to be metal of this density, in order to compose a mass of such mean density as we have found the Earth to possess by our experiment—or $\frac{4}{13}$, or between $\frac{1}{3}$ and $\frac{1}{4}$ of the whole magnitude, will be metal; and consequently $\frac{20}{31}$ or nearly $\frac{2}{3}$ of the diameter of the Earth, is the central or metalline part."

Of the Magnitude *of the* Earth.

The determination of the magnitude of the Earth seems to have engaged the attention, both of the ancient and modern astronomers and mathematicians. The most celebrated of the ancients, who attempted to ascertain the magnitude of the Earth by actual mensuration, were Anaximander, Cleomedes, Eratosthenes, Posidonius, Ptolemy, and Almamon, King of Arabia. Although the conclusions of the ancients differ very considerably from each other, yet the measures taken within the last 140 years, near the same place, agree very well.

The only person in England, who performed this problem with any tolerable degree of accuracy, was Mr. Richard Norwood, of which an account is given in his *Seaman's Practice*—the substance of which is as follows:—In the year 1633, he observed the apparent meridian altitude of the sun, at the time of the summer solstice near the Tower of London; and June 11, o. s. 1635, he observed the sun's apparent meridian altitude, near the middle of the city of York; from whence he found the difference of latitude between those places to be 2° 28′. Now, this being compared with the difference between the parallels of latitude of these places, deduced from actual mensuration, which was 9149 chains of 99 feet each, he found that one degree contained 367196, or in round numbers 367200 English feet. But as Norwood paced some of his distances,* and did not observe the vertical angles, or those of ascent and descent, with a degree of accuracy which is necessary in so important a problem,† the above measure of a degree is, therefore, only an approximation to the truth.

In order to obtain a more accurate determination of the magnitude of the Earth, mensurations have been taken at different times by several

* "Now touching the experiment, I confesse, that to have made it so exactly as were requisite, would have required much more time and expence than mine ability would reach unto: Yet, having made observation at *York*, as aforesaid, I measured (for the most part) the way from thence to *London*, and where I measured not, I paced; (wherein, through custome, I usually come very neer the truth) observing all the way as I came, with a circumferentor, all the principal angles of position or windings of the way, (with convenient allowance for other lesser windings, ascents and descents) and these I laid not down by a protractor after the usual manner, but framed a table much more exact and fit for this purpose."—Seaman's Practice, p. 7.

† "Now touching the angles of ascent and descent of hills and valleys, to have observed them exactly, would have required more time and charge than I could of myself bestow; yet I made allowance for such of them as were of most moment."—Seaman's Practice, p. 14

eminent

eminent French astronomers and mathematicians, particularly Messrs. Picard, Cassini, Maupertuis, de la Caille, Bouguer, &c. The near agreement of their measures, taken at the same place, is a proof of the accuracy of their operations; and the comparison of their measures, taken at different places, is a confirmation of Newton's theory of the spheroidal figure of the Earth.

The most accurate method, and that which is now usually practised, to find the length of a degree of latitude, or of any portion of the meridian, and hence the whole circumference of the Earth, is to measure, as accurately as possible, a base‡ of any convenient length, as about one-tenth of the portion of the meridian intended to be measured, and towards the middle of that portion: from this base, a series of triangles is to be extended, ending at the two extreme stations, from which the arch of the meridian intercepted between the parallels of latitude of these stations is to be computed: and the truth of the operation is, if possible, to be verified by means of bases measured near the end of each series of triangles. At each extreme station, the zenith distance of one or more stars is to be observed, and corected for precession, aberration, nutation, &c.; then, the difference between the zenith distances of the same star gives the correspondent celestial arch. Now,

As the celestial arch
Is to one degree,
So is the interval betweenthe two parallels deducedfrom mensuration,
To the length of a degree in the same measure.

In the year 1756, the Royal Academy of Sciences of Paris appointed eight astronomers to measure the length of a degree of latitude between Paris and Amiens. These gentlemen divided themselves into two companies, and each company measured a base separately. The result of their mensurations gave 57069 toises for the length of a degree; now, since 107 French feet are equal to 114 English feet, and 880 fathoms are contained in an English mile—hence the circumference of the Earth, expressed in English miles, is equal to $\frac{57069\times114\times360^\circ}{107\times880}=\frac{57069\times57\times9}{107\times11}=24873$. Hence also, one degree contains about 69.09 English miles, and a nautical or geographical mile contains 6080 feet: And since the proportion of the circumference of a circle to its diameter is as 355 to 113 nearly,‖ therefore the diameter of the earth is equal to 7917½ English miles.

For the purpose of ascertaining the distance run by a ship in any given

‡ The most accurate bases that have hitherto been measured are, perhaps, those of Hounslow Heath, and Romney Marsh—of which an account is given in the Phil. Transactions, for the years 1785 and 1790.

‖ If the three first odd figures 1, 3, and 5, be each put down twice—then, the three first of these will express the diameter, and the three last the circumference, nearly

given time, a half minute glass is commonly used, and a line divided acordingly; and since half a minute is the 120th part of an hour, therefore, the interval between two adjacent knots on the line should be $=\frac{6080}{120}=50$ feet 8 inches; but because it is safest to have the reckoning a-head of the ship, 48 feet are, therefore, commonly allowed to a knot. If this interval be divided decimally, or into ten equal parts, the computation will be rendered more easy and accurate: It may be observed, that the glass should run $29\frac{1}{2}''$, instead of $30''$, half a second being allowed for the time taken in turning the glass.

CHAP. II.

Definitions, Principles, &c.

DAY and night arise from the circumrotation of the Earth. That imaginary line about which the rotation is performed, is called the Axis, and its extremities are called Poles. That towards the most remote parts of Europe is called the *North Pole*, and its opposite the *South Pole*. The Earth's Axis being produced will point out the *Celestial Poles*.

The Equator is a great circle on the Earth, every point of which is equally distant from the Poles; it divides the Earth into two equal parts, called Hemispheres: that having the North Poles in its centre is called the *Northern Hemisphere*—and the other, the *Southern Hemisphere*. The plane of this circle being produced to the fixed stars, will point out the celestial Equator or Equinoctial. The Equator as well as all other great circles of the sphere, is divided into 360 equal parts, called *degrees*;* each degree is divided into 60 equal parts, called *minutes;* and the sexagesimal division is continued.

The Meridian of any place, is a semi-circle passing through that place, and terminating at the Poles of the Equator. The other half of this circle is called the *opposite Meridian.*

The Latitude of any place, is that portion of the Meridian of that place, which is contained between the Equator and the given place; and is either *North* or *South*, according as the given place is in the Northern or Southern Hemisphere; and, therefore, cannot exceed 90°.

* The ancients having no instruments by which they could make observations with any tolerable degree of accuracy, supposed the length of the year, or annual motion of the earth, to be completed in 360 days; and hence arose the division of the circumference of a circle into the same number of equal parts, which they called *degrees.*

The

The PARALLEL OF LATITUDE of any place, is a circle passing through that place, parallel to the Equator.

The DIFFERENCE OF LATITUDE between any two places, is an arch of a meridian intercepted between the corresponding parallels of latitude of those places. Hence, if the places lie between the Equator and the same Pole, their difference of latitude is found by subtracting the less latitude from the greater; but if they are on oppossite sides of the Equator, the difference of latitude is equal to the sum of the latitudes of both places.

The FIRST MERIDIAN is an imaginary semicircle, passing through any remarkable place, and is therefore arbitrary. Thus, the British esteem that to be the First Meridian, which passes through the Royal Observatory at Greenwich; and the French reckon for their First Meridian, that which passes through the Royal Observatory at Paris. Formerly, many French geographers reckoned the meridian of the island of Ferro to be their First Meridian; and others, that which was exactly 20 degrees to the west of the Paris Observatory. The Germans, again, considered the meridian of the Peak of Teneriffe to be the First Meridian. By this mode of reckoning, Europe, Asia, and Africa, are in east longitude; and North and South America, in west longitude. At present, the first meridian of any country is generally esteemed to be that which passes through the principal Observatory, or chief city of that country.

The LONGITUDE of any place is that portion of the Equator which is contained between the first meridian, and the meridian of that place; and is usually reckoned either *east* or *west*, according as the given place is on the east or west side of the first meridian; and, therefore, cannot exceed 180°*.

The DIFFERENCE OF LONGITUDE between any two places is the intercepted arch of the Equator between the meridians of those places, and cannot exceed 180°.

In order to illustrate the preceding definitions, let Pp (fig. 1st) represent the axis of the earth, EQ the equator, and PAp, PBp, the meridians of the two places A and B; then the portion AF, of the meridian of the place A, contained between it and the equator, will be the latitude of that place; in like manner, the intercepted arch BG, of the meridian PBp, will be the latitude of the place B—and the portion AH or BI, of the meridian passing through either of those places, contained between the parallels of latitude KAIL, MHBN, will be the difference of latitude: Hence, it is evident, by inspection of the figure, that the difference between the latitudes of any two places is equal to the difference, or sum, of their latitudes, according as the given places

* In order to prevent any ambiguity that might arise from the division of the longitude into east and west, some persons have reckoned it from the first meridian towards the east, until the first meridian again. Hence, 333° of longitude would, by the common method of reckoning, be 27° W.

are in the same, or in contrary hemispheres. Again, let PRp be the first meridian—then the arch of the equator RF, contained between the first meridian and the meridian of the place A, is the longitude of that place; RG is the longitude of the place G, and FG the difference of longitude of those places: Whence it is apparently obvious, that the difference of longitude between any two places, is equal either to the difference or sum of the longitudes of those places; according as they are on the same, or on contrary sides of the first meridian.

There are three different Horizons, the APPARENT, the SENSIBLE, and the TRUE.* The apparent or visible Horizon is the utmost apparent view of the sea or land. The sensible is a plane passing through the eye of an observer, perpendicular to a plumb line hanging freely: And the true or rational Horizon is a plane passing through the center of the Earth, parallel to the sensible Horizon.

Altitudes observed at sea, are measured from the visible Horizon. At land, when an astronomical quadrant is used, or when observations are taken with a Hadley's quadrant by the method of reflection, the altitude is measured from the sensible Horizon; and in either case, the altitude must be reduced to the true Horizon.

The ZENITH of any given place is the point immediately above that place, and is, therefore, the elevated pole of the Horizon: The NADIR is the other pole, or point diametrically opposite.

A VERTICAL is a great circle passing through the Zenith and Nadir; and, therefore, intersecting the Horizon at right angles.

The ALTITUDE of any celestial body is that portion of a Vertical, which is contained between its center and the true Horizon. The Meridian Altitude is the distance of the object from the true Horizon, when on the Meridian of the place of observation. When the observed Altitude is corrected for the depression of the Horizon, and the errors arising from the instrument, it is called the *apparent Altitude*; and when reduced to the true Horizon, by applying the parallax in Altitude, it is called the *true Altitude*. Altitudes are expressed in degrees, and parts of a degree.

The ZENITH DISTANCE of any object is its distance from the Zenith, or the complement of its Altitude.

The DECLINATION of any object is that portion of its meridian, which is contained between the equinoctial and the center of the object; and is either north or south, according as the object is between the equinoctial and the north or south pole.

The ECLIPTIC is that great circle in which the annual revolution of the Earth round the Sun is performed. It is so named, because Eclipses cannot happen but when the Moon is in, or near that circle. The inclination of the Ecliptic and Equinoctial is at present about 23° 28′; and by comparing ancient with modern observations, the obliquity of

* It may also be necessary to mention another Horizon, which may be called the REDUCED HORIZON. This is a plane passing through the earth's center, perpendicular to the radius answering to the reduced place of observation.

the

the Ecliptic is found to be diminishing—which diminution, in the present century, is about half a second yearly.†

The Ecliptic, like all other great circles of the sphere, is divided into 360°; and is further divided into twelve equal parts, called SIGNS:‡ each Sign, therefore, contains 30°. The names and characters of these Signs are as follow:

Aries,	♈	Cancer,	♋	Libra,	♎	Capricornus,	♑
Taurus,	♉	Leo,	♌	Scorpio,	♏	Aquarius,	♒
Gemini,	♊	Virgo,	♍	Sagittarius,	♐	Pisces,	♓

Since the Ecliptic and Equinoctial are great circles, they, therfore, bisect each other in two points, which are called the *Equinoctial Points.* The Sun is in one of these points in March, and in the other in September; hence, the first is called the *Vernal,* and the other the *Autumnal* Equinox—and that sign which begins at the Vernal Equinox is called *Aries.* Those points of the Ecliptic, which are equidistant from the equinoctial points, are called the *Solstitial Points;* the first the *summer,* and the second the *winter solstice.* That great circle which passes through the equinoctial points and the poles of the earth, is called the *Equinoctial Colure;* and the great circle which passes through the solstitial points and the poles of the earth, is called the *Solstitial Colure.*

When the Sun enters Aries, it is in the Equinoctial; and, therefore, has no declination. From thence it moves forward in the Ecliptic, according to the order of the signs, and advances towards the north pole, by a kind of retarded motion, till it enters Cancer, and is then most distant from the Equinoctial; and moving forward in the Ecliptic, the Sun apparently recedes from the north pole with an accelerated motion till it enters Libra, and being again in the Equinoctial, has no declination; the Sun moving through the signs Libra, Scorpio, and Sagittarius, enters Capricorn; and then its south declination is greatest, and is, therefore, most distant from the north pole; and moving forward through the signs Capricorn, Aquarius, and Pisces, again enters Aries: Hence, a period of the seasons is completed, and this period is called a Solar Year.

The signs Aries, Taurus, Gemini, Cancer, Leo, and Virgo, are called *Northern Signs,* because they are contained in that part of the Ecliptic which is between the Equinoctial and North Pole; and, there-

† It is also subject to two periodical inequalities; the first, arising from the attraction of the sun, is completed in six months, the maximum of which is only half a second; the other, called the *Nutation* of the Earth's Axis. is completed in a revolution of the Moon's Nodes, or 18 years 224 days, and is usually represented by supposing the pole of the earth to describe the periphery of an ellipse in a retrograde manner during a period of the Moon's Nodes. the greater axis being in the solstitial colure, and equal to 19″.1, and the least axis in the equinoctial colure, and equal to 14″.2; the greater axis being to the less, as the co-sine of the obliquity of the ecliptic, to the co-sine of double the obliquity.

‡ The division of the Ecliptic into twelve equal parts was the invention of Cleostratus Tenedius. See Pliny, lib. 2. cap. 8. For the Moon was supposed to complete a lunation in 30 days, and in that period the Sun would have completed a twelfth part of the Ecliptic, or 30°, which was therefore called a *Sign.*

fore, while the Sun is in these signs, its declination is *north*: the other six signs are called *Southern Signs*. The signs in the first and fourth quarters of the Ecliptic are called *Ascending Signs*; because, while the Sun is in these signs, it approaches the north pole—and, therefore, in the northern, temperate and frigid zones, the Sun's meridian altitude daily increases; or, which is the same, the Sun ascends to a greater height above the horizon every day. The signs in the second and third quarters of the Ecliptic are called *Descending Signs*.

The Longitude of the Sun, is that portion of the Ecliptic, which is contained between the Vernal Equinox and the Sun's center.

The Right ascension of any celestial object, is that arch of the Equinoctial, which is intercepted between its meridian, and the Vernal Equinox.

The TROPICS are circles parallel to the Equinoctial, whose distance therefrom, is equal to the obliquity of the Ecliptic. The Northern Tropic touches the Ecliptic at the begining of Cancer, and is, therefore, called the *Tropic of Cancer*; and the Southern Tropic touches the Ecliptic at the beginning of Capricorn, and is hence called the *Tropic of Capricorn*.

Circles about the poles of the Equinoctial, and passing through the poles of the Ecliptic, are called POLAR CIRCLES; the distance, therefore, of each Polar Circle from its respective Pole, is equal to the inclination of the Ecliptic and Equinoctial. That Circle which circumscribes the North Pole, is called the *Arctic* or *North Polar Circle*; and that towards the South Pole, the *Antartic*, or *South Polar Circle*.

That semicircle which passes through a Star, or any given point of the heavens, and the Poles of the Ecliptic, is called a CIRCLE of LATITUDE.

The REDUCED PLACE of a Star is that point of the Ecliptic, which is intersected by the circle of Latitude passing through that Star.

The LATITUDE of a Star is that portion of the circle of latitude contained between the Star and its reduced place—and is either *north* or *south*, according as the Star is between the Ecliptic and the north or south pole thereof.

The LONGITUDE of a Star is that portion of the Ecliptic, contained between the Vernal Equinox and the reduced place of the Star thus: PAp, is a Circle of declination; and PSR = ASB, the angle of position.

That semicircle which passes through a Star, and the Poles of the Equinoctial, is called A CIRCLE OF DECLINATION.

The ANGLE OF POSITION of a Star is the angle contained between the Circles of latitude and declination, which pass through that Star.

The *Nonagesimal Degree*, is that point of the Ecliptic, which is equidistant, or 90° from each of its intersections with the horizon.

The Altitude of the Nonagesimal, is an Arch of a Vertical Circle, contained between it and the horizon.

The Longitude of the Nonagesimal is that Arch of the Ecliptic which is contained between it and the Vernal Equinox.

Let

Let the Circle PEQ (fig. 2.) represent the Solstitial Colure, EQ the Equinoctial, P, p its Poles; ♋ ♑ the Ecliptic, whose Poles are R, r; and S the position of a Star with respect to these Circles: then is AS the declination, and ♈ A the right ascension of that Star; B is its reduced place: Hence, BS is the latitude, and ♈ B the longitude of the Star.

CHAP. III.

Of the Length *of the* Year.

THE orbit of the earth is an ellipse, in one of whose foci is the sun. That point of the earth's orbit which is most distant from the sun is called the *Aphelion*; and that which is nearest is called the *Perihelion*; and the line joining these is called the *Line of the Apsides*. Since, in common language, it is usual to attribute the motion of the earth to the sun, the first of these points is called the *Apogee* of the sun, and the second, the *Perigee*.

A Year is that period or portion of time, in which a revolution of the earth, with respect to some point in the heavens, is completed. As the equinoctial points and the line of the apsides have a slow motion, with respect to the fixed stars; there are, therefore, three different kinds of Years—a *Solar* or *Tropical*, a *Sidereal* or *Periodical*, and an *Anomalistic* Year.

A Solar or Tropical Year, is the interval of time between two successive returns of the sun, to the same equinoctial or solstitial point. The seasons are regulated by this Year.

A Sidereal or Periodical Year, is the interval of time between two successive returns of the earth to the same fixed star.

An Anomalistic Year, is the interval of time between two successive returns of the earth to the same apsis.

Of the Tropical Year.

The length of the Solar Year is ascertained by comparing the times of observation of two corresponding equinoxes; and the more distant these

these equinoxes are from each other, the more exact will the result be. To the length of the Year thus deduced, three corrections are to be applied, in order to obtain the exact length. The first, and most considerable of these corrections, arises from the motion of the apogee of the sun, which advances at the rate of 62″ annually; hence the mean anomaly, at the vernal equinox, being less than at the time of the preceding vernal equinox, the elliptic equation is also less—and that equation being additive, the sun's true longitude is, therefore, less; consequently, the mean length of the Year is less by the time answering to the above difference of the elliptic equation. Again, at the time of the autumnal equinox, the elliptic equation is less than at the preceding autumnal equinox, and being subtractive, the sun's longitude is therefore greater; and hence, the apparent length of the Year is less than the mean length. The above change of the elliptic equation reduced to time will, at the time of the vernal equinox, amount to 6″.5; and at that of the autumnal equinox to 8″.7. In the first case *subtractive*, and in the second *additive*, in order to obtain the mean length of the Year.

The second correction is in consequence of the precession of the equinoxes, being greater in the present age than formerly; and by comparing modern observations, the length of a mean Solar Year is found to be about 2″.6* less than that deduced from a comparison of modern with ancient observations, as those of Hipparchus.

The third and last correction of the length of the Year, arises from the attractions of the Moon, Jupiter, and Venus. This correction, however, becomes almost insensible, when distant observations of the equinoxes are compared.

In the year 145 before our æra, according to the astronomical mode of reckoning, Hipparchus observed the time of the vernal equinox at Alexandria; which, reduced to the meridian of Paris, was, 24th March, at 6h. 10′: and according to M. de la Lande's observations, made at Berlin, in the year 1752, the time of the equinox, reduced to the meridian of Paris, was, 19th March, or 8th old stile, at 16h. 42′. In this interval there are 1897 years, wanting 15d. 13h. 28′, of which 475 are leap years; and, therefore, in the above period there are 692864d. 10h. 32′, which being divided by 1897 years, gives 365d. 5h, 48′ 46″ nearly, the length of the Tropical Year.

Again, M. Cassini, at Bologna, upon the 28th March, 1657, at 0h. 5′ 1″ mean time, or 27th March, at 23h. 28′ 56″ mean time at Paris, from observations of the sun's altitude, found the declination to be 3° 15′ 12″ N.; and hence, the sun's longitude 0s. 8° 11′ 16″, or 0s. 8° 11′ 3″, allowance being made for the attraction of the planets, &c.: and from a series of five successive meridian altitudes of the sun, observed at Paris in 1760, M. de la Caille found the sun's longitude to be the same, 27th March, at 22h. 11′ 46″ mean time. The interval

* M. de la Lande's Astronomy, vol. i. page 300.

of

of time between these two observations is 37619d. 22h. 42′ 50″, which being divided by 103, gives 365d. 5h. 48′ 48″, the length of the Tropical Year. In these observations the sun was at its mean distance. M. de la Lande obtained the same results, from a comparison of nine equinoxes observed by Hipparchus, with late observations.

Of the Sidereal Year.

The method of finding the length of the Sidereal Year is to ascertain, from observation, the instants when the sun had twice the same position with respect to a given fixed star; and the interval of time between these instants, being divided by the number of revolutions, will give the length of the Sidereal Year.

From a series of observations made at Paris, by M. de la Hire, in 1684, the difference of longitude between the Sun and Sirius was 1° 21′ 59″, June 29th, at 0h. 2′ 50″ mean time; and from observations made at the Cape of Good Hope, by M. de la Caille, in 1785, the Sun was found to be in the same position with respect to Sirius 1st July, at 3h. 31′ 37″ mean time at Paris. The interval of time between these observations is 24472d. 3h. 28′ 47″, which increased by 26′ 27″, answering to the proper motion of Sirius in longitude, and divided by 67, the number of revolutions elapsed, gives 365d. 6h. 8′ 53″.2, the length of a Sidereal Year.

Again, upon the 21st December, 1684, at 23h, 59′ 50″ mean time at Paris, M. de la Hire found the difference of longitude between the Sun and Sirius to be 171° 55′ 45″; and M. de la Caille, from observations made at the Cape of Good Hope in 1751, found the difference of longitude to be the same upon the 24th December, at 4h. 7′ 36″ mean time, reduced to the meridian of Paris. The interval is 24472d. 4h. 7′ 46″, which increased by 24′ 44″, the time the Sun would apparently move a space equal to the proper motion of Sirius, and divided by 67, gives 365d. 6h. 9′ 26″.4, the length of the Sidereal Year: the mean of which and the former determination is 365d. 6h. 9′ 9″.8. As this mean is deduced from the results of observations made in nearly opposite points of the orbit, it may, therefore, be considered as an exceeding near approximation to the exact length of the Sidereal Year.

Since the same number of years are contained in the two intervals, the operation would have been shortened, by dividing half the sum of the intervals by the number of revolutions.

The length of the Sidereal Year is deduced with the greatest accuracy, from observations of the Sun, when at or near its mean distance from the earth—or rather, from the mean of the results, inferred from the comparison of observations made at nearly equal intervals of time, and in opposite parts of the orbit, as towards the apogee and perigee; because

because of the uniformity of the motion of the sun for several days when near the apsides*.

The length of the Tropical Year, and the precession of the equinoxes being given, the length of the Sidereal Year may be found as follows:

Reduce the precession of the equinoxes to time, at the rate of 59′ 8″ to 24 hours; which, being added to the length of the Tropical Year, will give the length of the Sidereal Year. Thus,

Let the length of the Tropical Year be 365d. 5h. 48′ 48″, and the precession of the equinoxes 50″.2†—required the length of the Sidereal Year? Now - 59′ 8″ : 50″.2 : : 24h. : 20′ 22″

Length of the Tropical Year, -	365	5	48	48
Length of the Sidereal Year, -	365	6	9	10

Of the ANOMALISTIC YEAR.

The length of the Anomalistic Year is determined by comparing the times of two passages of the sun through the same apside, as before; or it may be found by reducing the motion of the apogee to time, at the rate of 59′ 8″ to 24 hours, and adding it to the length of the Tropical Year.

According to M. de la Lande's tables, the annual motion of the line of the apsides is 62″.—Hence,

59′ 8″ : 62″ : : 24h. : 25′ 10″

Length of the Tropical Year, -	365	5	48	48
Length of the Anomalistic Year, -	365	6	13	58

Since cycles or periods, centuries, &c. contain some given number of years, we shall, therefore, conclude this chapter with the following definitions;

An ÆRA is a particular account, or reckoning of time, from some certain point; to which account, all remarkable events or actions are referred: as the Christian æra, which is used at present in Europe; the æra of Nabonassar, which has always been famous among astronomers, &c. An æra, therefore, has a beginning, but no end.

An EPOCH is the beginning, or first instant of an æra.

A PERIOD or CYCLE is a certain portion of time. The following are the principal periods or cycles.

The cycle of the SUN is a period of 28 years: in which space of time, all the varieties of the dominical letters will have happened;

* M. de la Caille's Leçons Elementaires d'Astronomie, page 239.

† Ditto, page 237, and Nautical Almanack, preface, p. 8.

and

and they will return in the same order as they did 28 years before*.

The cycle of the MOON, called also the Metonic cycle, from the name of its inventor, is a period of 19 years, containing all the variations of the days on which the new and full moons happen; after which, they will fall on the same days they did 19 years before. The number of years elapsed in this cycle is called the *prime*, or *golden number*.

The cycle of INDICTION is a period of 15 years, used by the Romans; as once in that period, a tax was collected among the inhabitants of those countries which they had conquered.

The VICTORIAN Period, so called from its inventor Victorinus, or Victorius, and employed by him to find Easter, is a period of 532 years, arising from the multiplication of the solar and lunar cycles. This is also sometimes called the *Dionysian* period.

The JULIAN Period, invented by Joseph Scaliger, contains 7980 years, being the product arising from the multiplication of the cycles of the sun, moon, and indiction. The commencement of the Christian æra, or year 0, according to the Dyonisian, or vulgar account, was the 4713th year of the Julian period.

A CENTURY is a period of one hundred years.

CHAP. IV.

Of the FIXED STARS.

THE Fixed Stars are so named, because they are observed to retain their relative places with respect to each other. Some stars appear to be of a sensible magnitude to the naked eye, but when viewed through a telescope, seem only as lucid points, without any apparent diameter; hence their immense distance from the solar system is inferred, and consequently, they emit their own light, otherwise they would be invisible. It is, therefore, reasonable to suppose them to be so many suns; diffusing light and heat, to planets revolving round them.

The number of fixed Stars appears very considerable to a person who looks inattentively at them; but, from a due contemplation, it will be found that, without the assistance of a telescope, there can very seldom be more than a thousand seen above the horizon at the same time: however, when the Stars are viewed with a good telescope,

* The first seven letters of the alphabet are placed in the Calendar opposite to the seven days of the week, and that letter which answers to Sunday is called the *dominical letter*. If the year contained exactly 365 days, a period of the dominical letters would be completed in seven years; but because every fourth year, which is called *bissextile* or *leap year*, contains 366 days, the period is therefore extended to four times seven, or 28 years.

their number appears almost infinite. Between the equinoctial and the parallel of 45° north declination, M. de la Lande observed the places of 8000 Stars, with an achromatic telescope of 8 feet. In a space 8° in length and 3° in breadth, Dr. Herschel reckoned 44000 small Stars; and in the year 1792, August 22, he found that in 41 minutes of time, no less than 258000 Stars had passed through the field of view of his telescope *.

The Stars, with respect to their apparent splendour, are divided into orders, called MAGNITUDES. The brightest are called Stars of the First Magnitude: the next to these in splendour, Stars of the Second Magnitude, and so on to those which are just perceptible to the naked eye, and which are called Stars of the Sixth Magnitude †. Those which cannot be discerned without the assistance of a telescope, are called *Telescopic Stars*, and are divided into orders of the Seventh, Eighth, Ninth, &c. Magnitudes accordingly. We are not, however, to infer from this, that the Stars can be exactly reduced to one or other of these magnitudes, for the star α Aquilæ is reckoned by some to be of the first magnitude, and others esteem it of the second; hence those Stars, whose magnitudes are doubtful, are generally marked in catalogues as partaking of both magnitudes—thus α Aquilæ is marked 1.2, signifying that it is either of the first or second, or rather between these magnitudes; and ν Scorpionis is marked 3.4, as being between these magnitudes; and the figure denoting the magnitude, to which the Star is nearest, is put first—thus, δ Scorpionis is marked 3. 2; signifying, therefore, that it is between the second and third magnitudes, but nearest the third. From what has been said of the magnitudes of the Stars, we are not to suppose that their sizes are in the ratio of their apparent magnitudes; they may perhaps be nearly of the same bulk, but the apparent magnitude of a Star depends on its distance.

The Stars, for the purpose of finding any one more readily, are divided into parcels called CONSTELLATIONS. These, in order to assist the imagination, are supposed to be circumscribed with some known figure, as that of a *man*, *woman*, *ship*, *sextant*, &c. and those Stars which lie between constellations are called UNFORMED STARS. As it would be an endless task to give a proper name to each Star, it has, therefore been customary to mark the Stars of each constellation, with the letters of the Greek alphabet, in such a manner, that the first letter is prefixed to the brightest Star, the second letter to the next in brightness, and so on ‡. Many of the brightest of the fixed Stars have also

* Phil. Transactions for 1795, p. 70.

† Instead of classing the Stars visible to the naked eye into *six* magnitudes, it, probably, would have been more proper to have divided them into *ten*—and the decimal division may easily be continued.

‡ Since many of the constellations contain more Stars than there are letters in the alphabet, it would have been found very convenient to have denoted them by the numeral characters; so that 1 being annexed to a Star would denote it to be the brightest Star in that constellation, 2 to the next in brightness, and so on. In like manner, it would be very satisfactory

also proper names—thus *α*, Bootæ, is also called *Arcturus*; *ε*, Virginis, is called *Vindemiatrix*, &c.

The celestial sphere is divided into three parts, the ZODIAC, and the NORTHERN and SOUTHERN HEMISPHERES.

The ZODIAC extends to about 8° on each side of the ecliptic, and contains the orbits of all the planets: there are twelve constellations in the zodiac. According to the ancients, there were 21 constellations in the northern hemisphere, and 15 in the southern; and consequently, 48 constellations in the zodiac and both hemispheres. Modern astronomers, however, by curtailing several of the ancient constellations of some of their stars, which they formed into new constellations; and by forming into constellations the unformed Stars, or those which lay between the ancient ones, have increased the number of constellations in the northern hemisphere to upwards of 40, and those in the southern to about 48; and consequently, there are upwards of 100 constellations in all. The names of these constellations are as follow:

ZODIACAL CONSTELLATIONS.

1	Aries,	The Ram.	7	Libra,	The Balance.
2	Taurus,	Bull.	8	Scorpio,	Scorpion.
3	Gemini,	Twins.	9	Sagittarius,	Archer.
4	Cancer,	Crab.	10	Capricornus,	Goat.
5	Leo,	Lion.	11	Aquarius,	Water Bearer.
6	Virgo,	Virgin.	12	Pisces,	Fishes.

NORTHERN CONSTELLATIONS.

1	Ursa Minor,	The Little Bear.	22	Triang. Borealis.	The Northern Triangle.
2	Ursa Major,	Great Bear.	23	Coma Berenices,	Berenice's Hair.
3	Draco,	Dragon.	24	Camelopardalus,	The Camelopard.
4	Cepheus,	Cepheus.	25	Monoceros,	Unicorn.
5	Bootes,	Bootes.	26	Triangulum Minus,	Little Triangle.
6	Corona Borealis,	The Northern Crown.	27	Lynx,	Lynx.
7	Hercules,	Hercules.	28	Leo Minor,	Little Lion.
8	Lyra,	The Harp.	29	Asterion et Chara,	Greyhounds.
9	Cygnis,	Swan.	30	Cerberus,	Cerberus,
10	Cassiopeia,	Cassiopeia.	31	Vulpecula et Anser,	The Fox and Goose.
11	Perseus,	Perseus.	32	Scutum Sobieski,	Sobieski's Shield.
12	Auriga,	The Waggoner.	33	Lacerta,	The Lizard.
13	Serpentarius,	Serpentarius.	34	Mons Mænalus,	A Mountain of Arcadia.
14	Serpens,	The Serpent.	35	Cor Caroli,	Charles' Heart.
15	Sagitta,	Arrow.	36	Renne,	The Rein Deer.
16	Aquila,	Eagle.	37	Le Messier,	M. Messier.
17	Antinous,	Antinous.	38	Taurus Regalis,	The Royal Bull.
18	Delphinus,	The Dolphin.	39	Friedrick's Ehre,	Frederick's Glory.
19	Equuhis,	Horse Head.	40	Tubus Herschelii Major,	Herschel's Great Telescope.
20	Pegasus,	Flying Horse.			
21	Andromeda,	Andromeda.			

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satisfactory if all the Stars in the heavens were again to be marked by numerals; so that 1 may be prefixed to the brightest Star in the heavens, 2 to the next in brightness, &c.

Thus 1. Canis Majoris. No. 1. Sirius.
2. Lyræ. 1. Vega, &c.

From this it would, therefore, be inferred, that Sirius was the brightest Star in the constellation Canis Major, and also the brightest Star in the heavens; that Vega was the brightest Star in Lyra, and the next in brightness to Sirius, &c.

SOUTHERN CONSTELLATIONS.

1	Cetus,	The Whale.
2	Orion,	Orion.
3	Eridanus,	River Eriadanus.
4	Lepus,	Hare.
5	Canis Major,	Great Dog.
6	Canis Minor,	Little Dog.
7	Argo Navis,	Ship Argo.
8	Hydra,	Hydra.
9	Crater,	Cup.
10	Corvus,	Crow.
11	Centaurus,	Centaur.
12	Lupus,	Wolf
13	Ara,	The Altar.
14	Corona Australis,	Southern Crown.
15	Piscis Australis,	Southern Fish.
16	Columba Noachi,	Noah's Dove.
17	Robet Carolinum,	Royal Oak.
18	Grus,	Crane.
19	Phœnix,	Phœnix.
20	Indus,	Indian.
21	Pavo,	Peacock.
22	Apus, ou *Avis Indica*,	Bird of Paradise.
23	Apis, ou *Musca*,	Bee or Fly.
24	Crux,	The Cross.
25	Chamelion,	Chamelion.
26	Triangulum Australis,	Southern Triangle,
27	Pisces Volans, ou Passer.	Flying Fish.
28	Dorado, ou Xiphias,	The Sword Fish.
29	Toucan,	American Goose.
30	Hydrus,	Water Snake.
31	Sextans,	Sextant.
32	Apparatus Sculptoris,	Apparatus of the Carver.
33	Fornax Chemica,	Chemical Furnace.
34	Horologium,	Clock.
35	Reticulus,	Reticulat. Rhomboid.
36	Cælum Sculptorium,	Graving Tool.
37	Equuleus Pictoris,	The Painter's Easel.
38	Pyxis Nautica,	Mariner's Compass.
39	Antlia Pneumatica,	Air Pump.
40	Octans,	Octant or Hadley's Quadrant.
41	Circinus,	A Pair of Compasses.
42	Norma,	The Square and Rule.
43	Telescopium,	Telescope
44	Microscopium,	Microscope,
45	Mons Mensæ,	Table Mountain.
46	Solitaire,	An Indian Bird.
47	Psalterium Georgianum,	The Georgian Psaltery.
48	Tubus Herschelii Minor,	Herschel's less Telescope.

Three more southern constellations have been lately added, viz. *Montgolfier's Balloon*, which is between Sagittarius, Capricon, the Southern Fish, and the Microscope; the *Press of Guttenberg*, between the great dog and the ship; and the *Cat*, between Hydra, the ship, compass and air-pump. The two first of these constellations were formed by astronomers at Gotha, in Upper Saxony, and the last by the late M. Jerome de la Lande.

The number of Stars of the first magnitude, in the zodiac, and in both hemispheres, do not amount to twenty.

By comparing the lengths of a sidereal and solar tropical years with each other, the former is found to exceed the latter by $20' 22''$. Hence, the intersection of the ecliptic and equator moves, in a retrogade direction, about $50''.2$ of a degree yearly; and this motion is called the *Precession* of the *Equinoxes;* therefore, a revolution of the equinoctial points will be completed in about 25816 years—this is called the Annus Magnus, or Great Year. It has also been called the *Platonic Year*, because Plato believed that after this period, all things would revert to the same state as before.

Since the equator changes its position with respect to the ecliptic, its axis will also be changeable; and its poles, during the annus magnus, will complete a period in a circular, or rather spiral orbit—therefore, on this account, the longitude, right ascension, and declination of every

every Star will be variable; and consequently, the pole of the equator cannot always be directed to the same star; or, which is the same, no Star can remain at or near the pole of the equinoctial, for any considerable space of time. The Star which at present is nearest the north pole is Alruccabah, a Star of the second magnitude, and the last in the tail of the constellation of Ursa Minor; the latitude of the Star is 66° 4′ 43″, and mean longitude, at the beginning of 1800, is 85° 46′ 10″. Now the nearest approach of this Star to the pole will be, when it is in the first of Cancer—which accordingly happens in the year 2103, at which time its distance from the pole will be about 29′ 55″. And since the fixed Stars complete a revolution about the axis of the ecliptic in 25816 years, therefore, any given fixed Star will perform half a revolution in half that time; and consequently, if that Star was before at its nearest appulse to the pole, it will now be the most remote from it possible; therefore, 12908 years after the year 2103, that is, in the year 15011, the present pole star will be 45° 33′ 5″ distant from the pole. About 2300 years before the common account of the commencement of the Christian æra, the star α Draconis, was the pole Star, being then within ten minutes of the pole.

By this motion of the fixed Stars it follows, that the zodiacal constellations have not retained their places with respect to the vernal equinox, but seem to have moved in consequentia, or towards the east, almost a whole sign, since the time of Hipparchus; so that the constellation Aries is now where Taurus then was, Taurus where Gemini. and so on. However, to avoid confusion, the spaces formerly occupied by the zodiacal constellations still retain their ancient names, and are called Anastra, or without their former Stars, whereas the spaces they now possess are called Stellata.

Besides the motion arising from the recession of the equinoctial points, the Stars have two apparent periodical motions, called the Aberration and Nutation; this latter is also sometimes called the Deviation of the fixed Stars. The first of these motions arises from the velocity of light, combined with that of the earth in its orbit; and hence, each of the Stars apparently describes an ellipse about its mean place, the longest diameter being 40″, and period a year. The second of these motions, or the Nutation, arises from the attraction of the moon upon the equatorial parts of the earth; by which the pole of the equator describes an ellipse about its mean place as a centre—the ratio of the axes of the ellipse being as 19″.1 to 14″.2: a period is completed in 18 years.—See note, p. 11.

Many of the principal fixed Stars are observed to change their position with respect to the adjacent Stars, and this change of place is called the Proper Motion of the Stars. The proper motion of *Sirius*, *Castor*, *Procyon*, *Pollux*, *Regulus*, *Arcturus*, and *Altair*, in right ascension, in 100 years, according to Dr. Maskelyne, are respectively -1′, 3″,-28″,-1′ 28″,-1′ 33″,-41″,-2′ 20″, and + 57″: the proper motion of Sirius in declination, in a century, is 2′, and that of Arcturus 3′ 21″, both

both tending to the South. All the Stars, except one, in his *Catalogue of the mean right ascensions of 36 principal fixed Stars, January 1, 1790* *, appear to have a proper motion in right ascension; but in the catalogue of the same stars, in the Greenwich observations of 1802, *Antares*, *α Herculis*, and 1 *α Capricorni*, have no proper motion in right ascension; and *Pollux* has none in declination.

As a necessary result of the proper motions of the Stars, Dr. Herschel infers the motion of the whole solar system in absolute space; which, in a great measure, seems to be confirmed from his comparison of the apparent relative motions of many of the Stars.

Several of the Stars have been known to disappear, as the Stars β and γ in the constellation Argo; In Dr. Maskelyne's Observations, vol. III. page 184, it is mentioned that the 55th of Hercules has disappeared, and in the Pleiades there were formerly seven Stars, but now six can only be observed. Again others, called New Stars, have appeared, and then entirely vanished. One of the most remarkable of the new Stars is that which appeared in the year 1572, about the beginning of November, which, with the Stats α, β, and γ, of Cassiopeia, formed a perfect rhombus. Its apparent magnitude exceeded that of Sirius or Lyra, and sometimes it could be seen in the day time. It continued in the same place during the time it was visible, which was 16 months. Its colour at first was a splendid white; it afterwards assumed a yellow colour, then red, like Mars or Aldebaran, and then became gradually more and more dull, until it entirely disappeared. The new star which appeared in Serpentarius, 10th October, 1604, was nearly as bright as the former; it entirely vanished, 8th October, 1605. Dr. Herschel has given a catalogue of stars which were formerly seen, but are now no longer visible; also, a catalogue of new stars †.

Periodical Stars are such as appear and disappear at regular intervals of time. The most remarkable of these stars is that in the neck of the constellation Cetus—according to Cassini, its period is 330 days; it is visible about 3 months, and invisible during the remaining part of its period. The period of χ Cygni is about 405d. 8h. There are two more periodical stars in the same constellation.

Variable or Changeable Stars are those whose apparent magnitudes are variable. Upwards of a hundred years ago, the star Algol was observed to be be changeable; and, according to Mr. Goodricke's observations, a period of its variations is completed in 2d. 20h. 49′ 3″, and M. de la Lande makes this period only 1″ less. Its greatest brightness is of the second magnitude, and least of the fourth. It changes from its greatest splendour to its least in about three hours and a half, and regains its highest magnitude in the same interval, which it retains during the remainder part of its period. The star β Lyræ completes a period of its variations in 12d. 19h. according to Mr. Goodricke; he also observed, that δ Cephei changed from the third to the fifth magnitude

* Dr. Maskelyne gave the author a copy of this catalogue in 1790.
† Philosophical Transactions for 1783.

nitude in 5d. 8h. 37½'. Mr. Pigott found the periodic time of η Antinoi to be 7d. 4h. 38'; When this star is at its greatest brightness, which it retains about 44h., it is of the third magnitude; and when at its least, the duration of which is about 30h., it is between the fourth and fifth magnitudes; its light decreases during about 62h., and the period of its increase is about 36. According to Dr. Herschel, the period of the changes of α Hercules is performed in 60 days.

Mr. Pigott suggests, that periodic stars may afford a ready means for determining accurately the differences of terrestrial longitudes. "This," says he, "would be a most satisfactory, useful, and profitable discovery, and may be the lot of those who have but a slight knowledge of astronomy, provided that with great exactness, and a good memory, a constant look out be given *."

The variation in the light of a fixed star has been accounted for, by supposing the variable star to have a large planet revolving about it, the earth being in the plane of the planet's orbit. Again, M. de Maupertuis conjectures, that in consequence of a very quick rotation of a variable star, the centrifugal force may make it assume the figure of a flat oblate spheroid, and that its inclination may be altered by the attractions of planets revolving about it; hence, when its plane is directed towards the earth, it will scarcely be visible, and it will be brightest when its flat side is opposed to the earth—in intermediate positions it will appear of intermediate magnitudes †. Lastly, it has been conjectured, that a part of the surface of a variable star is darker than the rest; and, therefore, in the time of the rotation of the star, a period of its phases will be completed. The following extracts, from papers by Dr Herschel, will serve to confirm the opinion of the rotatory motion of the stars:

"The rotatory motion of stars upon their axes is a capital feature in their resemblance to the sun. It appears to me now, that we cannot refuse to admit such a motion, and that indeed it may be as evidently proved as the diurnal motion of the earth. Dark spots, or large portions of the surface, less luminous than the rest, turned alternately in certain directions, either towards or from us, will account for all the phenomena of periodical changes in the lustre of the stars, so satisfactorily, that we certainly need not look out for any other cause ‡."

"That stars are suns can hardly admit of a doubt. Their immense distance would perfectly exclude them from our view, if the light they send us were not of the solar kind. Besides, the analogy may be traced much farther. The sun turns on its axis. So does the star Algol.—So do the stars called β Lyræ, δ Cephei, η Antinoi, ο Ceti, and many more, most probably all. From what other cause can we so probably account for their periodical changes? Again, our sun has spots on its surfaces; so has the star Algol; and so have the stars already named;

* Philosophical Transactions for 1797, page 133.

† Discours sur les differentes figures des astres, Paris, 1732.

‡ Phil. Transactions for 1796, page 455.

and

and probably every star in the heavens. On our sun these spots are changeable. So they are on the star *o* Ceti; as evidently appears from the irregularity of its changeable lustre, which is often broken in upon by accidental changes, while the general period continues unaltered. The same little deviations have been observed in other periodical stars, and ought to be ascribed to the same cause. But if stars are suns, and suns are inhabitable, we see at once what an extensive field for animation opens itself to our view *."

"When the biography of the stars, if I may be allowed the expression, is arrived to such perfection as to present us with a complete relation of all the incidents that have happened to the most eminent of them, we may then possibly not only be still more assured of their rotatory motion, but also perceive that they have other movements, such as nutations or changes in the inclination of their axes; which, added to bodies much flattened by quick rotatory motions, or surrounded by rings like Saturn, will easily account for many new phenomena that may then offer themselves to our extended views †."

Many of the fixed stars, which appear single to the naked eye, when viewed with a telescope, are found to consist of two, three, &c. stars, very near each other. These are called DOUBLE, TREBLE, &c. Stars.

Some of the principal of the double stars are *Castor*, *γ Virginis*, *Alphard*, *α Centauri*,‡ *α Herculis*, *Alruccabah*, *Vega*, *Regulis*, *ε Bootæ*, *γ Andromedæ*, &c.; *υ Lyræ* is a treble star, and *λ Orionis*, *β* and *ε Lyræ*, are quadruple stars. M. Mayer observed 72 double stars; and Dr. Herschel, from whose catalogue the greater part of the above are taken, has observed upwards of 700 double, treble, &c. stars, which he has arranged into six classes, according to their apparent distances.

From the observations of double stars made in opposite parts of the earth's orbit, Dr. Gregory and Mr. Emerson§ proposed to determine the annual parallax of the fixed stars, and hence their distance from the earth; and Dr. Herschel‖ has prosecuted this subject. The principles, however, upon which they proceeded are liable to objection.

A NEBULA is a portion of the heavens brighter than the circumjacent parts, which brightness arises from the combined light of a multitude of very distant stars, apparently near each other. The most considerable of the Nebulæ is the *Milky Way*, which is almost a great circle of the sphere, intersecting the ecliptic in nearly opposite points. From Cassiopeia, it proceeds towards the south, through the constellations Perseus, Auriga, Orion, Monoceros, Canis Major, Argo Navis, where the brightness is greatest, and Crux: it then returns to the north, through Circinus, Triangulum Australis, Ara,

* Phil. Transactions for 1795, page 68.

† Phil. Transactions for 1796, page 457.

‡ One of the stars of *α* Centauri is of the second magnitude, and the other of the fourth; their apparent distance is about 15″ or 16″.

§ Dr. Gregory's Astronomy, vol. 1. p. 500.—Emerson's Astronomy, p. 117, London, 1769.

‖ Phil. Transactions for 1782.—See also Galileo's Systema Cosmicum, 1632, p. 532.

and

and Norma, where it is divided into two branches, one of which passes through Scorpio, Serpentarius, Taurus, and Cygnus, to Cassiopeia; and the other through Telescopium, Sagittarius, Antinous, Aquila, Sagitta, Vulpecula, &c. to Cygnus, and continues to Cassiopeia. The next are those usually called the *Magellanic Clouds*; then *Præsepe* in Cancer, 9 *Orionis*, &c. M. de la Caille, at the Cape of Good Hope, observed 42 nebulæ, which he divided into three classes, each containing 14 nebulæ. In those of the first class he could perceive no stars. The nebulæ of the second consisted of distinct masses of stars; and those of the third contain stars of the sixth magnitude and under, with white spots similar to those in the first class. In the Connaissance des Tems for 1784, there is a catalogue of 103 nebulæ; and Dr. Herschel has given a catalogue of upwards of 1000, in the Philosophical Transactions for 1786.

In the southern hemisphere, to the eastward of the Crosiers, there is a part of the heavens of about 3° in diameter, of a deep black.

Nebulous Stars are those which are surrounded with a faint luminous atmosphere Dr. Herschel gives the following account of a nebulous star which he observed, November 13, 1790:—"A most singular phenomenon! A star of the eighth magnitude, with a faint luminous atmosphere, of a circular form, and of about 3′ diameter. The star is perfectly in the center, and the atmosphere is so diluted, faint, and equal throughout, that there can be no surmise of its consisting of stars; nor can there be a doubt of the evident connection between the atmosphere and the star. Another star, not much less in brightness, and in the same field of view with the above, was perfectly free from any such appearance."* In the Phil Transactions for 1802, Dr. Herschel has given a catalogue of 500 nebulous stars, and groups of stars.

The Zodiacal Light is a cone of light, having the sun for its base, and extending in the direction of the zodiac, and hence its name. It is seen, after the setting and before the rising of the sun, about the end of February and the beginning of March. This light is now understood to be the atmosphere of the sun; its breadth, at the horizon, is from 8° to 30°; and its length, along the zodiac, is sometimes 45°, and in favourable circumstances it will extend to 100°, where it terminates in a point. This light was observed by M. Cassini in 1683, from whom it had its name. It is, however, mentioned in a book published in England in 1661, entitled *Britannia Baconica*, by Mr. J. Childrey. A remarkable circumstance has been observed, namely, that the tail of a comet is not altered in passing through the zodiacal light.

The moon's orbit being contained within the zodiac, its motion is very rapid with respect to the sun, and those stars situated in that zone On this account, there is founded one of the best methods for finding the longitude at sea hitherto discovered. In order to ascertain the longitude of a ship, with as great accuracy as possible, by this method, the moon ought to be compared with those stars which are situated in or near its orbit; by which means as little as possible of the moon's

* Phil. Transactions for 1791.

proper motion will be lost. The stars used in the Nautical Almanac, for the purpose of determining the longitude at sea by the above method, are, *α Arietis*, *Aldebaran*, *Pollux*, *Regulus*, *Spica Virginis*, *Antares*, *α Aquilæ*, *Fomalhaut*, and *α Pegasi*. The distance between the moon and the sun, and one or more of these stars, according as they are in a proper position for observation, is given in the viii. ix. x. and xi. pages of the month, at the beginning of every third hour apparent time, by the meridian of Greenwich.

These stars may be easily known, as being the brightest of those lying near the Moon's path, or nearly in a line perpendicular to that joining the cusps of the Moon; the following directions for knowing them may, however, be of service.

The Pleiades, or, as they are more commonly called, the Seven Stars, although only six principal stars remain, are, it is presumed, universally known. Towards the S. E. and at the distance of nearly 14°, is the star *Aldebaran*, of the first magnitude, and of a reddish colour, which, together with a few small stars, form a triangular figure. Between the N. and E. of Aldebaran, and about the angular distance of 45°, is *Pollux*, in the constellation Gemini, and at a small distance to the N. is Castor. From Pollux a little to the S. of E. at the distance of about 37°, is *Regulus*, in the constellation Leo; and from thence, at the distance of about 54° towards the East, is *Spica Virginis*. From this star, and nearly in the same direction, at the distance of about 46°, is *Antares*, of the first magnitude. From Antares to *Altair*, or *α Aquilæ*, in a north easterly direction, the angular distance is nearly 61°. From Altair to *Fomalhaut*, in a south easterly direction, the distance is about $59\frac{1}{2}$° ; and from thence to *α Pegasi*, or Markab, the distance is about 45° in a northerly direction; and from α Pegasi to *α Arietis*, the distance is about $43\frac{3}{4}$° in a direction a little to the south of east: and Aldebaran is distant from α Arietis about $35\frac{1}{2}$°, nearly in the same direction, but inclining a little more to the south.

Some of the other principal fixed stars, which may be employed in finding the latitude, and the apparent time at the place of observation, may be known by their relative bearing and distance from those already described. The following few directions may, probably, be acceptable to some persons.

An imaginary line from the Pleiades through Aldebaran, at the distance of about 16° from that star, in a south-easterly direction, will pass through Bellatrix, of the second magnitude, in the constellation Orion; and towards the east, about $7\frac{1}{2}$° from Bellatrix, is Betelguese, of the first magnitude, in the same constellation. To the south of these stars, and nearly on a straight line, and at equal distances, are three stars, each of the second magnitude, called the Belt or Girdle of Orion: from the belt, towards the south is the Sword of Orion, in which is a remarkable nebula; a line from Betelguese, between the first and second stars in the belt of Orion, will pass through Rigel, of the first magnitude. From Betelguese, towards the east, at the distance of about 26°, is Procyon, between the first and second magnitudes, in the constellation Canis Minor. These two stars, and Sirius, of

of the first magnitude, in Canis Major, towards the south, form nearly an equilateral triangle.

From Aldebaran, in a direction a little to the east of north, and at the distance of about 31°, is *Capella*, of the first magnitude; these two stars and Castor form nearly an isosceles triangle, Capella being at the vertex. A line from Rigel through Capella produced will nearly pass through *Alruccabah*, or the pole star; the distance between the two former being about 54°, and that between Capella and Alruccabah 44°. The pole star is the last in the tail of the constellation Ursa Minor, which constellation contains seven principal stars, and is similar, but differently posited with respect to Ursa Major, or the Great Bear; the two westernmost stars of this constellation, when in the hemisphere south of the pole, are called the *Pointers;* as a line through them points out, or is nearly in a direction with the pole star.

Towards the south of Regulus, and inclining a little to the west, at the distance of about 23¼°, is *Alphard*, in the constellation Hydra. From Regulus to Deneb, the distance, in a direction to the north of east, is about 23¾°. From Spica Virginis to Arcturus, in a northerly direction, the distance is 33°; and nearly in a line between them is *Vindemiatrix*, in Virgo; to the north of this star, at the angular distance of about 27½ degrees, is *Cor Caroli* In a north easterly direction from Arcturus, at the distance of 19½° degrees, is *Alphacca*, in Corona Borealis; and from thence, nearly in the same direction, at the distance of about 39¾°, is *Vega*, or *α Lyra*, of the first magnitude. At the distance of 47° from Spica Virginis, towards the south, is the northernmost of four stars, forming a cross, and therefore, called the *Crosiers*.

Nearly 14° to the north-east of Altair, is the constellation Delphinus, in which are four principal stars, in form of a rhomboid; and this line being produced from Delphinus, in the same direction will pass through *Scheat*, a star of the second magnitude, in the constellation Pegasus. About 13° to the south of Scheat, is *Markab*, a star of the second magnitude, in the same constellation; nearly 16½° to the eastward of Markab, is *Algenib* or γ Pegasi, of the second magnitude; and about 14° to the eastward of Scheat, is α Andromedæ, or Alpheratz, a star of the third magnitude, in the head of Andromeda. These four stars form a figure which is usually called the *Square of Pegasus*.

From α Andromedæ, in a north easterly direction, at the distance of nearly 14⅗°, is *Mirach*; and 23¾° therefrom, in the same direction, is the variable star *Algol*. In a perpendicular direction from the middle of the line joining Mirach and Algol, towards the north, and at the distance of about one-eighth of that line, is *Almaach*. About 21½°, towards the north of Mirach is *Schedir* in Cassiopeia; this constellation contains five stars of the third magnitude, and is easily known. Between the south and west of the Pleiades, at the distance of 23°, or from Aldebaran 26°, is *Menkar*, of the second magnitude. Betelguese, Rigel, and *Achernar* are nearly in the same direction, the distance between the two last being 4½ times of that between the two first.

A celestial globe, or maps of the stars, or plans of the constellations, would be of great service to the navigator, who wishes more information in this particular department.

CHAP. IV.

Of the Planets.

A Planetary System is an assemblage of several bodies revolving about another body as a common center, which gives light and heat to all, and is, therefore, called a Sun; and the revolving bodies are called *Planets* and *Comets*. The path which a Planet or Comet describes in its motion round the sun, is called its *Orbit*.

Planets are of two kinds, *Primary* and *Secondary*. A primary planet regards the sun as its center of revolution; a secondary is a body revolving about a primary planet, and is called a *satellite*.

There are now known to be eleven primary planets, of which the names and characters are as follow:

Name.	*Character.*	*Name.*	*Character.*
Mercury,	☿	Saturn,	♄
Venus,	♀	Georgian,	♅
Earth,	⊖	Ceres,	⚳
Mars,	♂	Pallas,	⚴
Jupiter,	♃	Juno and Vesta.	

The planet Georgian was discovered by Dr. Herschel, March 13, 1781, who named it after his sovereign; but the French have given it the name of its discoverer: it has also been called Uranus, Urania, Minerva, &c. M. de la Lande is of opinion, that the 34th Tauri,* observed by Flamstead, December 13, 1690; and No. 964, in Mayer's Catalogue,† and observed by him, Sept. 25, 1756, were the Georgian planet.‡ Dr. Herschel is also of opinion, that this planet was observed by Flamstead, and mistaken for a small star.§

The planet Ceres was discovered by M. Piazzi, Astronomer Royal at Palermo in Sicily, upon the 1st of January 1801; Pallas, by Dr. Olbers, at Bremen, on the 28th of March 1802; the planet Juno was discovered by M. Harding, at Lilienthal near Bremen, upon the 1st of September 1804; and Vesta by Dr. Olbers, at Bremen, on the 29th of March 1807. The orbits of these four planets are contained between those of Mars and Jupiter; by comparing the great interval between the orbits of these two planets, with that between the orbits of any two adjacent planets formerly known, it was surmised by Mr. James Bernoulli, in his *Systema Cometarum*, Anno 1682, that there is a primary planet revolving about the sun, between Mars and Jupiter, whose period is about 4 years and 157 days, and its mean distance 2.583. This planet, from its smallness, and great distance, he supposes not to be visible to an observer on the earth; and, also, that it has several satellites belonging to it. Professor Maclaurin, upwards of seventy years ago, and lately C Loft, Esq. supposed that there must be, at least, one planet whose orbit is contained between those of

* Connaissance des Tems pour 1791, page 369.
† Astronomie, vol. ii. p. 143.
‡ Tables Astronomiques, p. 188.
§ Philosophical Transactions for 1797, p. 205.

Mars

Mars and Jupiter; and the late discoveries have now verified the conjectures of these gentlemen.

The number of satellites hitherto discovered belonging to these planets amount to eighteen; one of which belongs to the Earth, called the Moon; four belong to Jupiter, seven to Saturn, and six to the Georgian planet. Messrs. Cassini, Short, and Montaigne, observed a phenomenon, which induced them to believe that there was a satellite belonging to Venus.

Since the Sun is the lowest body in our system, and because the orbits of Mercury and Venus are contained within that of the Earth, they are, therefore, called inferior planets; the other planets, being always more distant than the earth from the Sun, are called superior planets. It may be observed, that all the planets hitherto discovered are superior with respect to Mercury, and inferior to the Georgian planet.

In the time of a planet's revolution round the Sun, a period of its seasons is completed, and this interval is called a Solar Year.

The periods, or sidereal revolutions of the several planets, together with their distances from the sun, expressed in English miles, are as follows:

Planets.	*Sidereal Revolution.*					*Mean Distance.**
Mercury, -	87d.	23h.	15′	43″	·6	- 35853400
Venus, -	224	16	49	10	.6	- 66995700
Earth, -	365	6	9	11	.6	- 92621000
Mars, -	686	23	30	35	.6	- 141125100
Jupiter, -	4332	14	27	10	.8	- 481887700
Saturn, -	10759	1	51	11	.2	- 883671000
Georgian,	83 years,	150d.	18h.			- 1767375000
Ceres, -	1681d.	12h.	14′	24″		- 256317000

The planet Pallas completes its revolution in 1680 days; mean distance from the Sun 2.76544, that of the Earth being 1.—Juno, in about 1588 days; its mean distance being 2.66072. M. Harding has denoted the character of this planet by a sceptre and star. The elements of the orbit of Vesta are as yet imperfectly known.

The planets Venus, the Earth, Mars, Jupiter, and Saturn, besides their annual motion round the Sun, have been observed to move round their axes from west to east, which motion is called their *rotation*. The time of rotation is measured by the return of any meridian to the same fixed star, which is called a *Sidereal Day*; and the return of the same meridian to the Sun is called a *Solar Day*. A terrestrial solar day is divided into 24 equal parts called *hours*; each hour is subdivided into 60 equal parts called *minutes*, and the sexagesimal division is continued. The times of rotation of the above planets, in mean solar time, are as follow:

Rotation of Mercury, -	25h	4′	0″	Jupiter, -	9h	55′	37″
Venus, -	23	21	19	Saturn, -	10	16	0
Earth, -	23	56	4	Ring of do.	10	32	15
Mars, -	24	39	22				

* These were computed on the assumption of the Sun's mean horizontal parallax being 8″ 8128, according to the determination of M. du Sejour, from the late transit of Venus. Vide Traité Analytique, &c. tom. i. p. 486.

The

The rotation of the Sun, according to *M. Cassini*, is performed in 25d. 14h. 8′; *M du Sejour* makes the time of rotation to be 25d. 13h. 44′; *M. de la Lande*, 25d. 10h. 0′. According to the observations of the author, in 1787, the time of the rotation of the sun is 25d. 7h. 52′.; and in Rowning's Philosophy, and many other works, it is stated at 25d. 6h. 0′.*

The time of rotation of a planet is discovered by observations on the spots on its disc; and though the nearness of Mercury to the Sun, and the great distance of the Georgian planet from the earth, have prevented the discovery of spots on their discs, and thereby the times of their rotation; yet from analogy it may be presumed, that each of these planets has a rotatory motion.

The rotation of the Moon is performed in 27d. 12h. 43′; and it was surmised, in the first edition of this work, that each of the satellites belonging to the other planets revolves about its axis: this is now confirmed by the late observation of Dr. Herschel; for, according to him,

The rotation of the First Satellite of Jupiter is performed in	1d.	18h	26′	.6
Second, . .	3	18	17	.9
Third, . .	7	3	59	.6
Fourth, . .	16	18	5	.1†

The fifth satellite of Saturn revolves on its axis in 79 days, and presents the same side to its primary, as the Moon does to the Earth; and that each of the satellites of the other planets has a rotatory motion, is scarcely to be doubted.

By means of the rotation of a planet, combined with its annual motion round the Sun, in the time that a period of the planet's seasons are completed, every point on its surface will have enjoyed the presence of the Sun for nearly an equal interval of time.

The figure of the Earth, Mars, Jupiter, and Saturn, is found by observation to be that of an oblate spheroid, the ratio of the equatoreal diameter to the axis of these planets being as 230 : 229, 1355 : 1272, 14 : 13,‡ and 11 : 10, respectively; and by analogy, the other planets are also of a spheroidal figure, the least diameter being that of rotation.

Saturn is surrounded with a thin circular ring, which appears to be double.§ If the semidiameter of Saturn be supposed to be divided into nine equal parts, then the radius of the interior edge of the ring is 15, and that of the exterior 21 of these parts; hence, the interval between the planet and the ring is equal to the breadth of the ring.

* Vol. ii. part. iv. p. 14.—See also Guthrie's Grammar, p. 3.—Imison's School of Arts, p. 297.—Adams' Geography, p. 4. &c.

† Philosophical Transactions for 1797, pages 343, 349.

‡ The proportion of the axes of Jupiter, agreeable to M. de la Place's calculations, is nearly as 69 : 74.

§ From Dr. Herschel's observations, the interval between the rings is 2839 miles.

The

The inclination of the plane of the ring to the ecliptic is 31° 20′; and the place of its ascending node, at the beginning of 1802, was 3s 21° 57′ 43″. The ring continues parallel to itself during a revolution of Saturn about the Sun. In that period, therefore, the plane of the ring is twice directed to the Sun, and that side of it is enlightened which is opposed to the Sun. The ring, being very thin, becomes invisible when its plane is directed either to the Sun, or to the Earth, or when its dark side is opposed to the Earth; that is, when the Sun and Earth are on opposite sides of the plane of the ring.* The rotation of the ring, as settled by Dr. Herschel, from observations made in the year 1789, on the apparent motions of five luminous and protuberant points on the ansæ of the ring, is 10h. 32′ 15″.4. The ring is so extremely thin, that, according to Dr. Herschel, it seemed not to be the fourth, at least not the third, part of the diameter of the third satellite, which was less than one second; and that the seventh satellite, when in the plane of the ring, appeared in the shape of a bead upon a thread, projecting on both sides of the same arm. The edge of the ring is either spherical or spheroidal. The light of the ring is generally brighter than that of the planet; and *M. du Sejour* is of opinion, that the southern side of the ring reflects more light than the northern. The ring is not wholly in the same plane. M. de la Place has deduced from theory, that the density of the ring is greater than that of Saturn; and Dr. Herschel observes, that since the ring casts a deep shadow upon the planet, is very sharply defined both in its outer and inner edges, and in brightness exceeds the planet itself, it seems to be almost proved, that its consistence cannot be less than that of the body of Saturn. The ring, as seen from Saturn, appears like a large luminous arch in the heavens, as if it did not belong to the planet.

The orbits of the planets are *ellipses*, in one of whose foci is the Sun. That point of the orbit of a planet which is farthest from the Sun, is called *Aphelion* or *Aphelium*; and the point nearest the Sun, the *Perihelion* or *Perihelium*. That point of the orbit of a planet which is most distant from the Earth is called the *Apogee* or *Apogæum*, and the point nearest the Earth is called the *Perigee* or *Perigæum*.

Since the orbits of the planets Mercury and Venus are contained within that of the Earth, they can never be seen in opposition to the Sun; but because the orbit of the Earth is contained within the orbits of the other planets, these planets may, therefore, be seen in opposition, or in any other position with respect to the Sun. The phenomena of an inferior planet, as observed from a superior planet, arising from their combined motions round the Sun, may be thus illustrated.

Let S (fig. 3.) represent the Sun, ABCDEFGH the orbit of an inferior planet, IKLMN the orbit of the earth, both being supposed in

* December 2nd, 1861, the earth will be so nearly in the plane of the ring, that it will disappear, and will continue invisible until the 24th of January 1862, when the earth will repass its plane. The ring will disappear a second time on the 2nd of June 1862, and reappear on the 17th of August, and will continue to be visible until the 4th of March 1878; it will, however, reappear on the 8th of that month.

the

the same plane, and the circle ♈♋♎♑ the ecliptic; now let the Earth in any point I of its orbit, when the inferior planet is at A, be in a right line between the Sun and the Earth, it is then said to be at its inferior conjunction; when at the point E, it is at its superior conjunction; and when at the points C or G, where a straight line from the Earth is a tangent to its orbit, it is then at its greatest eastern or western elongation—the elongation, at any other time, is measured by the angle contained between lines drawn from the center of the Earth to the centers of the Sun and planet. Now, if the planet be supposed to move forward according to the order of the signs, with the excess of its velocity above the Earth's motion, the Earth may be supposed to remain stationary at the point I; let the planet be at any point C of its orbit, hence its place referred to in the heavens is N, and when the planet is come to D, its apparent place is O; it has, therefore, apparently moved through the space NO: in like manner the planet having moved through the portion DEFG of its orbit, will have apparently described the space O♋PQ, and when the planet has moved from G to H, it will appear to have returned to P; and during the time it describes the portion HABC of its orbit, it will apparently move through the space P♋ON, contrary to the order of the signs; hence, during the time the planet is describing the western part of its orbit ABCDE, it sets sooner and rises earlier than the Sun, and may, therefore, be called a *Morning Star*; and during the time it is describing the eastern part of its orbit EFGHA, it rises and sets later than the sun—it will, therefore, be seen after sun-set, and of course called an *Evening Star*: during the time the planet is describing the superior part of its orbit, it moves according to the order of the signs, and is, therefore, said to be *direct*; but its motion, while describing the inferior part of its orbit, is apparently contrary to the order of the signs, as seen from the Earth, and it is hence said to be *retrograde*; when the planet is towards the points of greatest elongation, it is then either receding directly from, or approaching towards the Earth, and its apparent motion for some time is scarce perceptible; it is then said to be *stationary*.

While an inferior planet, as seen from the Earth, is moving according to the order of the signs, it is evident that the motion of the Earth will be direct as seen from that planet; when the planet is apparently retrograde, the Earth will be also retrograde; and when the planet is apparently stationary, the Earth will be also stationary, as seen from the planet. Hence, the phenomena of a superior planet is sufficiently obvious, and requires no further illustration.

Since, by observation, the orbit of each planet is found to be inclined to the ecliptic, it, therefore, intersects the ecliptic in two opposite points, called *Nodes*; that half of the planet's orbit, which is between the ecliptic and north pole, is called the northern part of its orbit; and the other half, the southern. That node which the planet enters, when advancing towards the north, is called the *Ascending Node*; and the other, the *Descending Node*; and an imaginary straight line joining these, is called the *Line of the Nodes*.

Let.

Let ABCD (fig. 4.) represent the orbit of the earth, and EFGH the orbit of an inferior planet, S the Sun, and EG the line of the nodes, E being the ascending, and G the descending node; also let A be the place of the earth in the ecliptic, and F the situation of the planet in its proper orbit; then, will M be the *Heliocentric* place of the planet, or its place as seen from the Sun; and N its *Geocentric* place, or point where it would be seen from the Earth; if FI be drawn perpendicular to the ecliptic, the angle FSI is the *Heliocentric Latitude*, and FAI the *Geocentric Latitude* of the planet; SI is called the *Curtate Distance*, and the difference between SF and SI, the *Curtation*.

The heliocentric and geocentric longitudes and latitudes of the planets, together with their declinations, and passages over the meridian in apparent time, are given in the Nautical Almanac, page iv. of the month. The declination and transit of a planet are useful in computing the latitude, and apparent time at sea, from its observed altitude.

The Periodic Time of a planet may be found, by observing the interval of time between two successive passages of the planet through the same node, allowance being made for the motion of the node during that interval; and to obtain a greater degree of accuracy, it will be proper to select two very distant observations, when the planet was in the same node; and the interval, after allowing for the motion of the node, being divided by the number of elapsed periods, will give the periodic time: and because the place of a planet is usually reckoned from the vernal equinox, therefore, the periodic time of revolution, with respect to the fixed stars, must be reduced to the periodic revolution, relatively to the equinoxes, by deducting a part proportional to the recession of the equinoxes in that interval. The time when an inferior planet is in its node is most accurately determined, by observing its transit over the Sun; and that of a superior planet, when in opposition, and at the same time in or near the node.

A Synodic Period of a planet, is the time elapsed between two successive returns of the planet to the same point of its orbit, as seen from the Earth.

The periodic time of an inferior planet may also be found, by dividing the product arising from the multiplication of the synodic period of the planet into the Earth's period, by the sum of these periods; and that of a superior planet is found by dividing the product of its synodic period into the Earth's, by the difference between these periods.

The Comparative Distance of an inferior planet may be deduced from the quantity of its greatest elongation; for, if the distance of the Earth from the Sun be supposed equal to unity, the distance of the planet from the Sun will be expressed by the sine of its greatest elongation, and its distance from the Earth at that time, by the cosine of elongation :—The distance of a superior plant from the Sun will be equal to the cosecant of the greatest elongation of the Earth, as seen from that planet, which is called the Parallactic angle; and the distance of the planet from the Earth is equal to the cotangent of that

 angle,

angle, the radius being unity. However, these being on the assumption of a circular orbit, will deviate a little from the truth.

Kepler, by comparing the periodic times and distances of the planets from the Sun with each other, found, that the squares of the periodic times are directly as the cubes of the mean distances; and since the periods of the planets may be very accurately found by observation, therefore, by the above law, the relative mean distance of the planets may be deduced with the greatest precision; and because the mean distance of the Earth from the Sun, in known measures, is pretty well ascertained by the late transit of Venus, therefore, the absolute mean distances of all the planets are also determined.

Since the planets receive their light from the Sun, it is evident, that whatever side is opposed thereto will be enlightened; therefore, at the inferior conjunction of Mercury or Venus, neither can be seen, because the unenlightened side is towards the Earth, unless at the time of a transit, and then it will appear like small black round spot upon the Sun's disc; but at the superior conjunction, the enlightened hemisphere of the planet is towards the Earth, consequently the whole of its disc will be visible; and when the planet is at its greatest elongation, one half of it only can be visible, because the plane of the terminator, or circle bounding light and darkness, is directed towards the Earth.—Hence, when either Mercury or Venus is in the inferior part of its orbit, less than its half appears enlightened, but more, when in the superior part of its orbit. The distance of Jupiter, Saturn, and the Georgian planet, being very considerable, in respect of the Earth's distance from the Sun, the enlightened discs of these planets are, to sense, always opposed to the Earth; but because of the proximity of Mars, it is observed to be full and gibbous alternately.

The magnitude of Mercury is about a 15th part of the Earth; Venus a 9th nearly; Mars a 7th part; Ceres 1-400th part of the magnitude of the Earth; and the other late discovered planets probably still less. The magnitude of Jupiter is 1281 times, Saturn 995 times, and the Georgian planet 80½ times, that of the Earth: and the Sun is 1384462 times larger than the Earth.

The principal planets are easily known from the fixed stars, by the steadiness of their light; and, if viewed with a good telescope, they appear to be of a considerable magnitude. Their apparent motion among the fixed stars serves also to distinguish them.

The first of the planets that was discovered to belong to the solar system is probably Venus. This was about 710 years before the Christian æra. The greatest elongation of this planet is about 48°, when it is in the inferior part of its orbit, and when the elongation is about 40°, it is apparently of the greatest brightness. In this position, which happens about 36 days before and after its inferior conjunction, it transits the meridian 2h. 31′ before and after the Sun. Venus has often been seen in the day time without the assistance of a telescope, and is bright enough to cast a shadow of night. The period of this phenomenon is eight years; the apparent diameter of Venus, is 39″, and the enlightened part 10¼″, or little

more

more than one-fourth. When Venus is in the western part of its orbit, it rises before the Sun, and is called *Phosphorus* or the *Morning Star*, or *Helal-ben-shahar*, that is, *Helal son of the morning*. When this planet is in the eastern part of its orbit, it sets after the Sun, and is called *Hesperus* or the *Evening Star*.

It may be remarked, that the times of revolution and rotation of the planets, and their mean distances from the Sun, remain constant; and their secular inequalities are periodical. The stability of the solar system is a wonderful phenomenon, and the most worthy of the contemplation of the philosopher.

CHAP. V.

Of the Moon.

The Moon, though only a satellite belonging to the earth, is, next to the Sun, the most conspicuous body in the heavens, both on account of its apparent magnitude, and diversity of aspect.

By contemplating the Moon during the course of a month, it is observed to put on a variety of appearances. When the Moon is first observed a little to the eastward of the Sun, a small portion of its disc is enlightened, and that portion is next the Sun; the enlightened part increases more and more until it appears perfectly round—the western part then becomes invisible, and the enlightened part gradually diminishes as it approaches the Sun, and entirely disappears when in conjunction with that object, except at that time, it happens to pass directly between the observer and the Sun. These various appearance are in general called the *Phases* of the Moon.

By comparing the Moon's phases and its elongations from the Sun with each other, it is found, that the same phase corresponds to the same elongation. Hence it evidently appears, that the Moon is an opake body, receiving its light from the Sun, and, therefore, that part only is enlightened which is opposed to the Sun.

Since the Sun is much larger than the Moon, it is evident that more than a hemisphere is continually enlightened; it is also manifest, that a complete hemisphere of the Moon cannot be seen by a spectator on the earth; but the difference in both cases is so very trifling, that it may be neglected, in the following illustration of the phases of that satellite.

Let S (fig. 5.) represent the sun, E the earth, and ACFH the orbit of the moon, in which it revolves from west to east, according to the order of the letters. When the moon is at A, it is in *conjunction* with the sun; and as its dark side is opposed to the earth, no part, therefore.

therefore of the moon can be seen, unless at that time it happens to pass over the sun's disc;—this phase is called *new moon*, and it is said to be in *syzigy*. When the moon is at B, or 45° from the sun, it is said to be in its *first octant;* in this case, part of the moon's enlightened disc is towards the earth, and it appears in the form of a crescent, as represented at L; the curve bounding the enlightened and dark parts of the moon, or terminator, being an ellipse. When the moon is at the point C, where the angle of elongation is 90°, it is said to be in *quadrature*, and the terminator is now apparently a straight line; one half, therefore, of the moon's enlightened disc is seen from the earth, as represented at M: when at D, the moon is in its *second octant*, and as much is now visible as was invisible when it was at B; and since more than half its disc is visible, as seen from the earth, the moon is, therefore, said to be *gibbous:* and when the moon is at F, it is in *opposition*, and appears to be completely illuminated; and is now, as well as when in conjunction, said to be in *syzigy*. The moon having passed the opposition, and come to G the third octant, is again gibbous, as represented by the phase O: when at H, the moon is again in *quadrature*, and appears half full, as at P; and having come to I, the *fourth octant*, it appears like a crescent, as at Q; and lastly, when to A, it is in conjunction, and the dark side being opposite to the earth, the moon, therefore, becomes invisible.

During the time the moon is moving from A to C, it is said to be in the *first quarter*, from C to F in the *second*, from F to H in the *third*, and from H to A in the *fourth quarter;* when the moon is at A, it comes to the meridian at noon, and when at F, at midnight; therefore, while the moon is in the part ACF of its orbit, it comes to the meridian between noon and midnight, and the western part of the moon's disc is enlightened. During the time the moon is describing the other part of its orbit FHA, it comes to the meridian between midnight and noon, and the eastern part of its disc is visible; when the moon is in the first and fourth quarters, it appears in form of a crescent, and when in the second and third quarters, it is gibbous.

Some days before and after new moon, besides the crescent, the remaining part of its disc is faintly seen; the reason of this will evidently appear, by considering, that when the moon is in those parts of its orbit, the enlightened side of the earth is opposed to the moon; and since the quantity of light reflected by each body is supposed proportional to the squares of their diameters, therefore, as the earth is much greater than the moon, the quantity of light reflected by the earth to the moon is so considerable, as to render the dark part visible.

The interval of time between two successive new or full moons is called a *synodic* period, or usually a *lunation*, and is completed in 29d. 12h. 44′ 2″.8; a revolution of the moon, with respect to the fixed stars, is completed in 27d. 7h. 43′ 11″.5; and with respect to the vernal equinox, in 27d. 7h. 43′ 4″.6; the anomalistic revolution of the moon is performed in 27d. 13h. 18′ 33″.9; and with respect to the node, in 27d. 5h. 5′ 35″.6; and since the same side of the moon is always turned towards the earth, it, therefore, turns round its axis, in

in the same time that it completes a revolution, with respect to the fixed stars.

From the foregoing illustration of the moon's phases it will appear, that if its orbit coincided with the ecliptic, it would pass directly between the Earth and the Sun at every new moon, and being an opaque body, would intercept part of the sun's disc, and hence cause an eclipse of the sun; and at the time of opposition, the moon would pass through the shadow of the earth, and, therefore, the direct rays of the sun being intercepted by the earth, an eclipse of the moon would take place.

But because the moon's orbit makes an angle with the ecliptic, of about 5° 8′ 49″, these circles, therefore, intersect each other in two opposite points called Nodes, and the common intersection of the plane of these circles is called the Line of the Nodes; and if this line were always coincident with the syzigial line, the moon would be eclipsed at every opposition, and the sun at every new moon; but the mean motion of the nodes is about 19° 19′ 43″ yearly, in a retrograde direction.

It hence obviously follows, that an eclipse cannot happen, but when the moon is within certain limits of the node, which distance, in a lunar eclipse, is determinable by the inclination of the moon's orbit to the ecliptic, and the sum of the semidiameters of the moon, and that section of the earth's shadow through which the moon passes; and if at the time of opposition, the moon's latitude is less than the difference between its semidiameter and that of the shadow, the moon will be totally eclipsed with continuance; and the less the latitude of the moon, the longer is the duration of the eclipse. If the moon's latitude be equal to the above difference, the eclipse will be total without continuance; if greater than the above difference, but less than their sum, the eclipse will be partial; and if it is equal to, or exceeds that sum, no eclipse can happen at that time. The limit within which a solar eclipse can happen, is inferred from the inclination of the moon's orbit, and the sum of the moon's horizontal parallax, and the semidiameters of the sun and moon; and, therefore, if the moon's latitude exceds this last sum, no part of the earth can be eclipsed; but if less, the eclipse may be total or annular at one place, partial at a second, and at a third no part of the sun may be obscured. A *total* eclipse of the sun is when, at the time of the greatest obscuration, the Sun is completely hid by the Moon; an *annular*, when the Sun is observed to appear like a luminous ring round the Moon; and a *partial* eclipse, when part of the Sun is obscured.

There cannot be more than *seven*, nor less than *two* eclipses in a year: and in 18 years there are commonly 70 eclipses, of which 41 are of the sun, and 29 of the moon.

Because the mean motion of the lunar nodes is about 19° 19′ 43″ yearly, a revolution is completed in about 18 years 224 days; and if in that time an exact number of lunations were contained, there would be a regular return of the same eclipse at the end of every period of the nodes; but this is by no means the case—however, in 18y. 11d. 7h. 42′ 31″, leap year being four times contained; but if five times, then in 18y. 10d. 7h. 42′ 31″, the node, after being in conjunction with the

Sun

Sun and Moon, returns so near to the conjunction as to be only 28′ 12″ distant therefrom; hence, if to the mean time of any eclipse, 18y. 11d. 7h. 42′ 31″, or 18y. 10d. 7h. 42′ 31″, be added, according as there are four or five leap years in the period, the sum will be the mean time of an eclipse nearly. The period of 521 Julian years, 3h. 5′, is more accurate; and that of 2362 years, 16d. 5h. 5′, is still more exact.

Of the LUNAR IRREGULARITIES.

The Lunar Irregularities, previous to the time of Sir Isaac Newton, exercised the skill of the most eminent astronomers; but that celebrated mathematician having applied his admirable principle of universal gravitation to the moon, has left a theory of the lunar motions which, in its present improved state, is sufficient to ascertain the moon's longitude within less than half a minute of a degree. An explanation of these irregularities does not come directly within the present plan, but perhaps may be treated of in a following work. We shall, however, in this place, briefly mention a few of the principal equations.

The orbit of the Moon is a variable ellipse, having the Earth in one of its foci. Hence the elliptic equation, answering to any given mean anomaly, is variable; this equation is divided into two parts, the one called the *Equation of the Center*, and the other the *Evection*. The mean elliptic eqnation, when greatest, amounts to 6° 18′ 32″, and the quantity of the greatest mean evection is 1° 20′ 28″.

The true longitude of the moon is found to precede the mean, while the moon is moving from the syzigy to the quadrature, and to be behind it, from the quadrature to the following syzigy. Hence arises an equation, depending upon the moon's distance from the syzigy, called the *Variation*. The greatest mean variation is 36′ 59″. 8.

When the earth is in aphelion, the motion of the moon is found to be accelerated, and retarded when in perihelion. The equation for correcting this irregularity is called the *Annual Equation*, which, when greatest, amounts to 11′ 9″, and whose argument is, therefore, the sun's mean anomaly. The time of the revolution of the moon with respect to its apogee, and also to its node, being longer when the earth is in aphelion, than when it is in perihelion, Newton, therefore, added two other annual equations, each of these, as well as the former, being proportional to the elliptic equation of the earth's orbit, as they depend on the distance of the earth from the sun; Newton made the first of these equations, or that of the Moon's mean anomaly, to be 20′, and the second, 9′ 30." In Mayer's Tables, improved by Mason, these are 21′ 42″, or the annual equation of the Moon's node, and 9′ 12″, respectively.

Several other equations are also necessary, in order to compute the moon's longitude to any tolerable degree of exactness; some of which serve to correct irregularities to which most of the former are liable, on account of the change of distance of the sun and moon from the earth, their relative positions, &c.

The lunar nodes move in a very irregular manner, being stationary when

when the moon is in quadrature, or without latitude; but in other cases, their motion is retrograde. The nodes complete a revolution in 18 years, 223d. 7h. 13′ 18″.

The motion of the moon's apsides is irregular, being direct when the moon is in the syzigies, and retrograde when in the quadratures. The direct motion is quickest when the line of the apsides coincides with the syzigies, and the retrograde motion is quickest, when the above line coincides with the quadratures; however, the direct motion exceeds the retrograde, and a revolution of the line of the apsides, according to the order of the signs of the ecliptic, is completed in 8y. 312d. 11h. 11′ 39″.

The inclination of the moon's orbit is also variable; it is greatest when the line of the nodes coincides with that of the quadratures, and least, when that line coincides with the syzigies.

The time of the moon's revolution is also found to be subject to a periodical inequality, called the *Secular Equation*, which arises from the Sun's action on the Moon, combined with the variation of the excentricity of the Earth's orbit; the Moon's motion is accelerated, as this excentricity diminishes.

According to M. de la Place, the secular equation of the moon is $= 11''.135i + 0''04398^3$, wherein i represents the number of centuries elapsed since 1700, and the sign is to be changed if the given time is before 1700. According to M. de Lambre's comparison of modern observations, the mean secular motion of the moon, as given by M. Mayer, is too great by 25″. In Mayer's last Tables, the secular equation of the moon is 9″; therefore, the mean motion of these tables should be corrected by a quantity equal to $-25''i + 2.135i^2 + 0.04398i^3$. The nodes and apogee are also subject to a secular equation. It hence evidently appears, that the computation of the moon's place from tables is very tedious; and hence also appears the value of the Nautical Almanac, published annually by order of the Commissioners of Longitude, under the inspection of the Astronomer Royal, which contains every article relating to the Moon, that may be of service either to the practical astronomer or navigator. The Moon's longitude and latitude are given at noon and midnight, in page v. of the month; its age and passage over the meridian, and right ascension and declination at noon and midnight, are contained in page VI. of the month; its semidiameter and horizontal parallax, at noon and midnight, in page VII.; and its distance from the sun and proper stars, at every third hour, by the meridian of Greenwich, are contained in pages VIII. IX. X. XI. of the month. The Connaissance des Tems is also a valuable publication.

Of the HORIZONTAL MOON.

That the moon is imagined to appear much larger when in or near the horizon, than when at any considerable degree of elevation, is an observation familiar to every person: whereas, the semidiameter of the

the moon really subtends a less angle in the first case, than in the second, by the quantity of the augmentation.*

To account for this seeming paradox, has exercised the skill of many eminent astronomers and philosophers, who have given various solutions for that purpose. These, however, we may probably have occasion to mention in another work—and, therefore, we shall confine ourselves to that solution which is now generally received. For this purpose, it will be necessary to premise the following lemmata.

I.

The nearer that any object is to the eye of an observer, it will appear under a greater angle.

II.

Let there be two objects of the same magnitude, and placed at equal distances from an observer; but from some illusion, one of them is imagined to be more distant than the other; then, that object which is judged to be at the greatest distance, will be considered as the largest of the two.

III.

The expanse, or firmament, from whatever cause, whether from the appearance of clouds in the atmosphere, these towards the zenith being nigher than those near the horizon; from the appearance of a number of interposed objects between the observer and the most distant parts of the horizon; the greater quantity of vapours near the horizon, which render objects fainter than when elevated, or otherwise; is imagined to be a small portion of a spherical surface,† the nearest point being the zenith, and the most distant the horizon.

Now, since the heavenly bodies are, to imagination, disposed on the surface of this circular or vaulted arch; and since, therefore, an object is judged to be more distant when in or near the horizon, than it is when at any considerable degree of elevation; it hence follows, that of two objects of the same magnitude, and at equal distances from an observer, that which is apparently farthest distant will, to imagination, be the largest of the two. Now, let one of these objects be near the horizon, and the other near the zenith; then, in consequence of the vaulted appearance of the sky, the former will be imagined to be more distant than the latter. Hence, when the moon is in the horizon, being then supposed to be more distant from the observer than when in

* The augmentation is contained in Table xxxi.

† P. Asclepi, in his treatise *De Apparente Objectorum Distantia et Magnitudine*, &c. printed at Rome in 1769, attributes the apparent magnitude of the horizontal moon to the apparent figure of the sky, and not to a series of interposed objects, as adopted by M. de la Lande, in his Astronomie, vol. ii. page 202. There is a geometrical construction of this vaulted appearance of the sky, with the method of computation, given in Robins' *Mathematical Tracts*, vol. ii. p. 245; in which work, this subject is treated very fully. Euler, in his *Letters to a German Princess*, vol. ii. p. 494, accounts for the flattened form of the sky, to arise from objects appearing less brilliant when in the horizon, than at any considerable elevation.

any

any other position, that object will, therefore, be judged to be largest; and, to imagination, will continually decrease in magnitude until it has attained the meridian, when it will be supposed to be least. However, that the moon is not larger when in the horizon, than when in the meridian, may be inferred from the following simple experiment *: Roll up a sheet of paper in form of a tube, of such a size, that the moon when in the horizon may appear to fill it exactly; then observe the moon when in the meridian, or at any considerable degree of elevation, and, in place of appearing less, will, when viewed through the same tube, appear a little greater, in consequence of being nearer to the observer.

It is upon account of this vaulted appearance of the sky, that reckoning, by estimation, any small number of degrees from the zenith towards the horizon, and the same number from the horizon towards the zenith, the former will actually be a greater arch than the latter. Also, the distance between any two stars, when observed near the horizon, is imagined to be much greater than the same distance when the stars are near the zenith This will appear evident by observing the stars Castor and Pollux, when in or near these two situations.

That the altitude of an object is apparently greater than the truth, may be found by first estimating the altitude of that object, and then ascertaining its true altitude by observation. In this manner, Dr. Smith found the true altitude of an object to be only 23°, which, according to estimation, appeared to be 45°; and hence, he inferred the horizontal distance of the spherical segment to be three or four times greater than the vertical distance: and supposing the diameter of the moon to be divided into 100 equal parts, when in the horizon, he infers, that the diameter at other altitudes will be imagined to contain less of these parts, as follows:

Let at alt. 0°,	the moon's diameter	be divided	into	100 equal parts;
Then at alt. 15°	the diameter	will be imagined	to be	68 of these parts
30	—	—	—	50
45	—	—	—	40
60	—	—	—	34
75	—	—	—	31
90	—	—	—	30

According to Mr. Robins, the horizontal distance will never be less than four times the perpendicular distance, when 45° by estimation measure only 20°; but if the altitude be 28°, it will be little more than two and a half.

Of the Harvest Moon.

In these northern latitudes, towards the time of the autumnal equinox, from two to three days before, till as long after the time of full moon, the retardation, or difference between the times of the rising of the moon, in that interval, is only about two hours; whereas, at the

* Ferguson's Astronomy, p. 99.—M. de la Lande's Astronomie, vol ii. p. 201.

opposite time of the year, and in an equal interval, this difference in the time of rising will amount to seven or eight hours. Hence, since the time of harvest is towards the autumnal equinox, this phenomenon has obtained the name of the *Harvest Moon.*

In order to understand the reason of this phenomenon, it may be observed, or rather shown, by a common globe, that equal portions of the ecliptic rise in unequal portions of time, and this difference increases with the latitude. Thus, that portion of the ecliptic, having the beginning of Aries in its middle, rises in a much less interval of time, than an equal portion of the ecliptic in the middle of which is the first of Libra.

At the time of harvest, the moon, when full, is in Pisces or Aries; and, therefore, since any portion of the ecliptic, as the arch described by the moon in a day, will rise in a much shorter interval of time, than an equal portion of the opposite part of the ecliptic; at the time of the autumnal equinox, therefore, the retardation or difference between the times of the rising of the moon, on two successive evenings, when near the opposition, will be less than at any other time of the year.

If the moon's orbit coincided with the ecliptic, and that orbit at the same time invariable in form and position, the interval in the time of rising in two successive nights, under the same circumstances, would take place every year. But the case is otherwise; for the moon's orbit is inclined to the ecliptic in an angle of about 5° 8′ 49″; and, therefore, the interval of time in the rising of the moon, on two successive nights, will vary according to the position of the moon's nodes at that time; that interval being least when the moon's ascending node is in Aries, and greatest when in Libra: it is hence obvious, that the harvest moons are more or less beneficial, according to the position of the moon's nodes. And since a revolution of the nodes is completed in about 19 years, in that interval, therefore, a period of the most and least beneficial harvest moons will be completed.

In the following table * are contained the years, in which the harvest moons are most and least beneficial to the inhabitants of the northern hemisphere.

HARVEST MOONS *most beneficial in the Years*

1798	1799	1800	1801	1802	1803	1804	1805	1806	
1816	1817	1818	1819	1820	1821	1822	1823	1824	1825*
1835	1836	1837	1838	1839	1840	1841	1842	1843	
1853	1854	1855	1856	1857	1858	1859	1860	1861	1862

HARVEST MOONS *least beneficial in the Years*

1807	1808	1809	1810	1811	1812	1813	1814	1815
1826	1827	1828	1829	1830	1831	1832	1833	1834
1844	1845	1846	1847	1848	1849	1850	1851	1852
1863	1864	1865	1866	1867	1868	1869	1870	1871

* Ferguson's Astronomy, page 214.

The

The phenomena of the harvest moon may also be easily explained, from the simple consideration of the change of the moon's declination. The change of declination of the moon, as well as of every celestial object, is greatest when it is in the equinoctial. If, therefore, the moon be in or near the equinoctial, and tending towards the elevated pole, that object would, evidently, rise sooner on the following night, it being supposed to pass the meridian at the same hour and minute on both days, in like manner as the sun. But, because the moon is about 50 minutes later in passing the meridian each day, therefore, what is gained in the time of rising by the change of declination, is more than compensated by the diurnal retardation of the moon.—Now, although this takes place in every lunation, yet it only happens at that full moon which is towards the end of September, and therefore, about the time of harvest, and hence its name. The following full moon is usually denominated by the name of the *Hunters' Moon*.

The figure of the moon is a spheroid, the longest diameter being directed to the earth; * and hence, the same side of the moon is always opposed to the earth.

That the moon has an atmosphere was affirmed by many astronomers, and is now very satisfactorily proved by M. Schroeter. † The mountains in the moons are visible to the naked eye; volcanoes also have been lately discovered; and that the moon is inhabited, was asserted by Anaxagoras about 2300 years ago.

The mean horizontal parallax of the moon is $57' 39''$; hence its mean distance from the earth is about 236000 English miles. ‡

The ratio of the semidiameter of the moon to its horizontal parallax, according to M. de la Lande, is as $16' 23''.3 : 60'$; or in round numbers, nearly as $3 : 11$. Hence, the earth appears thirteen times larger to the moon than the object does to the earth; and hence, the reason of the faint light in the dark part of the moon soon after the conjunction, which light is, by the French, called the *Lumière Cendree*. The quantity of the moon's visible surface, at any given time, is equal to half the area of the moon's disc, multiplied by the versed sine of the elongation at that time. From the above ratio also, the magnitude of the moon is a 49th part of that of the earth, and quantity of matter a 66th part. A heavy body, at the surface of the moon, will fall three feet during the first second; and the length of a pendulum vibrating seconds will be $7\frac{1}{3}$ inches; and the proportion of moon-light to day-light as 1 to 96000.

The right ascension and declination of a planet, are deduced from observations of its zenith distance, and the time of its transit over the meridian; from which, and the obliquity of the ecliptic, the longitude and latitude of the planet are found by computation. But the longitude and latitude of a planet are found directly by astronomical tables, for any

* Newton's Principia, page 431. Amst. 1714, or vol. iii. page 547; Geneva, 1742.

† Philosophical Transactions, 1792.

‡ The semidiameter of the earth is $3958\frac{3}{4}$ English miles—see page 7, hence the distance $= 3958\frac{3}{4} \times$ co-secant $57' 39'' = 236077$ miles.

 give.

given time; with which, therefore, and the obliquity of the ecliptic, the right ascension and declination of the planet are to be found by computation. The two following problems, communicated to the Author by Dr. Maskelyne, without their demonstrations, will be found very useful for the above purpose.

PROBLEM I.

Given (AR) the right ascension, declination, and (O) the obliquity of the ecliptic, to find the longitude and latitude, true to the nearest second by Taylor's Logarithms, or to the nearest 10″ by Gardiner's Logarithms, or to the nearest minute by Sherwin's or Hutton's Logarithms, without proportioning.

Tan. dec. — S. AR = tan. A, north or south, as declination is.
Call O in first six signs of AR, south; in last six signs, north.
A + O = B.
A, less than 45° co. ar. cos. A + cos. B + tan. AR
A, more than 45°. tan. A + co. ar. sine A + cos. B + tan. AR } =
tan. long. of the same kind as AR, unless B be more than 90°, when the quantity found of the same kind as AR, must be taken from twelve signs.

Long. nearer III. and IX. signs, than O. and VI. signs, sine long. + tangent of B.
Long. nearer O. and VI. signs, than III. and IX. signs, tan. long. + sine long. + tangent of B. } =
tangent of the latitude of same name as B.

EXAMPLE.

Let the Moon's right ascen. be 93° 58′, declin. 28° 27′ N. and obliquity of the ecliptic 23° 28′. Required its longitude and latitude?

Dec.	28°	27′N.		tang.	9.7338601		
R.A.	93	58		- sine -	9.9989584	tang.	11.1590023
A. -	28	30	27″N.	tan.	9.7349017	ar.co-s.	0.0561324
O. E.	23	28	s.				
B.	5	2	27N.	- - co-sine -	9.9983170	tan.	8.9455023
Lon.	93	30	2	- - tangent -	11.2134517	sine	9.9991889
Lat.	5	1	53 N.	- - tangent - -			8.9446912

PROBLEM II.

Given the long. and lat. of a fixed star or planet, and (O) the obliquity of the ecliptic, to find the right ascension and declination.

Tan. lat. — sine long. = tan. A, north or south, as the latitude is.
Call O north in six first signs, and south in six last signs.
A + O = B.

A, less

A, less than 45°, co. ar. cos. A + cos. B + tang. longitude.
A, more than 45°, tan. A + co. ar. sine A + cos. B + tan. long. } =
tan. right ascension of the same kind as longitude, unless B be more than 90°, when the quantity found of same kind as longitude must be subtracted from twelve signs.

AR nearer III and IX signs than O and VI signs, sine AR + tan. B.
AR nearer O & VI signs than III & IX signs, tan. AR + cos. AR + tan. B } =
tangent of declination of same title as B, true to the nearest second by Taylor's Logarithms, to nearest 10″ by Gardiner's Logarithms, or to nearest minute by Sherwin's or Hutton's Logarithms, without proportioning.

Example.

Let the Moon's long. be 7s. 14° 26′ 21″, and lat. 4° 0′ 34″ N. and the obliquity of the ecliptic 23° 27′ 48″. Required the right ascension and declination?

Lat. ☽ 4° 0′ 34″ tan. 8.8456713
Lon. 224 26 21 sine 9.8451920 - tan. - 9.9914974

A = 5 43 0.7 tan. 9.0004793 ar. co. cos. 0.0021654
O = 23 27 48. S.

B = 17 44 47.3 S. — co-s. — 9.9788260 tan. 9.5051970

RA. 223 11 11.2 — tan. — 9.9724888 sine 9.8352940

Decl. 12 21 14.6 S. — — tangent — — — 9.3404910

Remarks.

I.

The latitude and longitude of the moon, or a star, may be found by the following formulæ:

Co-v. s. Lat. = v. s. R. Ascen. from ♑ × s. co-decl. × s. ob. ecl.
+ v. s. co-decl. ∽ ob. ecl.

Co-s. Longitude from ♈, or ♎ =
secant lat. × co-s. decl. × co-s. R. Ascen. from ♈, or ♎.

II.

The right ascension and declination may be found by the following formulæ:

Co-v. s. Decl. = v. s. long. from ♋ × s. co. lat. × s. obliq. of the ecliptic + v. s. co. lat. ∽ ob. ecliptic

Co-s. Right Ascension from ♈, or ♎, =
secant decl. × co-s. lat. × co-s. long. from ♈, or ♎.

CHAP.

CHAP. VI.

Of COMETS.

COMETS* are bodies belonging to the solar system, revolving about the Sun in very excentric orbits, and are only visible when near their perihelion.

As the orbits of Comets are elliptical, in one of whose foci is the Sun; they are, therefore, subject to the same laws as a planet; that is, they describe equal areas, in equal times, and the squares of their periodic times, are as the cubes of their mean distances, &c. As few Comets have been observed to have any diurnal parallax, they must, therefore, be more distant from the Earth than the Moon; but in consequence of an annual parallax, they descend within the orbits of some of the planets; and, similar to them, receive their light from the Sun. The motion of Comets is either direct, or retrograde.

From observation it is ascertained, that a Comet consists of a solid body, called the *Nucleus*, or *Head*, and an *atmosphere*; from the form of which, different names have been given to these bodies; such as hairy, when the atmosphere is equally diffused round the nucleus; and those with a train, or tail, which is nearly in a direction opposite to the Sun; and a Comet is said to be *tailed*, or *bearded*, according as it is approaching to, or receding from the Sun: in the first case, the train follows, and in the latter it precedes the Comet. In Britain, some Comets have been observed without any appearance of an atmosphere, or tail, at the same time, that in countries more favourable for observation, a coma or tail has been visible. From these phenomena, combined with its motion, a comet is easily distinguished from a fixed star.

The tail of a Comet has commonly a small degree of curvature, of which that in the end of 1807 is a recent instance, and the convex side is more distinct, or better defined than the concave; the tail is so extremely thin, that the fixed stars may be seen through it, in the same manner as through the Aurora Borealis, and, therefore, has a strong resemblance to that phenomenon, and also to the electric fluid; and Dr. Hugh Hamilton of Dublin observes, that a spectator, at a distance from the earth, would see the Aurora Borealis in the form of a tail opposite to the Sun, as the tail of a Comet lies. The tail of a Comet is observed to be larger after it has passed its perihelion, than it

* The word Comet is derived from Κομητης, *Cometa* from Κομη, *Coma* a head of hair.

was

was when the Comet was descending towards the Sun. The tails of some Comets have been observed to be of a very considerable length; the tail of the Comet of 1618 appeared under an angle of 104°; that of 1680, subtended an angle of 70°; and, according to Mr. Maclaurin, the tail extended from the head to a distance scarcely inferior to the vast distance of the Sun from the earth.

Dr. Halley was the first who predicted the return of a Comet. By comparing the elements of the orbits of the Comets of 1456, 1531, 1607, and 1682, he supposed them to be the same Comet, and that it would return about the end of the year 1758, or the beginning of 1759, which prediction was accordingly verified.

The number of Comets belonging to the solar system is very uncertain. Riccioli enumerated 154 previous to the year 1618. Lubienietzki states the number to be 415 in the year 1665; and some late writers have increased the number to upwards of 700.

In order to calculate the place of a Comet, the elements of its orbit must be previously ascertained; these are, the time when the Comet was in its perihelion, the place of its perihelion, the perihelion distance, the place of its ascending node, and the inclination of its orbit to the ecliptic. From these data the place of a Comet may be easily found by tables computed expressly for that purpose; as those by *Dr. Halley, M. Bockhart de Saron, M. du Séjour, M. de Lambre.* *

Mr. Whiston supposed that the deluge was occasioned by the tail of the Comet of 1680 in its descent towards the Sun; and he also suggested that the general conflagration will arise from the tail of the same Comet in its ascent from the the Sun. The interval between two successive periods is supposed to be 12,000 years, if the periodic time of the above-mentioned Comet is 575 years, there will be 21 revolutions in 12,075 years.

Dr. Halley observes, that in the late descent of the Comet of 1680, its true path left the orbits of *Saturn* and *Jupiter* below itself a little towards the south: it approached much nearer to the paths of *Venus* and *Mercury*, and much nearer still to that of *Mars*. But as it was passing thro' the plane of the ecliptic, viz. to the southern node, it came so near the path of the *Earth*, that had it come towards the Sun 31 days later than it did, it had scarce left our globe one semidiameter of the *Sun* towards the north; and, without doubt, by its centripetal force (which, with the great *Newton*, I suppose proportional to the bulk, or quantity of matter in the Comet), it would have produced some change in in the situation and species of the *Earth's* orbit, and in the length of the year. But may the great good God avert a shock or contact of such great bodies moving with such forces (which, however, is by no means impossible), lest this most beautiful order of things be entirely destroyed, and reduced into its ancient chaos: and Mr. Maclaurin, speaking of the same Comet, says; "it is not to be doubted but that, while so many Comets pass among the orbits of the planets, and carry

* In *M. de La Lande's Astronomie*, vol. iii. page 251, there is a table of the elements of the orbits of 78 Comets, and in Dr. Rees's Dictionary, one of 97 Comets.

such

such immense tails with them, we should have been called by very extraordinary consequences, to attend these bodies long ago, if their motions in the universe had not been at first designed and produced by a Being of sufficient skill to foresee their distant consequences." *M. du Séjour* observes, that it is very improper to instill terror into the minds of men without any just reason. The Comet of 1770 approached nearer to the earth than any hitherto observed, and produced no sensible effect either upon the motion of the Earth, or upon its inhabitants.

Dr. Halley mentions, in his Synopsis of Comets, printed in 1705, and also in his Astronomical Tables, that as it is more than probable, that the rest of the Comets described in his Catalogue, will return after having finished their periods, whence their periodic times being given, the axes, and from thence the species of their elliptic orbits will be also given; in order, therefore, to render the tediousness of these operose calculations as easy as possible to future astronomers, he calculated a general table of the motions of Comets, according to the parabolic hypothesis, wherein are contained the double areas of the segments, the logarithms of the right and versed sines, with their differences, and the versed sines themselves to every fifth part of the degrees of the excentric anomaly.

The reason of taking notice of Comets, is that of corresponding observations which may be made on them at different places, their relative situation in longitude may be accurately determined, if their velocity is considerable, which has been the case with many: The Comet of 1472, observed by Regiomontanus, described an arch of 40° in one day, in the circum-polar parts of the northern hemisphere;* the remarkable Comet that appeared in 1664 and 1665, moved over a space of 20° in one day, and described almost six signs before it disappeared; and the motion of the Comet, in longitude, of 1760, from the 7th to the 8th of January, was 41°¼. See upon this subject *Sir Isaac Newton's Principia; Maupertuis' Essays on Comets; M. de la Lande's Astronomie, tome* iii. *p.* 221; *Traité Analytique des Mouvemens Apparens des Corps Celéstes, par M. Dionis du Séjour, tome* ii. *p.* 416; *Sir Henry Englefield, Bart. M. Pingre,* &c.

* Riccioli Almagest.

BOOK

BOOK II.

CONTAINING

An Account of the Instruments for observing Altitudes and angular Distances at Sea, and of the Corrections to be applied to Observations made with those Instruments.

CHAP. I.

Of HADLEY'S QUADRANT.

THE first account we have of an instrument for measuring angles by reflection, is in Sprat's History of the Royal Society, page 246, which is as follows: " A new instrument for taking angles by reflection, by which means the eye at the same time sees the two objects, both as touching in the same point, though distant almost a semicircle; which is of great use for making exact observations at sea." The inventor of this instrument is supposed to be Dr. Hook. Other instruments of a similar nature were invented by Sir Isaac Newton, Dr. Halley, and Mr. Street.* We are, however, indebted to John Hadley, Esq. for the first account of this admirable instrument, which he communicated to the Royal Society, May 13, 1731. This account was published in the Transactions of that Society, No. 420. In the same paper he describes another instrument, having a third speculum, and the positions of the specula and telescope altered, whose use is to observe the Sun's altitude by means of the opposite part of the horizon. An instrument of a similar kind was invented by Mr. Godfrey of Pennsylvania; and Mr. Logan, Chief Justice of that province, transmitted a description of it to England, before or about the time Mr. Hadley's account was published in the Philosophical Transactions.

The instruments in use for measuring altitudes at sea, previous to the invention of Hadley's Quadrant, were, the Cross Staff and Davis'

* In Mr. Edward Harrison's *Idea Longitudinis*, printed at London in the year 1696, page 50, he says, " Mr. *Street*'s way," (of finding the Longitude) is unknown to me, but we suppose his way by the *Moon's* motion also, by his contriving an *Instrument* for taking *Angles* by *Reflection* : I have seen the Instrument.

Quadrant. The advantages possessed by the latter for observations of the Sun's altitude, brought it into more general use, especially among the British navigators. Observations, however, made with that instrument, though in the hands of a good observer, were found liable to error, when the sea was agitated; and it was found impossible to observe the Sun's altitude, when the motion of the ship was very considerable. Hence, appears the superior excellence of Hadley's Quadrant, which is designed to be of use, where the motion of the observer, or any circumstance occasioning an unsteadiness in the common instruments, render the observations difficult or uncertain.

The form of this instrument, according to the present mode of construction, is an octagonal sector of a circle, and, therefore, the arch contains 45°; but because of the double reflection, the limb is divided into 90°; and, therefore, this instrument is usually called a Quadrant —of which the following are the principal parts:

1. An Octant, consisting of an arch or limb, connected with two radii.
2. An Index, moveable round a center, and accompanied with a dividing scale, to show the observed altitude, or angular distance.
3. A Speculum, or Index Glass, placed on the Index in such a manner, that its plane is over the center of motion of the Index.
4. Two Horizon Glasses, with their adjusters; H (fig. 6.) being the fore-horizon glass, and G the back-horizon glass.
5. A set of coloured Glasses, to prevent the effect of the solar rays on the eye of the observer.
6. Two sight Vanes.

Of the Octant.

The Octant consists of two radii, a limb, and two braces, which last are intended to strengthen the instrument, and prevent it from warping. The arch or limb, although only an eighth part of a circle, is divided into 90°, on account of the double reflection. These degrees are numbered from right to left, and each is commonly divided into three equal parts. Hence, one of these intervals contains twenty minutes; and by means of a scale at the end of the index, divided into twenty equal parts, an observation may be easily read off to the nearest minute. The graduation on the limb is continued a few degrees to the right of O; this portion is usually called the *arch of excess*, and is found very convenient for several purposes.

Of the Index.

The Index is a flat bar, commonly made of brass, moveable round the center of the intrument, and broader towards the axis of motion; at the other end of the index, a piece of brass turns up, behind the limb, having a spring to make the dividing scale lie close to the limb, and

and a screw to fasten it in any position. Some quadrants have an adjusting screw affixed to the lower part of the index, by which means the index is made to move gently along the limb, without those sudden startings to which it is liable when moved by hand, and which may render the observation uncertain to two or three minutes. It may be observed, that when the index is moved by hand, it should be taken hold of by the lower part, and not by the middle, as is sometimes the practice.

That part of the index, which moves along the limb, has a small scale attached to one side of a rectangular aperture, called a Dividing Scale,* but most commonly a Vernier or Nonius, from the names of the

* This method of division was first clearly explained by Clavius, in Lemma 1st of his Treatise on Astrolabes, printed at Mayence in 1611; it was afterwards published by Pierre Vernier, who claimed it as his own, in a small tract, intitled, "La Construction, l'Usage, & les Proprietés du Quadrant Nouveau de Mathematique," &c. printed at Brussels, 1631. This method of division may be thus illustrated:

Let two equal lines or circular arcs be so divided, that the number of equal divisions in the one, is one more than the number of equal divisions in the other; that is, let the one be divided into n, and the other into $n+1$ parts, and let a part of the former be called unity.

Then, $n : 1 :: n+1 : \frac{n+1}{n} =$ a part of the other.

And, $\frac{n+1}{n} - 1 = \frac{n+1-n}{n} = \frac{1}{n} =$ difference of the parts.

Or, let the one be divided into n, and the other into $n-1$ parts;

Then, $n : 1 :: n-1 : \frac{n-1}{n} =$ a part of the other.

And, $1 - \frac{n-1}{n} = \frac{n-n+1}{n} = \frac{1}{n} =$ the difference of the parts.

Now since in common quadrants, the Vernier is divided into 20 equal parts, whereof the limb contains either 19 or 21, the absolute length of both being the same, it is hence evident, that each division on the dividing scale must answer to 1 minute; for in the present case n represents 20, and, therefore, $\frac{1}{n} = \frac{1}{20}$ of a division on the limb: but each divison on the limb is 20 minutes, and $\frac{1}{20}$ of 20 minutes is 1 minute.

Let a degree on the limb be divided into four equal parts, and let the dividing scale contain a space equal to $7\frac{1}{4}$ degrees, or 29 of these parts, but divided into 30 equal parts, then will the dividing scale show half minutes, for $\frac{1}{30}$ of $\frac{60'}{4} = \frac{1}{2}$ min. or 30″. And lastly, if a degree is divided into six equal parts, and the dividing scale contains 59, or 61 of these parts but is divided into 60 equal divisions, then will each division shown by the dividing scale be 10″; for $\frac{1}{60}$ of $\frac{60}{6} = \frac{1}{6}$ of a minute $=$ 10″.

This method of division has been unjustly ascribed to Petrus Nonius; for Nonius' method is very different from Vernier's, as may be seen in his Treatise, "De Crepusculis," printed at Lisbon, 1522, and also in his Treatise, "De Arte atque Ratione Navigandi."

Nonius' method consists in describing within the same quadrant 45 concentric arches, dividing the outermost into 90 equal parts, the next within into 89, the next into 88, and so on till the innermost was divided into 46 equal parts only: by which means, in most observations, the plumb line or index must cross one or other of these circles very near a point of division—whence, by computation, the degrees and minutes of the intercepted arch might be easily inferred. However, this method of division gave way to that of diagonals, published by Thomas Diggs in his Treatise, "Alæ seu Scalæ Mathematicæ," printed at London in 1573, who says it was invented by a Richard Chanseler, a famous mathematical

the supposed inventors. It usually contains a space equal to 21 or 19 divisions of the limb, and is divided into 20 equal parts; hence, the difference between a division on the limb, and a division on the dividing scale, is one-twentieth of a division on the limb, or one minute; and, therefore, if any division on the dividing scale is in the same straight line with a division on the limb, then no other division on the dividing scale can coincide with a division on the limb, the extreme divisions excepted.

Sometime ago, it was usual to reckon the divisions on the Vernier from its middle towards the right, and from the left towards the middle; and since this scale usually contains 20 minutes, therefore, the first ten minutes were contained on the right of the middle line, and numbered 5.10, and the other ten on the left, and marked 10.15. However, this method of division being found inconvenient, a much more commodious method has been lately introduced, wherein the divisions are numbered from right to left; hence the degree and minute, pointed out by the dividing scale, may be easily found thus:

Observe what minute on the dividing scale coincides with a division on the limb; then this minute being added to the degree and part of a degree on the limb, immediately preceding the first division, on the dividing scale, will be the degree and minute required.

Thus, suppose the fourteenth minute on the dividing scale coincided with a division on the limb, and that the preceding division on the limb to the first on the dividing scale was 56° 40′, hence the division pointed out by the Vernier will be 56° 54′. If a magnifying glass be used, the coincidence of the divisions may be more accurately observed.

Of the Speculum.

The Index Glass, or Speculum, is a rectangular piece of plane glass, one of whose planes is for the most part completely silvered.

The intention of this glass is to reflect the image of the observed object, either to the fore or back-horizon glass, and from thence to the eye of the observer.

This glass is set in a brass frame, and is placed perpendicular to the plane of the instrument, immediately over the center of motion of the index, by means of two screws in a small plate of brass, connected with the under part of the brass frame, and projecting at right angles from the back of that frame: one of these screws takes into, and the other presses against the index.

The Index Glass, according to the common method of setting it in its frame, is often liable to be bent; and hence the difference between the true and observed altitudes might be considerable. This source of error, however, is now removed by Mr. Dollond, who contrived the

mathematical instrument-maker. This method of diagonals was applied to Davis' Quadrants, and to the first of Hadley's; but now, since the dividing scale or Vernier has been introduced, the diagonal method is as little esteemed as that of Nonius after the invention of diagonals.

frame

frame so, that the glass lies on three points, which, by means of a spring and crew, press the glass against other three points immediately opposite to the former; and since the position of a plane can be adapted to any three points, therefore, the plane of the glass cannot be bent, though the pressure be considerable.

As every silvered glass, although its planes be perfectly parallel to each other, gives two reflections, one from the fore surface of the glass, being the faintest, and the other from the silvered surface, which is the brightest, it is, therefore, evident, that errors may arise from these double reflections; in order to prevent which, Dr. Maskelyne proposed, that the back of the upper half of the index glass, or that farthest from the plane of the instrument, instead of being silvered as usual, should be ground rough, and painted black, the line joining the silvered and blacked parts being parallel to the plane of the instrument.—Hence, when the object to be observed is bright, its reflection is to be taken from this half of the glass, the reflection from the silvered part being prevented by a screen set before it; but if the object is not sufficiently bright, the screen may be turned down, and its reflection taken from the silvered part as usual.

Of the Horizon Glasses.

There are two horizon glasses, one for the fore, and the other for the back observation, which are, therefore, named the fore and back-horizon glasses. Each glass is set in a brass frame, having an axis passing through the frame of the instrument, provided with an apparatus, by which it is capable of receiving a small motion, so as to be set in a proper position with respect to the index glass; and in the pedestal of each glass are two sunk screws, one on each side of the glass, so that by screwing up the one, and unscrewing the other, the glass may be set perpendicular to the plane of the instrument. The lower part of the fore-horizon glass is silvered, and the upper part is left transparent, through which one of the objects is to be seen directly, the other being seen by reflection from the silvered part of the glass. Both upper and lower parts of the back-horizon glass are silvered, and in the middle is a transparent slit, parallel to the plane of the instrument, through which the direct object is to be viewed.

Of the Coloured Glasses.

There are commonly three coloured glasses, two of which are tinged red, and the other green, by which the eye of the observer is preserved from the effects of the solar rays; and the glare of the Moon, in night observations, is taken away. Each of these glasses is set in a separate cell, and so connected, that either one or more may be used at the time of observation. When the back observation is to be used, the stem of the glasses is put into another perforation between the horizon glasses, made on purpose to receive it. The two red glasses are

are adapted to observations of the Sun; and, therefore, in order to render them useful on all occasions, the one is tinged darker than the other, so that either, or both, may be used according to the brightness of the Sun; other degrees of shade may be also obtained by using the green glass with either of the former. The green glass is particularly useful in observations of the Moon; it may be also used in observing the Sun's altitude, when that object is very much obscured.

Of the Sight Vanes.

The Sight Vanes are pieces of brass, standing perpendicular to the plane of the instrument. That vane which is opposite to the fore-horizon glass, is called the fore-sight vane, and the other, the back-sight vane. There are two holes in the fore-sight vane, the lower of which, and the upper edge of the silvered part of the fore-horizon glass, are equidistant from the plane of the instrument, and the other is opposite to the middle of the transparent part of that glass. The back-sight vane has only one perforation, which is exactly opposite to the middle of the transparent slit in the horizon glass to which it belongs.

Fig. 6. represents the quadrant, wherein DB, CD, are the two radii or sides; BC the limb; P, Q, the braces; DV the index moveable about the center D, upon which is the index glass A; and at the other extremity, the dividing scale, or vernier, V; H is the horizon glass for the fore observation, and G that for the back observation; E and F are the correspondent sight vanes; and I, the coloured glasses, whose stem is to be put into the hole K, when the back observation is to be used.

Adjustments *of* Hadley's Quadrant.

Previous to observation, the quadrant should be carefully adjusted,* otherwise it will be impossible to deduce acccurate conclusions from the observations. For this purpose, the mirrors must be set perpendicular to the plane of the instrument, the fore-horizon glass must be set parallel to the speculum, and the planes of the speculum and back-horizon glass produced, must be perpendicular to each other when the index is at O. Hence the number of adjustments is five; however, three of these being once made, are not so liable to alteration

* We may observe, that since Davis's Quadrant is not capable of adjustment, it therefore becomes necessary to find its error, which was usually determined by observations made at a place whose latitude had been previously well ascertained, or by comparison with another instrument whose error was known; this error was to be allowed in all succeeding observations made with the same quadrant; and according as the error tended to increase or diminish the ship's northern latitude, the quadrant was, therefore, said to be *northerly* or *southerly*, by a quantity equal thereto. But Hadley's quadrant is constructed so as to be capable of being perfectly adjusted; hence, when a person says his quadrant is northerly or southerly, it may be inferred, that he neither understands the principles of the instrument, nor for what purpose it is adjusted.

as

as the other two, which last should be always examined before every observation.

Adjustment I.

To set the Index Glass perpendicular to the Plane of the Instrument.

Set the index to about 45°, and hold the quadrant so, that its plane may be nearly parallel to the horizon, the limb being from the observer; then look into the index glass, and observe if the portion of the limb seen by reflection appears in the same plane with that seen directly; if they do, the speculum is perpendicular to the plane of the quadrant; if, otherwise, the error is to be taken away, by turning the screws behind the speculum in contrary directions, and both screws must be left equally tight.

This adjustment may be performed in a more accurate manner, by means of two adjusting tools, fig. 20. on each of which is a line drawn parallel to its base; these lines are exactly at the same height.

Let the quadrant be laid on a table, with its face upwards, and the index placed towards the middle of the limb; then place the tools on the limb, one at each end, direct the sight to the index glass, and let the position of the index be altered till the half of the direct tool is intercepted by the farther edge of the speculum, and half of the other tool reflected from the same part of the speculum, the other half being off the glass. Now, if the index glass is perpendicular to the plane of the quadrant, the direct and reflected lines on the tools will be in the same straight line; if not, the lines will appear broken, and the coincidence is to made perfect by means of the screws behind the speculum.

Adjustment II.

To set the Fore-Horizon Glass perpendicular to the Plane of the Quadrant.

Set the index to O; hold the plane of the quadrant parallel to the horizon, and apply the eye to the lower hole, in the sight vane; then direct the sight to the horizon, or to any other well defined object; and if the horizons seen directly, and in the silvered part of the glass, are apparently in the same straight line, the fore-horizon glass is perpendicular to the plane of the instrument; if not, one of the horizons will appear higher than the other: now, if the horizon seen by reflection is higher than the same seen directly, unscrew the nearest screw in the pedestal of the glass, and screw up that on the farther side of the glass, till the direct and reflected horizons appear to be in the same straight line. But if the reflected is lower than that seen directly, unscrew the farthest, and screw up the nearest screw, till the coincidence of the horizons is perfect; observing to leave both screws

screws equally tight; and the fore-horizon glass will be perpendicular to the plane of the quadrant.

Adjustment III.

*To set the Fore-Horizon Glass parallel to the Index Glass, the Index being at Zero.**

Hold the plane of the quadrant in a vertical position, with the arch lowermost, the index being previously set exactly to O: now the eye being applied to the hole in the vane next the quadrant, and the sight directed to a well defined part of the horizon, or any well defined distant object,† then if the horizon seen in the silvered part of the glass, coincides with that seen through the transparent part, the horizon glass is adjusted; but if the horizons do not coincide, unscrew the milled screw, in the middle of the lever on the other side of the quadrant, and turn the nut at the end of the lever, till both horizons coincide: then fix the lever in this position, by tightening the milled screw.

If this adjustment be again examined, it will perhaps be found imperfect; in this case, therefore, it remains, either to repeat the adjustment, or rather, to find the error of adjustment, or, as it is usually called, the *index error*, which may be done thus:

Let the sight be directed through the vane to the horizon, move the index till the reflected horizon coincides with that seen directly, and the difference between O on the limb, and O on the vernier, is the index error; which is additive when the beginning of the vernier is to the right of O on the limb, otherwise subtractive.

Adjustment IV.

To set the Back-Horizon Glass perpendicular to the Plane of the Instrument.

Put the index to O, hold the plane of the quadrant parallel to the horizon, and direct the sight towards the horizon, through the back-sight vane. Now, if the horizon reflected from the silvered part of the glass, is in the same straight line with that seen through the transparent part, the glass is perpendicular to the plane of the instrument. If the horizons do not unite, turn the sunk screws in the pedestal of the glass, till both appear to make one continued straight line.

Adjustment V.

To set the Back-Horizon Glass perpendicular to the Plane of the Index Glass produced, the Index being at Zero.

Let the index be put as much to the right of O as twice the dip of the horizon amounts to; hold the quadrant in a vertical position, and

* The point from which the divisions on the limb are numbered, or O.
† The Sun or a bright star are the best objects for this purpose.

apply

apply the eye to the back-horizon vane; then, if the reflected horizon coincides with that seen directly, the glass is adjusted; otherwise, the screw, in the middle of the lever, on the other side of the quadrant, must be unturned, and the nut at its extremity turned, till both horizons coincide. We may observe, that the reflected horizon will be inverted, that is, the sea will be apparently uppermost, and the sky lowermost.

The back-horizon glass may be easily adjusted at land as follows: place three rods perpendicular to the horizon, in a sandy beach, or level field, so as to be exactly in the same straight line, and nearly equidistant from each other; the distance between two adjacent rods being not much less than a quarter of a mile; if the distance be greater, so much the better: then put the index of the quadrant exactly at O, and let the observer be stationed at the middle rod, with the plane of the quadrant parallel to the horizon, and the index glass above the rod.—Now direct the sight to one of the extreme rods, and if the reflected image of the other rod coincides with that seen directly, the glass is adjusted; if the coincidence is not perfect, unturn the milled screw, in the middle of the lever belonging to this glass, and turn the nut at the end of the lever, till both rods appear in one, and the glass will be adjusted; then fasten the milled screw.

This adjustment, as well as that of the fore-horizon glass, is subject to a small error, which is chiefly produced in fastening the milled screw in the middle of the lever; it, therefore, becomes necessary to find the error of adjustment, which may be done, by again examining the coincidence of the objects as before, after the milled screw is fastened, and making it perfect by moving the index; and the interval between the beginning of the index scale, and zero on the limb, will be the error of adjustment, or the index error for the back observation, to be applied as formerly. In the above determination, we have supposed the rods to be in the same straight line, which can be very easily effected; however, if these rods are not exactly in a right line, the error of adjustment may be thus found.

Let one of the extreme rods be called the *first*, and the other the *second rod*; now, the observer being placed at the middle rod as before, let the sight be directed to the first rod; make the reflected image of the second rod coincide with it, by moving the index, and mark the division pointed out by the scale. Then let the observer turn half round, direct the sight to the second rod, and move the index till the first, seen by reflection, coincides with it, and again mark the angle shown by the index scale*. Now, if one of these angles be measured on the limb, and the other on the arch of excess, half their difference will be the error of adjustment, or index error; but if both angles be measured either on the limb, or on the arch of excess, half their sum is the index error; subtractive, from altitudes given by the quadrant,

* Or invert the position of the quadrant, by turning its face downwards; hold its plane level as before, and direct the sight again to the first rod; move the index till the second, seen by reflection, coincides with it, and read off the angle shown by the dividing scale.

when the angle shown on the limb exceeds that on the arch of excess, or if both angles be measured on the limb: in other cases, this error is to be added. If the observation is that of the angular distance between any two objects, then the supplement of that given by the quadrant, corrected as above, will be the angle required.

Various other methods have been proposed for effecting the adjustment of the back-horizon glass, some of which we shall briefly mention. 1st, In Mr. Dollond's method, an index is applied to the back-horizon glass, by which it may be moved, so as to be parallel to the index glass, when O on the index scale coincides with O on the limb. When this is effected, the index of the back-horizon glass is to be moved exactly 90° from its former position, which is known by means of an arch divided for that purpose; and then, the plane of the back-horizon glass will be perpendicular to the plane of the index glass produced. 2d, In Mr. Blair's method, the under edge of the index glass is ground and polished, so as to be at right angles to the plane of that glass; and hence, the back-horizon glass is adjusted, by making the direct and reflected horizons agree, the index being at O, exactly in the same manner as the fore-horizon glass is rendered parallel to the speculum.* Mr. Gilkerson, Postern-row, Tower-hill, London, has a new method of adjusting the index glass, by means of a screw at the upper end of that glass, for which a patent was obtained.

Use of Hadley's Quadrant in Observing Altitudes at Sea.

The altitude of any object is determined by the position of the index on the limb, when by reflection that object appears to be in the horizon. In observing altitudes, it will be found most convenient to hold the quadrant with the left hand, and move the index with the right. Hence, the left hand is to be applied to the farther radius, towards the end of the curved brace, and the right hand to the lower part of the index.

There are two different methods of taking observations with Hadley's quadrant. In the first of these the face of the observer is directed towards that part of the horizon immediately under the Sun, and is, therefore, called the *Fore Observation*. In the other method, the observer's back is to the Sun, and it is hence called the *Back Observation*. This last method of observation is to be used only, when the horizon under the Sun is obscured, or rendered indistinct by fog, or any other impediment.

If the object, whose altitude is to be observed, be the Sun, and if so bright, that its image may be seen in the transparent part of the fore-horizon glass, the eye is then to be applied to the upper hole in the fore-sight vane, otherwise to the lower hole; and in this case, the quadrant is to be so held, that the Sun may be bissected by the line joining the silvered and transparent parts of the glass. Hence, one half of the Sun will be reflected from the silvered part of the glass.—The Moon is to be kept as nearly as possible in the same position, and

* See also the article Navigation, by the Author, in the Encyclopædia Britannica, vol. xii. page 726, third edition.

the

the image of a star is to be reflected from that part of the silvered part of the glass, which is adjacent to the line of separation of the two parts.

In observing the altitude of any object, the quadrant must be held perpendicular to the horizon, and so, that its plane produced would pass through that object. In observations of the Sun, this latter position is easily obtained, provided the altitude is low, by holding the quadrant in such a manner, that the shadow of the farther radius may fall on that next the observer; in this case, the sight is to be directed to the horizon. If the observed object is not the Sun, it will be proper to direct the sight to the object; the quadrant being held so, that the image of the object may appear in the silvered part of the glass, as before mentioned, and the index is to be moved till either limb of the object appears in contact with the horizon; but because that part of the horizon, which is immediately under the object, is not exactly known; it, therefore, becomes necessary to give the quadrant a slow vibratory motion, the axis of which being that of sight, the observer, at the same time, turning himself about upon his heel*, so as to keep the object always in that part of the horizon glass, which is at the same distance as the eye from the plane of the quadrant. By this means the reflected object will describe an arch of a parallel circle round the true Sun, whose convex side will be downwards in the fore observation, and upwards in the back; and, therefore, when by moving the index, the lowest point of the arch in the fore observation, or highest in the back, is made to touch the horizon, the quadrant will stand in a vertical plane, and the altitude above the visible horizon will be properly observed.

It is often found, in measuring the altitude of any object, or the distance between two fixed objects, that a different angle† will be obtained, by finishing the observation with a motion of the index in contrary directions. Thus the angle, obtained by moving the index according to the order of the divisions on the limb, is always found to be greater, than when the index is moved in a retrograde direction, or contrary to the order of the divisions. As this error arises from the bending of the index, it may be obviated by taking care to move the index the same way, both in the adjusting, and in taking the observations.

It frequently happens, that when the object is nearly in contact with the horizon, the observer, by endeavouring to make this contact better, often pushes the index too far, and thereby renders the contact worse than before. In order to remove this inconvenience, the best quadrants are provided with a screw at the lower end of the index, called an *adjusting*, or *tangent screw*; by which a slow motion may be given to the index. Hence, those sudden starts, to which it is liable when moved by hand, are prevented.

A small magnifying glass will be found of great service in reading off the observed angle; in doing which, the limb may be held towards the observer, and the eye should be in a plane perpendicular to the quadrant, passing through the coinciding divisions.

* Dr. Maskelyne's Remarks on Hadley's Quadrant, Nautical Almanac, 1774.

† An excellent account of this error is given in the Nautical Almanac for 1788.

To take Altitudes by the Fore Observation.

I.

Of the Sun.

Turn down either of the coloured glasses, before the horizon glass, according to the brightness of the Sun; and if the Sun is bright enough to be seen in the transparent part of the glass, let the eye be placed at the upper hole in the sight vane, otherwise at the hole next the plane of the quadrant; direct the sight to that part of the horizon which is under the Sun, and move the index till the coloured image of the Sun appears in the horizon glass; then give the quadrant a slow vibratory motion about the axis of vision; move the index till the Sun's lower or upper limb, at the lowest part of the arch, described by this motion, is in contact with the horizon, and the degrees and minutes, shown by the index on the limb of the quadrant, will be the observed altitude of the Sun.

II.

Of the Moon.

Put the index to O, turn down the green-tinged glass, place the eye at the lower hole in the sight vane, and direct the sight to the Moon; which being found in the silvered part of the horizon glass, move the index gradually, and follow the Moon's reflected image, till the enlightened limb is in contact with the horizon, at the lower part of the arch described by the vibratory motion, as in the last article; and the index will show the observed altitude of that limb of the Moon, which was brought in contact with the horizon. If the observation is made in the day time, the coloured glass is unnecessary.

III.

Of a Star *or* Planet.

Put the index to O, direct the sight to the star, through the lower hole in the sight vane, and transparent part of the horizon glass; move the plane of the quadrant a very little to the left, and the image of the star will be seen in the silvered part of the glass; then move the index, and the image of the star will descend; continue this motion of the index till the star is in contact with the horizon, at the lowest part of the arch described; and the degrees and minutes, shown by the index on the limb, will be the observed altitude of the star.

When that part of the horizon, under the object whose altitude is intended to be observed, is obscured or rendered indistinct, by fog or any other impediment, the back observation becomes necessary; we would, however, advise the observer, before he depends entirely on the altitude taken by the back observation, to accustom himself to this mode of observing, and to ascertain the accuracy to which he is capable of attaining, by comparing altitudes, taken by this method, with those of the same object, taken at the same time by the fore observation, allowance being previously made for the dip, and index error.

To

To take Altitudes by the Back Observation.

I.

Of the Sun.

Put the stem of the coloured glasses into the place made to receive it between the horizon glasses; turn down either, according to the brightness of the Sun, and hold the instrument vertically; then direct the sight through the hole in the back-sight vane, and transparent slit in the horizon glass, to that part of the horizon which is opposite to the Sun; move the index till the Sun is in the silvered part of the glass, and by giving the quadrant a vibratory motion, whose axis is that of vision, the image of the Sun will describe an arch whose convex side is upwards; now bring the limb of the Sun, when in the upper part of the arch, in contact with the horizon; and the degrees and minutes shown by the index on the limb will be the altitude of the other limb of the Sun.

II.

Of the Moon.

The altitude of the Moon is observed in the same manner as that of the Sun, with this difference only, that the use of the coloured glass is unnecessary, unless the Moon is very bright; and that the enlightened limb, whether it be the upper or lower, is to be brought into contact with the horizon. In the day time, the Moon's altitude, may be observed as directed in the next article.

III.

Of a Star *or* Planet.

Look directly to the star, through the vane and transparent slit in the horizon glass; move the index till the opposite horizon, with respect to the star, is seen in the silvered part of the glass, and make the contact perfect as formerly. If the altitude of the star is known nearly, the index may be set to that altitude, the sight directed to the opposite horizon, and the observation made as before.

Unless observations are made accurately, the conclusions to be deduced therefrom cannot be depended on; and since a single altitude is not to be so much relied on, as the mean of several, taken at short intervals of time; therefore, four or five altitudes should be taken at nearly equal intervals of time, and their sum, divided by their number, will be the mean altitude which the object would probably have had at the mean interval, or the instant arising from the division of the sum of the times of observation by their number. If the intervals between the successive observations are short, and nearly equal, the corresponding differences of altitude will also be either nearly equal, or gradually increasing or decreasing. Hence, an erroneous altitude may easily be discovered; thus, if the altitudes of an object were 19° 58′, 20° 4′, 20° 11′, and 20° 15, and the corresponding times of observation, 9h. 15′ 1″, 9h. 15′ 45″, 9h. 16′ 36″, and 9h. 17′ 30″, respectively; it

it may hence be presumed, that the three first altitudes are tolerably accurate, but that the last should have been about 20° 18′, and, therefore, such altitude should be absolutely rejected.

This method of deducing the mean altitude of the same limb of the Sun is, however, attended with an error arising from the irradiation, or apparent spreading of its rays beyond the true limb; and therefore, in order to diminish, if not wholly to destroy it, the mean of an equal number of altitudes of each limb, observed at nearly equal intervals of time, should be taken. As this error amounts only to a few seconds, it, therefore, becomes insensible in observations taken at sea.

Examination of a Quadrant.

If a quadrant be carefully made by a good artist, it may be very safely depended on; but as many quadrants are made by workmen careless of their reputation, it therefore becomes necessary to examine any instrument very particularly before it is purchased; or, if it be already purchased, to be able to make allowance, in subsequent observations, for any error to which it may be liable. For this purpose, it is hoped, the following remarks will be found of service.

The frame of the quadrant ought to be made of well seasoned wood, otherwise the action of the Sun, at the time of observation, will be ready to alter its plane; from this cause, an error may be produced in the observed angle. This alteration may be observed, by looking along the plane of the instrument. The frame should also be entirely free of rents, and every joint should be as close as possible.

The length of the scale on the limb should be adapted to the radius of the instrument; this may be examined in the following manner. Construct a scale of equal parts, as accurately as possible, upon which measure the length of the radius of the instrument, or distance, between the center of motion and the exterior circular line bounding the divisions; then will the extent between 0° and 90° be equal to the diameter of the instrument $\times .38628$. If, therefore, the correspondent number be taken from the scale of equal parts, and applied to that on the limb, it should reach from 0° to 90.

If the diameter of the instrument be divided decimally, the extent between any two divisions on the limb, will be equal to the sine of one fourth of the intercepted angle.

The divisions on the limb may be also verified, as follows. In a sandy beach, or level piece of ground, let two rods be placed in a vertical position, at a convenient and nearly equal distance from the observer; adjust the quadrant by the left hand rod, and observe, as accurately as possible, the angular distance between these rods; let this observation be repeated, and the mean of these measures will be the correct angle between the rods; now measure the distance between the center of the index glass and each rod, and also the distance between the rods as accurately as possible. Hence, the angle at the center of the index glass may be found by computation, the difference between which and that by observation, will be the error of the scale at that particular division. By increasing and diminishing the distance

between

between the rods, and, proceeding as above, it will be known if the error is proportionable to the arch; and, in any case, a table of errors may be drawn out.

The intermediate divisions may also be examined, by means of the index scale, provided it be of the proper length; which may be easily known, by multiplying the radius of the instrument by .05526, if that scale takes in 6° 20′, but by .06108, if it is equal to 7°. Now let one of the extreme divisions on the index scale be put to any degree, and the other extreme division will coincide with a division on the limb; and in this manner may the whole scale be verified. To assist the eye, while examining the coincidence of the divisions, a magnifying glass will be absolutely necessary; and it should be so held, that its axis may be perpendicular to the plane of the instrument, and over the division to be examined.

After having examined, by this or any other method, the length of the whole arch, and also of the intermediate divisions; if they are found erroneous, a table, containing the errors at each degree, should be constructed; which table is to be constantly used in correcting altitudes observed with this instrument.

The accuracy of the divisions of the index scale may be easily examined, by observing if the differences of coincidence between these divisions and those on the limb be regular, and that no two divisions on the index coincide, at the same time, with two on the limb, the extreme divisions excepted.

The index should be of a sufficient strength to prevent it, as much as possible, from bending, when moved along the limb; it should be entirely free of any shake or play at the center, nor should its motion there be stiff. Both surfaces of each mirror should be perfect planes, and strictly parallel to each other: the first of these requisites may be verified by means of two distant objects, as follows: move the index till both objects are exactly in contact at the upper edge of the silvered part of the horizon glass; now, the plane of the quadrant being still directed to the same objects, move it, in its own plane, so as to make the united images move along the line of separation of the horizon glass, and if in this motion the images continue united, the reflecting surface is a good plane, otherwise, the plane is imperfect. The parallelism of the planes of any of the reflecting surfaces may be examined, by viewing the image of the flame of a candle reflected obliquely from the glass; for, if that image appears single and well defined about the edge, the surfaces of the glass are parallel to each other; but if the reflected image is repeated, the planes of the glass are inclined, and if produced would meet. If this glass was constructed so as to be capable of reversion, the inclination of its planes might be found, and hence a table drawn up, to exhibit the error answering to any given angle.

The coloured glasses should be free of veins, and have their planes true and parallel. The veins in a coloured glass may be discovered by viewing the Sun through it, and the want of parallelism of the planes may be found in the following manner.

Observe

Observe the Sun's altitude when very near the meridian, and fasten the index by the screw for that purpose; then turn the coloured glass so, that the plane which was next the index glass may now be next the horizon glass, and if the altitude is the same, the planes of the glass are parallel to each other; if not, make the contact of the Sun's limb and horizon perfect, and half the difference of altitude will be the error of the coloured glass. The less the meridian altitude of the Sun, the more accurately this verification will be made; and it should not be attempted when the Sun is near the zenith.

CHAP. II.

Of the SEXTANT.

THE Sextant, as its name implies, is the sixth part of a circle, and, therefore, contains 60°; but, because of the double reflection, it is divided into 120°. It is constructed on the same principles as the quadrant, and may be said to be that instrument extended. As the sextant is particularly intended to measure the distance of the Moon from the Sun, or a fixed star, in order to find the longitude of a place by lunar observations; and as this distance must be obtained as accurately as possible, it is, therefore, constructed with more care, and provided with some additional appendages, that are wanting in the quadrant.

This instrument is commonly made of metal; the index and limb are of brass, but the frame is of a harder composition; the only wooden part belonging to it is a handle attached to the back, by which, when observing the distance between two objects, it is to be held with one hand, while the other is regulating the motion of the index.

A degree on the limb of this instrument, is commonly divided into three equal parts; each of which, therefore, contains 20′, and the index scale is divided so as to show half minutes: in some sextants, the degree is divided into six equal parts, hence each contains 10′, and the vernier shows 10″.

That end of the index next the limb is furnished with an *adjusting*, or, as it is sometimes called, a *tangent screw*, by which the index may be moved slowly and regularly, and, therefore, the contact of the limbs of any two objects may be made as perfect as the eye is able to distinguish, when assisted with one of the telescopes which accompany this instrument.

In some sextants, the lower half of the index glass, or that next the plane of the instrument, is silvered as usual, and the back surface of the other half is ground and painted black. A thin plate of brass, equal

equal in length and breadth to the silvered part of the speculum, and moving on an axis, is to be raised before that part of the index glass, when the Sun is very bright; in which case, the reflection is to be taken from the polished surface of the upper half of that glass, so that any error which might probably arise from the want of parallelism in the planes of the glass, is by this means avoided. This plate is also painted black.

A sextant is not fitted up for the back observation; and in some, the horizon glass is so fixed to the instrument, as to be incapable of adjustment. In instruments of this construction, it becomes absolutely necessary to find the index error.

The coloured glasses are similar to those applied to a common Hadley's quadrant, and are usually four in number; a sextant is, however, generally provided with three more, to be placed occasionally on the farther side of the horizon glass. These are particularly useful in observing the Sun's altitude by reflection at land; or in finding the index error by means of the Sun.

There are two telescopes belonging to the sextant, one of which shows objects in their natural position; and the other is of the common astronomical construction, and, therefore, shewing objects inverted. A tube, without glasses, also accompanies this instrument. By means of these, the line of sight may be rendered parallel to the plane of the sextant; and the contact of the limbs of any two objects more accurately observed. The tube, or either telescope, is to be screwed into a brass ring, which is connected with another brass ring, by means of two screws, in such a manner, as to raise or lower the telescope, in order that the line of collimation may be directed to a proper part of the horizon glass.

A circular head, containing a plate, in which are three coloured glasses, and an aperture without glass, sometimes accompanies the sextant. This head is to be screwed on the eye end of the tube, or on that of either telescope; the edge of the plate projects a little beyond the head on one side, and is moveable by the finger, so as the perforation, or either coloured glass, may be brought opposite to the circular aperture in the head.

Fig. 7th is a plan of the sextant, as described above; fig. 8th is one of the telescopes; fig. 9th is the field of view of the telescope, with the two parallel wires; fig. 10th is the tube; and fig. 11th is a section of the instrument at the line AB, in which D is a section of the handle, at its proper distance from the plane of the instrument.

Adjustments *of the* Sextant.

The adjustments of a sextant are, to set the mirrors perpendicular to its plane and parallel to each other, when the index is at zero, and to rectify the position of the line of collimation. The deviation of each of these from its true position might indeed be found, and the resulting error of observation, from thence computed; but this by no means ought to be admitted, unless in cases of absolute necessity.

Adjustment I.

To set the Index Glass perpendicular to the Plane of the Sextant.

Put the index to about 60°, and hold the plane of the sextant nearly parallel to the horizon, the limb being from the observer; then direct the sight to the speculum, and if the reflected limb of the instrument appears to be exactly in the same plane with that seen directly, the glass is perpendicular to the plane of the instrument. But if the limbs are not in the same plane, turn the screws in the projecting plane behind the speculum, till they are apparently so, and the glass will be adjusted. This adjustment might be effected by means of the adjusting tool.

Adjustment. II.

To set the Horizon Glass perpendicular to the Plane of the Sextant.

The index glass being previously adjusted, set the beginning of the divisions on the index to zero on the limb, and hold the plane of the instrument in a horizontal position; then direct the sight to the horizon glass, and if the reflected horizon is apparently in the same straight line with that seen directly, the glass is perpendicular to the plane of the sextant; otherwise, turn the adjusting screw of the horizon glass, at the back of the instrument, till the coincidence of the reflected and direct horizons is perfect.

Adjustment III.

To set the Horizon Glass parallel to the Speculum.

Set the first division on the index to zero on the limb; fasten the index in this position, and make the coincidence of these divisions perfect, by means of the adjusting screw at the end of the index, the eye being assisted with the magnifying lens; screw the telescope in its support, and turn the screw belonging thereto, at the back of the instrument, till the field of the telescope is bisected by the line which separates the silvered and transparent parts of the horizon glass; now hold the sextant vertically, direct the sight to the horizon, and if the reflected and direct horizons do not coincide, release the screw which lies nearly behind the support of the telescope, and turn the nut at the extremity of the lever, till the coincidence of the horizons is perfect; and the horizon glass will be adjusted.

After the screw which contains the lever in its place is fastened, it will be proper to examine this adjustment; if the coincidence of the horizons is not perfect, the adjustment is to be repeated till it is so; but as it is difficult to obtain an exact coincidence by this means, the horizons may be brought to coincide, by turning the adjusting screw of the index, and the difference between the two zeros is the index error.

The index error may also be found, by measuring the diameter of the Sun or Moon twice, with a motion of the index, in contrary directions. If both measures are taken either to the right or left of O on the limb, half their sum will be the index error, and is additive or subtractive

tractive accordingly; but if one of the measures is taken to the right, and the other to the left of O, half their difference is the index error, being additive when the diameter measured to the right of O exceeds that measured to the left; otherwise, subtractive. Since in some sextants the horizon glass cannot be adjusted, the index error must, therefore, be found; and in this case it may be considered as a constant quantity, to be applied to all angles measured with the same instrument.

In altitudes observed at land by the method of reflection, the double altitude affected by the index error is given by the instrument; in this case, it will be found convenient to call half the index error, the correction, which being applied to half the angle given by this instrument, will give the apparent altitude of the object.

Adjustment IV.

*To make the Line of Collimation * parallel to the Plane of the Sextant.*

Turn the eye end of the telescope, containing the two parallel wires, till the wires are parallel to the plane of the instrument; and let two distant objects be selected, as two stars of the first magnitude, or the Sun and Moon, whose distance may not be less than 90° or 100°; make the contact of the limbs of these objects as perfect as possible; at the wire nearest the plane of the instrument; fix the index in this position; move the sextant till the objects are at the other wire, and if the same limbs are in contact, the axis is adjusted; but if the limbs are either apparently separated, or partly cover each other, correct half the error by the screws in the circular part of the supporter, one of which is above, and the other between the telescope and sextant; turn the adjusting screw at the end of the index, till the limbs are in contact; then bring the objects to the wire next the instrument, and if the limbs are in contact, the axis of vision of the telescope is parallel to the plane of the instrument; if not, proceed as at the other wire, and continue till no error remains.

If the Moon is one of the objects, this adjustment must be performed as quick as possible; otherwise, allowance must be made for the Moon's apparent motion in the interval.

This adjustment may also be performed as follows: lay the sextant on one end of a large table, whose surface is an exact plane; then take the adjusting tool, and raise the circular hole to the height of the middle of the eye end of the telescope, and place it at the other end of the table: direct the telescope to the tool, make the wires parallel to the plane of the instrument, and adjust the telescope to as distinct vision as possible. Now, if the circular hole in the tool is apparently in the middle between the wires, the axis of the telescope is parallel to

* The line of collimation is an imaginary straight line, joining the center of refractions of the object glass of a telescope, and either the intersection of the cross wires, or middle between the parallel wires.

the

the plane of the instrument; if not, one half of the error is to be corrected, by raising or lowering the slide containing the circular hole, and the other half by the screws in the socket containing the telescope. For the sake of accuracy, this adjustment is to be repeated, by making the circular part of the tool of the same height as the eye end of the telescope, and proceeding as before, until no error remains.

This adjustment may also be made, by looking directly to the Sun through the telescope; then, if the shadow of the farther radius exactly covers the hither radius, or middle verticle bar, at the same time that the Sun appears in the middle between the wires, the telescope is right; otherwise, it must be adjusted by the wires in the circular part of the supporter.

The error of the line of collimation being given, the resulting error of observation may be found, by entering table xxii. with the above error at the top, and the observed distance in the side column.

It is sometimes necessary to know the angular distance between the wires of the telescope; to find which, let these wires be placed perpendicular to the instrument, hold the sextant verticle, direct the sight to the horizon, and move the sextant in its own plane, till the horizon and upper wire coincide; keep the sextant in this position, and move the index till the reflected horizon is covered by the lower wire, and the division shown by the index on the limb, corrected by the index error, will be the angular distance between the wires.

Use *of the* Sextant.

The sextant is particularly adapted to measure the angular distance between the Moon and the Sun, or a fixed star. When the distance between the Moon and either of these objects is to be observed, the sextant must be held so, that its plane produced may pass through the eye of the observer and both objects; and the reflected image of the most luminous of the two, is to be brought in contact with the other seen directly. To effect this, therefore, it is evident, that, when the brightest object is to the right of the other, the face of the sextant must be held upwards; but if to the left, downwards. When the face of the sextant is held upwards, the instrument should be supported with the right hand, and the index moved with the left; but when the face of the sextant is from the observer, it should be held with the left hand, and the motion of the index regulated by the right hand.

Sometimes a sitting posture will be found very convenient for the observer, particularly when the reflected object is to the right of the direct one; in this case, the instrument is to be supported by the right hand, the elbow may rest on the right knee, the right leg at the same time resting on the left knee.

If the sextant is provided with a ball and socket, and a staff, one of whose ends is attached thereto, and the other resting in a belt fastened round the waist of the observer; the greater part of the weight of the instrument may, by this means, be supported by his body.

To

To observe the Distance between the Moon and any celestial Object.

I.

Between the Sun *and* Moon.

Put the telescope in its place, and the wires parallel to the plane of the instrument; then, if the index glass is half silvered and half blacked, and if the sun is very bright, raise the plate before the silvered part of the speculum; direct the telescope either to the transparent part of the horizon glass, or to the line which separates the silvered and transparent parts of that glass, according to the brightness of the Sun, and turn down one of the coloured glasses; then hold the sextant * so, that its plane produced may pass through the Sun and Moon, having its face either upwards or downwards, according as the Sun is to the right or left of the Moon; direct the sight through the telescope to the Moon, and move the index till the limb of the Sun is nearly in contact with the enlightened limb of the Moon; now fasten the index, and by a slow motion of the instrument make the image of the Sun move alternately past the Moon, and, when in that position where the limbs are nearest each other, make the coincidence of the limbs perfect, by means of the adjusting screw. This being effected, read off the degrees and part of a degree pointed out on the limb by the index, using the magnifying glass; and thus the angular distance between the limbs of the Sun and Moon is obtained.

II.

Between the Moon *and* Star.

Direct the middle of the field of the telescope, to the line of separation of the silvered and transparent parts of the horizon glass; if the Moon is very bright, turn down the lightest coloured glass, and hold the sextant so, that its plane may be parallel to that passing through the eye of the observer and both objects; with its face upwards, if the Moon is to the right of the star; but if to the left, the face of the sextant is to be held from the observer. Now, direct the sight through the telescope to the star, and move the index, till the moon appears by reflection to be nearly in contact with the star; fasten the index, and turn the adjusting screw, till the coincidence of the star and enlightened limb of the Moon is perfect; and the degrees and parts of a degree, shown by the index, will be the observed distance between the Moon's enlightened limb and the star.

The contact of the limbs must always be observed in the middle between the parallel wires.

* A sextant, under the name of a *double* or *improved* Sextant, has been lately invented by Mr. E. Hoppe, Church-street, Minories, London. This instrument has a second arch, with an index, by which means observations may be multiplied, and the errors of adjustment accurately ascertained.

It

It is sometimes difficult, for those not much accustomed to observations of this kind, to find the reflected image in the horizon glass; it will, perhaps, be found more convenient to look directly to the object, and by moving the index, to make its image coincide with that seen directly; or, if the distance between the objects be known nearly, the index may be set to this distance; the sight being directed to one of the objects, and the sextant held as formerly described, and the other object will be seen in the field of the telescope.

CHAP. III.

Of the Circular Instrument *of* Reflexion.

THIS instrument was invented by the late celebrated astronomer, Professor Mayer; and since, has been greatly improved by the Chevalier de Borda, M. Jean Hyacinth de Magellan, Captain Mendoza, Mr. Troughton, and others. The circular instrument, or, as it is sometimes called, the *multiplying circle*, is evidently preferable to a quadrant or sextant, in as much as the observation of the distance between any two objects, may be repeated round the whole of the limb, and the errors of the instrument thereby diminished, or rendered almost insensible.

This instrument consists of the following parts; a circular ring or limb, two moveable indices, two mirrors, a telescope, coloured glasses, &c.

The limb of the instrument is a complete circle of metal, and is connected with a perforated central plate by six radii; it is divided into 720 degrees, because of the double reflection; each degree is divided into three equal parts, and the division is carried to minutes, or lower, by means of the index scale, as usual.

The two indices are moveable round the same axis, which passes exactly through the center of the instrument; the first index carries the central mirror, and the other, the telescope and horizon glass; each being provided with an adjusting screw for regulating its motion, and a scale for shewing the divisions on the limb.

The central mirror is placed on the first index, immediately above the center of the instrument, and its plane is inclined to the middle line of the index about 30°. The four screws in its pedestal, for making

making its plane perpendicular to that of the instrument, have square heads, and are, therefore, easily turned either way by a key for that purpose.

The horizon glass is placed on the second index near the limb, so that as few as possible, may be intercepted, of the rays proceeding from the reflected object, when to the left. The perpendicular position of this glass is rectified in the same manner, as that of the horizon glass of a sextant, to which it is similar. It has also another small motion, whereby its plane may be disposed so, as to make proper angles with the axis of the telescope, and a line joining its center, and that of the central mirror.

The telescope is attached to the other end of the index. It is an achromatic astronomical one, and therefore inverts the observed objects; it has two parallel wires in the common focus of the glasses, distant from each other between two and three degrees, and which, at the time of observation, must be placed parallel to the plane of the instrument. This is easily done, by making the mark on the eye piece, coincide with that of the tube. The telescope is moveable by two screws in a vertical direction with regard to the plane of the instrument, but is not capable of receiving a lateral motion.

To this instrument belong two sets of coloured glasses, each containing four, and differing in shade from each other. The glasses of the larger set, which belong to the central mirror, should have each about half the degree of shade with which the correspondent glass, of the set belonging to the horizon mirror, is tinged; because the ray from the luminous object passes twice through the coloured glass placed before the central mirror, and only once through the other coloured glass. These glasses are kept tight in their places, by small pressing screws, and make an angle of about 85° with the plane of the instrument; by which means, the image from the coloured glass is not reflected to the telescope. When the angle to be measured is between 5° and 35°, one of the glasses of the largest set is to be used; in other cases, one of the less set is to be placed before the horizon glass.

The handle is of wood, and is screwed to the back of the instrument, immediately under the center, with which it is to be held at the time of observation.

Fig. 12. is a plan of the instrument, wherein the limb is represented by the divided circular plate; A, is the central mirror; *a,a* the places which receive the parts *a,a* of the coloured glass, fig. 15. EF the first or central index, with its scale and adjusting screw; MN the second or horizon index; GH the telescope; I,K the screws for moving it to, and from, the plane of the instrument; C the place of the coloured glass, fig. 14.; and D its place in particular observations.

Fig. 13. is a section of the instrument, wherein the several parts are referred to by the same letters as in fig. 12; fig. 14 represents one of the horizon coloured glasses; and fig. 15. one of the central coloured glasses; fig. 16. is the key for turning the adjusting screws of the mirrors; fig. 17. is the handle; fig. 18. a section of one of the radii,

radii, towards its middle; fig. 19. the *ventelle*, which is only used in terrestrial observations, for diminishing the light of the direct object, and where place at the time of observation is D; fig. 20. is the tool for adjusting the central mirror vertically, and for rectifying the position of the telescope with regard to the plane of the instrument. There is another tool exactly of the same size of the former; the height of these is nearly equal to that of the middle of the central mirror.

ADJUSTMENTS *of the* CIRCULAR INSTRUMENT.

The tendency of these adjustments is, to make the mirrors perpendicular to the plane of the instrument; to make the line of collimation parallel thereto; to prevent any false light from entering the telescope, &c. Some of these adjustments being once made, will continue perfect, for a considerable space of time, provided the instrument receives no injury; but it will be necessary to examine others from time to time.

ADJUSTMENT I.

To set the Horizon Glass so, that none of the Rays from the central Mirror shall be reflected to the Telescope from the Horizon Mirror, without passing through the coloured Glass belonging to this last Mirror.

Place the coloured glass before the horizon mirror, direct the telescope to the silvered part of that mirror, and make it nearly parallel to the plane of the instrument; move the first index, and if the rays from the central mirror to the horizon glass, and from thence to the telescope, have all the same degree of shade with that of the coloured glass used, the horizon glass is in its proper position; otherwise, the pedestal of this glass must be turned, till the uncoloured images disappear.

ADJUSTMENT II.

To set the central Mirror perpendicular to the Plane of the Instrument.

Place the two adjusting tools on the limb, about 350° of the instrument distant, one on each side of the division on the left, answering to the plane of the central mirror produced; then, the eye being placed at the upper edge of the nearest tool, move the central index, till only half of the reflected image of this tool is seen in the central mirror towards the left, and move the other, till its half to the right is hid by the same edge of the mirror; then if the upper edges of both tools are apparently in the same straight line, the central mirror is perpendicular to the plane of the instrument; otherwise, bring them into this position, by the screws in the pedestal of the mirror.

This adjustment may be performed in the same manner as used in setting the speculum of a sextant or quadrant perpendicular.

Hence

Hence, by examining this adjustment in different parts of the limb, it will be known if the limb be in the same plane.

Adjustment III.

To set the Horizon Mirror perpendicular to the Plane of the Instrument.

The central mirror being previously adjusted, direct the sight through the telescope to any well defined object; then if, by moving the central index, the reflected image passes exactly over the object seen directly, the mirror is perpendicular; if not, its position must be rectified by means of the screws in the pedestal of the glass.

A planet, or star of the first magnitude, will be a good object for this purpose; the Sun will also be found very convenient for making this adjustment; in which case, one of the coloured glasses must be put on the farther side of the horizon mirror.

This adjustment may be also performed by observing, if the horizon of the sea, seen in the silvered part of the horizon glass, and by the edge of that glass, appears to make one continued straight line.

Adjustment IV.

To make the Line of Collimation parallel to the Plane of the Instrument.

Lay the instrument horizontally on a table; place the two adjusting tools on the limb, towards the extremities of one of the diameters of the instrument; and, at about 15 or 20 feet distant, let a well defined mark be placed, so as to be in the same straight line with the tops of the tools; then raise or lower the telescope, till the plane passing through its axis and the tops of the tools is parallel to the plane of the instrument, and direct it to the fixed object; turn either or both of the screws of the telescope, till the mark is apparently in the middle between the wires; then is the axis of the telescope parallel to the plane of the instrument, and the difference, if any, between the divisions pointed out by the indices of the screws, will be the error of the indices. Hence, this adjustment may, in future, be easily rectified.

In this process, the eye tube must be placed so as to obtain distinct vision.

Adjustment V.

To find that Division, to which the second Index being placed, the Mirrors will be parallel, the central Index being at Zero.

Having placed the first index exactly to O, direct the telescope to the horizon mirror so, that its field may be bisected by the line joining the silvered and transparent parts of that mirror; hold the instrument vertically, and move the second index, till the direct and

reflected horizons agree; and the division shown by the index will be that required.

This adjustment may also be performed by measuring the diameter of the Sun or Moon in contrary directions. The middle between the divisions on the limb will be that required; or it may be effected, by making the reflected and direct images of a star or planet coincide.

Since it very often happens, that the contact of the objects cannot be observed exactly in the middle of the field of the telescope, it, therefore, becomes necessary to know the interval between the point where the contact was observed, and the middle of the field, expressed in minutes. This may be estimated, by knowing the angular distance between the wires, which may be thus determined:

Turn round the eye-piece of the telescope, till the wires are perpendicular to the plane of the instrument, and put the first index to O: direct the telescope to any well defined object, and move the horizon index, till the reflected and direct objects are coincident; then make one of the wires coincide with the object, and turn the central index, till the reflected image of the object coincides with the other wire; and the arch passed over by that index will be the angular distance between the wires.

The interval between the point where the contact of the limbs was observed, and the middle of the field, being estimated, the corresponding error of observation is found directly from Table XXII.

Verification of the Parallelism of the Surfaces of the Glasses.

I.

Of the CENTRAL MIRROR.

Select two well defined objects, whose angular distance exceeds 90° or 100°; now the instrument being accurately adjusted, let several observations of the angular distance between the objects be taken, in the manner hereafter to be described, and from thence find the mean angular distance. Then take out the central mirror, and turn it so, that the edge which was formerly uppermost may now be next the plane of the instrument; rectify its position, and take an equal number of observations of the angular distance of the same two objects; now half the difference between the mean of these, and that of the former, will be the error of the mirror answering to the observed angle. If the first mean exceeds the second, the error is subtractive; otherwise additive, the mirror being in its first position.

The error corresponding to any other angle may be found thus. Add together the proportional logarithm of the error found as above, the proportional logarithm of the error answering to the given distance from col. 4. tab. XXI. and the arithmetical complement of the proportional logarithm of the error corresponding to the observed distance from the same table; the sum, rejecting 10 in the index, will be the proportional logarithm of the required error. Hence a particular table

table of errors for a given mirror, whose planes are inclined, may be constructed.

Since the angle between the plane of the horizon glass and axis of the telescope produced, is constant in all observations, therefore, no error arises from the want of parallelism in its surfaces.

II.

Of the Coloured Glasses.

Place one of the coloured glasses on this, and another on the farther side of the horizon mirror, with respect to the telescope; set the first index to O, direct the telescope to the Sun, and move the second index till the limbs of the direct and reflected images coincide; observe at least five or six contacts, and from thence find the mean point of division answering thereto; then turn the nearest coloured glass, so that the surface which was from the horizon mirror may now be towards it; observe the contact of the same two limbs as often as before, and find the mean division; then half the difference between these two means will be the error of the nearest coloured glass, which is additive or subtractive, according as the first mean is less or greater than the second, the coloured glass being in its first position. In like manner, the parallelism of the surfaces of the other coloured glasses may be examined, and that of the green glass may be verified by means of the Moon.

As in cross observations, the errors arising from the coloured glass are nearly counterbalanced, therefore, in this case the above error may be neglected. This error will also vanish in observations to the right, or to the left, by taking an equal number, and changing the position of the coloured glass at each observation.

Use *of the* Circular Instrument.

There are three different methods of observing the angular distance between two objects with this instrument. The observation taken by the first method is named *an Observation to the Right;* that by the second, *an Observation to the Left;* and the last is called *a Cross Observation.*

An observation to the right is that, wherein the object, whose image is to be reflected, and the central mirror are on the same side of the telescope. An observation to the left, when the object to be reflected, and the central mirror, are on opposite sides of the telescope; which, in both cases, is supposed to be directed to the other object: and a cross observation is the combination of two successive observations, the one being to the right, and the other to the left.

General Precepts,

For observing the Angular Distance between any two Objects.

Adjust the instrument as before directed; then put the first index to O, and the other index so, that both mirrors may be parallel; hold

the instrument in such a position, that its plane produced may pass through both objects, having its face upwards, if the reflected object is to the left of the other, but downwards, if to the right; direct the telescope to the object which is apparently the least luminous, and move the second index according to the order of the divisions, till the limb of the reflected object is in contact with that of the object seen directly; fasten the second index, and make the coincidence of the limbs perfect, by means of the adjusting screw; and the arch passed over by that index, will be nearly equal to the distance between the limbs of the objects. Now move the first index towards the second, by a quantity equal to twice the measured distance; invert the position of the instrument, by directing its face to the opposite point; direct the telescope to the faintest object, and both will be seen in the field; make the contact of the same two limbs perfect, and half the angle shown by the central index on the limb will be the angular distance between the limbs of the objects.

If the Sun is one of the objects, a coloured glass must be placed before the central mirror, when the distance is between 5° and 35°. In other distances, one of the coloured glasses belonging to the horizon mirror is to be used.

In order to render the observations as accurate as possible, it will be necessary to continue measuring the angular distance as above, till the central index has passed once or twice over the limb; and the degrees and parts of a degree, contained in the revolutions and excess of a revolution, being divided by twice the number of cross observations, will be the angular distance between the objects. On this, the principal advantage of the circular instrument depends.

It might seem superfluous to enlarge upon the use of this instrument; however, as some may wish to have instructions for every particular operation, it is, therefore, thought proper to subjoin the following.

Particular Precepts.

I.

To observe the Distance between the Sun and Moon.

1st, The Sun *being to the* Right *of the* Moon.

Prepare the instrument for observation, as directed above; set a proper coloured glass before the central mirror, if the distance between the objects is less than 35°, but if above that quantity, place a coloured glass before the horizon mirror; make the mirrors parallel or nearly so, the first index being at O, and hold the instrument so, that its plane may be directed to the objects, with its face downwards, or from the observer; direct the sight through the telescope to the Moon; move the second index, according to the order of the divisions on the limb, till the nearest limbs of the Sun and Moon are almost in contact; fasten that index, and make the coincidence of the limbs

limbs

limbs perfect, by the adjusting screw belonging thereto; then invert the instrument, and move the central index towards the second, by a quantity equal to twice the arch passed over by that index; direct the plane of the instrument to the objects, look directly to the Moon, and the Sun will be seen in the field of the telescope; fasten the central index, and make the contact of the same two limbs exact, by means of the adjusting screw; then half the arch passed over by the central index will be the distance between the nearest limbs of the Sun and Moon.

2d, *The* Sun *being to the* Left *of the* Moon.

The instrument being previously prepared, hold it with its face upwards, so that its plane may pass through both objects; direct the telescope to the Moon, and make its limb coincide with the nearest limb of the Sun's reflected image, by moving the second index; now put the instrument in an opposite position, direct its plane to the objects, and the sight to the Moon, the central index being previously moved towards the second, by a quantity equal to twice the measured distance; and make the same two limbs that were before observed, coincide exactly, by turning the adjusting screw of the first index; then half the angle shown by the first index will be the angular distance between the observed limbs of the Sun and Moon.

II.

To observe the Angular Distance between the Moon and a fixed Star or Planet.

1st, *The* Star *being to the* Right *of the* Moon.

In this case, the star is to be considered as the direct object, and the enlightened limb of the Moon's reflected image is to be brought in contact with the star or planet, both by a direct and inverted position of the instrument, exactly in the same manner as described in the last article. If the Moon's reflected image is very bright, the lightest tinged glass is to be used.

2d, *The* Star *being to the* Left *of the* Moon.

Proceed in the same manner as directed for observing the distance between the Sun and Moon, the Sun being to the right of the Moon, and using the lightest tinged glass, if necessary.

Beside the instruments already described, for measuring the angular distance between the Moon and the Sun, or a fixed star, several others have been proposed for the same purpose, particularly the *Megameter*, by M. de Charnieres, which is constructed on the same principle as the object glass of the micrometer; but since this instrument does not measure angles above 10°, and because the distances given

in

in the Nautical Almanac always exceed that quantity, we, therefore, shall not enter upon a description of it, but refer to his treatise, entitled, *Theorie et Pratique des Longitudes en Mer*, printed at Paris in 1772. A description of Mr. Garrard's *Antimeter, or Reflecting Circle*, may be seen in his pamphlet, published in 1785; and in the article Antimeter, by the Author of this work, in the Supplement to the Encyclopedia Britannica.

CHAP. IV.

Of the Manner *of taking a complete Set of* Lunar Observations.

In order to take a set of lunar observations, in a regular and accurate manner, three assistants will be necessary, whereof two are to observe the altitudes of the Moon, and of the Sun or star, at the same time that the distance is taken by the principal observer; the other assistant, having a watch showing seconds, is to mark the time when these observations are made. If the Sun or star is at a proper distance from the meridian, the time may be inferred from the altitude observed at the same time with the distance, and, therefore, the watch is not necessary; but if the object with which the Moon is compared, be too near the meridian, the watch is absolutely necessary, in order to connect the observations for ascertaining the apparent time at the ship, and the longitude, with each other. This last assistant must be provided with a pencil and paper, to write down the observations as they are taken, which may be as follows:

The sextant and quadrants being accurately adjusted, or the error of adjustment found, set their indices to the estimate distance and altitudes respectively. Now, let the principal observer place his assistants in the most convenient situation possible, and desire them to be prepared when he is ready to observe the distance; then all are to begin to observe at the same time, and when the principal observer has brought the nearest limbs of the Sun and Moon, or the enlightened limb of the Moon and a star into contact, he is to ask the other observers, if they are all ready; and being answered in the affirmative, then, as soon as he has obtained a perfect coincidence of the limbs of the objects, he is to make it known to his assistants, by calling out any particular word, as *now*, or *done*. *—The person having the watch is

* The French call out *top*, which is to the same import.

immediately

immediately to write down, first the second, and then the minute of observation, the hour being previously marked; the principal observer is to read off the distance, using the magnifying glass, which he is to communicate to the time assistant, that it may be wrote after the time of observation. The other assistants are, in like manner, to report the observed altitudes of the limbs of the objects separately. Hence, a complete set of observations is obtained. In this manner, let these observations be repeated, till at least four or five different sets are observed, which may be easily accomplished in the space of 6 or 8 minutes; and these sets ought to be taken at nearly equal intervals of time. The mean of each particular observation is to be taken; that is, the sum of each is separately to be divided by the number of sets; and the longitude of the ship is to be inferred from thence, in the same manner as from a single set, but much more to be depended on.

To illustrate the above, the following observations, taken in the manner described, are subjoined.

☽ March 2, 1789. Height of the eye 18 feet.

Times per watch.	Dist. ☉ and ☽'s nearest limbs.	Alt. ☽'s up. limb.	Alt. ☉'s low. limb.
23h. 8′ 31	71° 50′ 10″	19° 5′	24° 56′
11 18	52 20	19 26	25 1
13 15	53 0	19 45	25 5
15 1	53 50	20 2	25 9
16 36	54 30	20 15	25 13
Sum 14 41	14 50	213	24
Mean 23 12 56	71 52 58	19 42.6	25 4.8

The astronomical method of reckoning the time has been followed, in order to make it agree with the Nautical Almanac. The mean of the above times reckoned nautically would be, March 3d, 11h. 12′ 56″, A. M.; and if the above mean had been March 3d, 1h. 13′; then, by the nautical account, it would be, March 4th, 1h. 13′, P. M. Hence, in the present case, the astronomical method of reckoning time seems preferable to the nautical method.

An expert observer may, in case of necessity, diminish the number of his assistants, and might even be capable of making all the necessary observations himself, with a tolerable degree of accuracy, as follows:

Let two or three altitudes of the limbs of the Sun and Moon be taken, and write down the time of each; then observe five or six distances between the limbs of these objects; and lastly, several altitudes of each are to be again observed. Now, find the mean of each particular set of observations, and reduce the altitudes to the time of the mean distance, which may be done by even proportion, the motion in altitude being supposed uniform during the interval between the observations; and hence a complete set will be obtained.

The

The following observations being given, it is required to reduce the altitudes to the time of the mean distance.

Times per watch.					
0,h. 24′ 31″	Alt. ☉'s l. limb =	54° 6′	}	Means, 0,h. 25′ 37″—54° 5′
25 41	— — — —	54 5		
26 40	— — — —	54 4		
27 50	Alt. ☽'s upper limb	20 2	}	0,h. 28′ 47″—20° 3.3
28 44	— — — —	20 3		
29 46	— — — —	20 5		
31 10	Dist. nearest limbs	73 13	}	0,h. 33′ 43″—73° 13′ 57″
32 50	— — — —	73 13½		
33 30	— — — —	73 14		
35 0	— — — —	73 14½		
36 40	— — — —	73 14¾		
37 15	Alt. ☉'s l. limb	53 54	}	0,h. 38′ 24″—53° 52′.7
38 20	— — — —	53 53		
39 38	— — — —	53 51		
41 12	Alt. ☽'s up. limb	20 57	}	0,h. 42′ 14″—21° 4′.3
42 4	— — — —	21 4		
42 36	— — — —	21 12		

0,h. 25′ 37″ — —	0,h. 25′ 37″ — —	54° 5′	
0,h. 38 24 — —	0,h. 33 43 — —	53 52.7	
12 47 :	8 6	: :	12.3 8′
Sun's altitude at 0h. 25′ 37″	— — —		54 5
* Reduced altitude of Sun's lower limb	—		53 57

0,h. 28′ 47″ —	0,h. 28′ 47″ —	20° 3′.3	
0,h. 42 14 —	0,h. 33 43 —	21 4.3	
13 27 :	4 56 : :	1 1	: 22′.3
Moon's altitude at 0h. 28′ 47″	— — —		20 3.3
Reduced altitude of the Moon's upper limb	— —		20 25.6

We have now obtained the following set of observations:

Times p. watch.	Dist. nearest limbs of ☉ and ☽	Alt. ☉'s l. limb.	Alt. ☽'s up. limb.
0,h. 33′ 43″	73° 13′ 57″	53° 57′	20° 25.6.

One set of altitudes of the Sun and Moon would be sufficient, provided the azimuths of the objects be known,† as then the change of altitude, in the interval between the observations of the altitude and distance, may be found by computation as follows:

To the constant logarithm 8.8239, add the log. secant of the latitude, the log. co-secant of the azimuth, and the proportional logarithm of the interval of time between the observations of the altitude and distance; the sum, rejecting radius, will be the proportional logarithm of the corresponding change of altitude.

* When the object is near the meridian, as the Sun in the present example, the change of altitude is not proportionable to the time. However, as these altitudes are to be used only in clearing the apparent distance, and because no material error can arise in the distance, from an error of a few minutes in the altitude; therefore, the rule of proportion will be still found sufficiently accurate for the above purpose, when the interval of time is short. The change of altitude may be more accurately computed by the rule given in the text.

† Sine of azimuth = secant altitude. Co-sine latitude. Sine dist. object from the meridian, to radius 1.

If

If the Sun is near the meridian at the time when the distance is observed, the apparent time cannot be deduced from thence with a sufficient degree of accuracy; if, therefore, the error of the watch is not known, altitudes of that object, when at a proper distance from the meridian, should have been taken before the observation of the distance, or must be taken afterwards, for the express purpose of ascertaining its error. The apparent time might, indeed, be inferred from the Moon's altitude; but, for several reasons, it is not so much to be depended upon, as that deduced from the Sun's altitude.

If, in places where the Sun rises and sets daily, the observations are taken any time between half an hour after sun-rise and ten o'clock in the forenoon, or between two in the afternoon, and half an hour before sun-set, the apparent time may be inferred from the Sun's altitude. In this case a watch is not absolutely necessary.

A set of observations may be taken with accuracy during the time of the evening or morning twilight; and the observer, though not much acquainted with the stars, will not find it difficult to distinguish the star with which the Moon is to be compared; for the time of observation, and the ship's longitude by account, being known, the estimate time at Greenwich may be found; and by entering the Nautical Almanac with the reduced time, the distance between the Moon and the given star will be found nearly. Now, set the index of the sextant to this distance, and hold the plane of the instrument so, as to be nearly at right angles to the line joining the Moon's cusps; then, having found the Moon in the silvered part of the horizon glass, give the sextant a slow vibratory motion, the axis of which is that of vision; and the star, which is usually one of the brightest in that part of the heavens, will be seen in the transparent part of the horizon glass; the Moon, by the above motion, apparently moving alternately up and down with respect to the star. But if the star is known, from which the Moon's distance is to be observed, the most convenient, and, at the same time, the most expeditious method will be, to find the Moon in the silvered part of the horizon glass, the index being at zero, and then to move the index until the enlightened limb of the Moon is in contact with the star.—See page 69.

In many places of the Earth, the Sun and Moon will be nearly in a vertical position at some particular time of the day; when they are in this situation, and the Sun highest, the distance between them may be observed with greater accuracy than in any inclined position: after new Moon, the Sun will be highest before noon, and before new Moon, in the afternoon. This is also applicable to the distance between the Moon and a star, the Moon being highest.

In taking the means of the several observations, those which are evidently doubtful, or erroneous, ought absolutely to be rejected. A doubtful altitude or distance may be easily discovered, by observing, if the successive differences of altitude or distance be proportional to those of the times of observation, which are supposed to be accurately marked. If, however, the time, and two of the other observations be

correct, the error of the third may be inferred, by the method given in page 62. It may also be observed, that in order to attain the greatest accuracy in deducing the mean from a series of observations, they ought to be taken at, as nearly as possible, equal intervals of time, as *one, one and a half*, or *two minutes*.

CHAP. V.

Of the Corrections to be applied to the Altitude of an Object observed at Sea, and to the observed Distance between two Objects.

THE altitude of the limb of a celestial body, observed at sea, quires four separate corrections, independent of those arising from the errors of the instrument, in order to obtain the true altitude of its center. These are on account of the semidiameters, Depression of the Horizon, Refraction, and Parallax. The semidiameter and parallax, however, vanish, when the observed object is a fixed star. The distance between the limbs of the Sun and Moon, or between a star and the Moon's limb, is affected by refraction and parallax; but the distance between two stars is affected by refraction only.

Of the SEMIDIAMETERS *of the* SUN, MOON, *and* PLANETS.

The semidiameter of the Sun is given in the Nautical Almanac, page III. of the month for the instant of apparent noon of every sixth day, and that of the Moon in page VII. to the time of noon and midnight, by the meridian of Greenwich. These semidiameters may be reduced to any intermediate time, and to any other meridian, by proportion.

The Moon's semidiameter, as given in the Ephemeris, is the angle under which it would be seen when in the horizon, or from the center of the Earth; but the apparent magnitude of an object is inversely as its distance from the observer; and since the Moon is nearer an observer, by a complete semidiameter of the Earth, when in the zenith, than it is when in the horizon; and because the semidiameter of the Earth has a sensible ratio to the distance of the Moon from its center, the semidiameter of the Moon will be apparently increased by a quantity equal to $\frac{s^2}{15^\circ\ 46'\ 45''-s}$, wherein s is the horizontal semidiameter of the Moon. This quantity is called the greatest *augmentation*; and the augmentation at any given altitude will be equal to the sine of that altitude, the radius being the greatest augmentation. Table XXXI. contains the augmentation of the Moon's semidiameter, to every third degree of apparent altitude of the Moon; and Table

XXX.

XXX. contains the augmentation adapted to the altitude of the Moon, and to the longitude of the nonagesimal, which is useful in the calculation of eclipses and occultations.

The distance of the Sun from the Earth being immense, when compared with the Earth's radius, the augmentation of the Sun's semidiameter is, therefore, insensible.

When the altitude of the lower limb of the Sun or Moon is observed, its semidiameter must be added thereto; but if the upper limb be observed, the semidiameter must be subtracted, in order to obtain the central altitude. The contrary rule is to be applied to altitudes taken by the back observation. In altitudes of the Sun, taken for the purpose of ascertaining the latitude, and when no great precision is required, 16 minutes may be allowed. If a greater degree of accuracy is sought, the semidiameter may be taken from the Nautical Almanac to the nearest tenth of a minute. But when altitudes are observed at land with a good instrument, the semidiameter is to be taken out to the nearest tenth of a second.

If the distance between the nearest limbs of the Sun and Moon is observed, their apparent semidiameters must be added thereto, in order to obtain the central distance. If the nearest limb of the one of these objects is compared with the remote limb of the other, the apparent semidiameter of the first must be added to, and that of the second subtracted from, the observed distance. If the distance between the Moon and a fixed star be observed, the Moon's augmented semidiameter must be added to, or subtracted from, the observed distance, according as the nearest or remote limb of the Moon is compared with the star.

The apparent semidiameter of a planet is variable. This variation depends on the distance of the planet from the Earth; and since the apparent semidiameter of any of the planets never exceeds half a minute, the altitude of the center may, therefore, be observed with sufficient accuracy. A fixed star has no sensible magnitude, even when viewed with a very powerful telescope.

Of the Depression *of the* Horizon.

The depression, or, as it is commonly called, the *dip of the horizon*, is the angle contained between the sensible and apparent horizons, the angular point being the eye of the observer.

Since the altitude of any object, observed at sea, is measured from the apparent horizon, and as this horizon is below the sensible, by a quantity depending on the height of the eye, the altitude of a celestial object, as taken by the fore observation, is therefore greater, but less when the back observation is used, by a quantity equal to the angle contained between these horizons.

Illustration.

Let ABD, (fig. 21.) be a section of the Earth, whose plane produced passes through the observer and object, and let BE be the height of the observer's eye above the Earth's surface; hence FEG will be the

the sensible horizon, EHI the apparent horizon, and therefore, the angle FEI the depression of the apparent, below the sensible horizon. Let O be any object whose altitude is to be observed, which is done by bringing its image in contact with the ray from the eye to the apparent horizon produced; and therefore the angle OEI is the observed altitude, being greater than the angle OEF, the altitude unaffected by dip, by the angle FEI. In the back observation, the observed altitude is OEK, to which the angle HEF = GEL must be added, to obtain the altitude above the sensible horizon FE.

The dip may be computed by the following rule.

To the constant logarithm 0.4236, add the proportional logarithm of the height of the eye above the sea, in feet; half the sum will be the proportional logarithm of the dip of the horizon.

Example.

Required the Dip of the Horizon, answering to an Elevation of 100 *Feet above the Surface of the Sea?*

Constant logarithm, - - -	0.4236
Given height, 100 feet, = 1′ 40″ P. L.	2.0334
Sum	2.4570
Dip of the horizon, 10′ 38″ P. L. -	1.2285

The dip is affected by Terrestrial Refraction, which, according to Dr. Maskelyne, amounts to about one tenth of the whole: Bouguer and Lambert suppose it to be $\frac{1}{9}$ and $\frac{1}{14}$ respectively; this last is also what M. le Gendre deduced from several experiments.* According to General Roy, it varies from $\frac{1}{3}$ to $\frac{1}{24}$ of the comprehended arch:† and later observations have shown, that it varies from $\frac{1}{7}$ to $\frac{1}{34}$;‡ and that, at a mean, it may be taken at $\frac{1}{15}$ of the contained arch ||.

Since the figure of the Earth is that of an oblate spheriod, the radius of curvature is, therefore, variable with the latitude; it hence follows, that, speaking strictly, no single table of dip can answer in all places.—Tables of dip should be constructed, so as to answer to the latitude of the place, and azimuth of the observed object. It, however, may be observed, that the difference of dip arising from the above cause, is so inconsiderable, as to have been hitherto neglected.

Of Refraction.

The refraction of any celestial body is the difference between its apparent place, and that wherein it would be seen, if the space between the observer and object was either a void, or of an uniform density.

* Memoires Acad. Sciences pour 1787.

† Philosophical Transactions for 1790, p. 236.

‡ Ditto for 1797, pages 468, 469.

§ Ditto - page 472.

The

The first person who appears to have written on refraction is Al Hazen, an Arabian, about the year 1080; and the first who constructed a table of refractions, and applied that table to the practice of astronomy, was Tycho Brahe, towards the year 1570.

The course of a ray of light from any luminous body is rectilineal, while it moves through a medium* of an uniform density. If a medium of a different density be interposed, the ray will be bent from its original course at the junction of the two media, and assume a new rectilineal direction, provided this last medium be of an equal density throughout; but if the medium be of a different density, and so as to become either more dense or continually rarer, the ray will describe a curve. Now, the density of the atmosphere increases, as it approaches the Earth's surface; hence, a ray from any celestial body will describe a curve, in passing through the atmosphere to the eye of an observer; and the angle contained between the tangent of this curve at the eye, and a straight line from the eye to the object, is called the *Refraction*.

ILLUSTRATION.

Let the circle ABC (fig. 22.) represent a section of the Earth; DEF the surrounding atmosphere; S the apparent place of a star; and R the point wherein it would be seen by an observer at A, if the intervening space was a void, or of an uniform density: now let the ray RD be incident on the surface of the atmosphere at D, which, therefore, instead of proceeding in a rectilineal direction to E, will be refracted to A in the direction of the curve DA, because the density of the atmosphere increases as it approaches the Earth's surface; hence, an observer at A will see the object at S, the line AS being a tangent to the curve at A. The arch RS is called the Refraction of the object in altitude, and is evidently to be subtracted from the observed altitude, to obtain the true.

Since the refraction is greatest at the horizon, and as it decreases very rapidly as the altitude increases; therefore, the Sun and Moon, when in or near the horizon, appear to be of an elliptical figure.† Upon this account, therefore, a correction becomes necessary, when the distance between the Sun and Moon, or the Moon and a fixed star, is measured in any other direction than that parallel to the horizon; the Sun or Moon being supposed to be near that circle. This correction is contained in Table XXXII.

Physical Cause of Refraction.

LEMMA 1st.—A ray of light from any luminous object moves in a rectilineal direction, in a medium of an uniform density.

* Any transparent substance, as the atmosphere, water, glass, &c. is called a medium.

† For accounts of some very remarkable effects of the refraction near the horizon, the reader is referred to a paper, by Captain Huddart, in the Phil. Transactions for 1797, part I.; to another, by Mr. Latham, in the Transactions for 1798; and to a third paper, by Mr. Vince, in the Transactions for 1799, part I. See also Dr. Hutton's Dictionary, vol. II. page 352. The author has also observed many remarkable instances of refraction, particularly on the promontory called Buchanness, on the East coast of Scotland.

2d.

2d.—A ray, passing out of one medium into another of a different density, will be bent out of its rectilineal direction, at the junction of the media; and if the medium from which the ray proceeds be rarer than that into which it enters, it will be attracted towards the perpendicular at the point of incidence. This is evident, from the course of a ray out of air into water, &c. For the proof of these, the writers on optics may be consulted.

It is demonstrable, that every body is endowed with an attractive power, which reaches to some distance beyond its surface, as that of Cohesion, Magnetism, &c. Now, a ray of light from any of the heavenly bodies will, as its entrance into the terrestrial atmosphere, be attracted towards the denser parts; and since the density of the atmosphere increases, the nearer the Earth's surface, therefore, the ray, as it approaches the observer, will be more and more attracted, its velocity accelerated, and of course its rectilineal direction changed. Hence, that portion of the ray contained between an observer and the extremity of the atmosphere, will be a curve, except in that case, when the ray is perpendicular to the refracting medium.

Several eminent mathematicians, particularly Messrs. Bournoulli, Bouguer, Euler, Mayer, Séjour, Simpson, &c. have investigated the law of this curve, and have given rules for computing the refraction corresponding to any given altitude. We are, however, indebted to the celebrated Dr. Bradley for the following rule,* which he deduced from his observations; namely, *That the refraction at any altitude is to 57″, in the direct ratio of the tangent of the apparent zenith distance, lessened by three times the estimate refraction to the radius.* If a greater degree of accuracy is required, the computation must be repeated, using the refraction thus found, in place of that by estimation, or taken from the tables.

Example.

Required the mean Refraction,† answering to 10° of apparent Altitude, the corresponding Refraction by Cassini's Tables being 5′ 28″ ?

Apparent altitude, - -	10° 0′ 0″		
Thrice given refraction, =	16 24	Constant log.	2.2775
Sum,	10 16 24	Tangent, -	9.2583
Approximate refraction, - -	5 14	P. Log. -	1.5358
	3		
Thrice approx. refraction, -	15 42		
Apparent altitude, -	10 0 0	Constant log.	2.2775
Sum,	10 15 42	Tangent, -	9.2578
Mean refraction, - - -	5 15	P. Log. -	1.5353

If

* Explanation and Use of Dr. Makelyne's Astronomical Tables, page 5. Requisite Tables, 1st. edit. p. 129.

† The mean refraction, answering to any given apparent altitude, may be computed by

If the density of the atmosphere remained constantly the same, it might be naturally concluded, that the refraction answering to any given altitude would also continue invariable; but this is by no means the case; for it is found by observation, that the density of the atmosphere is variable, and, according to the experiments of Mr Hauskbee and others, the refractive power is found to be proportionable to its density, but the density of the atmosphere is directly as its compressing force, and inversely as its heat: and since the weight and heat of the atmosphere are shewn by the barometer and thermometer respectively, it hence follows, that the mean refraction may be reduced to the true, by allowing for the difference between the actual and mean heights of the mercury in these instruments.

The refractive power of the atmosphere is as its weight, while its heat remains the same; hence, if a table of refraction be adapted to the mean heat of the atmosphere, as 55°, and to a given altitude of the barometer, as 30 inches, then the mean refraction will be to the true, as 30 inches is to the observed height of the barometer,* the thermometer being at 55°.

The change of refraction, arising from a given change in the heat of the atmosphere, is determined by observation. M. de la Caille makes the change of refraction, answering to a change of 10° in Reaumur's thermometer, to be $\frac{1}{27}$ of the whole; M. Mayer makes this change to be $\frac{1}{22}$. According to Dr. Bradley, *the true refraction is to the*

by the following rule, deduced from a formula given by M. du Séjour, in the first vol. of his Traité Analytique des Mouvemens apparens des Corps Célestes, p. 635.

To the constant log. 9.9992828, add the log co-sine of the apparent altitude; the sum, rejecting radius, will be the log. co-sine of an arch. One sixth of the difference between this arch and the apparent altitude will be the refraction.

EXAMPLE.

Let the apparent altitude be 10°, required the corresponding refraction?

Constant logarithm,				9.9992828
Apparent altitude,	10° 0′ 0″	-	Co-sine	9.9933515
Arch	10 31 22	-	Co-sine	9.9926348

$$\frac{31\ 22}{6} = 5'\ 13''.7 = \text{mean refraction.}$$

* The mean refraction is adapted to an altitude of 30 inches of the barometer, and 55° of Fahrenheit's thermometer. If, therefore, the barometer be at the standard, but the thermometer higher than 55°, it is evident, the mercury in the barometer will be expanded, and consequently, the observed height will exceed the true; otherwise, it will be less than the true height. Again, since the expansion of the mercury is at its height; hence the higher the barometer, the greater is the expansion answering to a given change in the thermometer. It, therefore, becomes necessary to correct the observed altitude of the barometer.

It is found, that the expansion of the mercury in the barometer, answering to a change of 1° in the thermometer, is about .00304 of an inch. Now let the difference between the height of the thermometer and 55° be called a, and the observed height of the barometer b.

Then 1° : a° :: .00304 : .00304a

And 30 : b :: .00304a : .000101ab = correction.

Hence, if a cypher be prefixed to the observed height of the barometer, and the whole considered as a decimal, then the product of this quantity by the decimal .101, and by the difference between the height of the thermometer and 55°, will be the correction; which is additive, when the thermometer is above 55°, otherwise subtractive.

mean,

mean, in a direct ratio of the altitude of the barometer to 29.6 inches; and in an inverse ratio of the altitude of the thermometer increased by 350 *to the number* 400. Agreeable to this rule, Table VIII. was computed; from which the corrections of refraction, answering to a given altitude of the barometer and thermometer are found by inspection.

Since the altitudes of the barometer and thermometer, are observed at one extremity only of the curve described by the ray, in its passage through the atmosphere; it is hence evident, that these corrections serve only to approximate to the true refraction.

Of PARALLAX.*

That part of the heavens in which a planet or any celestial object would appear, if viewed from the surface of the Earth, is called its *Apparent Place:* and the point wherein it would be seen at the same instant from the Earth's center, is called its *True Place.* The difference between the true and apparent places, is called the *Parallax in Altitude.*

ILLUSTRATION.

Let C (fig. 23.) represent the center of the Earth, A the place of an observer on its surface, whose sensible horizon AE, true horizon CF, and zenith Z; also let ZKEF be a great circle in the heavens, and GHD the apparent diurnal path of a planet, arising from the rotation of the Earth. Now let the planet be in any point H of its diurnal path, then will I be its apparent place referred to in the heavens, and K its true place; the difference between these or the arch KI is called the parallax in altitude. If the planet be in the zenith, its true and apparent places coincide, and therefore the parallax vanishes; but if the planet be in the horizon at D, then the arch LE, which is the measure of the angle LDE = ADC, will be the horizontal parallax. Hence, also, the horizontal parallax of a planet is the angle which the Earth's semidiameter, when viewed directly, subtends at that planet; and the parallax in altitude, is the apparent angular distance of the observer from the center of the Earth, as seen from the planet's center.

The more elevated a planet is above the horizon, the less is the parallax, its distance from the Earth's center continuing the same. When the planet is in the zenith, it has no parallax, but when in the horizon, its diurnal parallax is greatest.

Since the apparent place of a planet is more distant from the zenith than its true place, it is therefore evident, that the parallax in altitude must be added to the observed altitude, in order to obtain the true altitude of the planet, as seen from the Earth's center.

The horizontal parallax being given, the parallax corresponding to any given altitude may be found by the following rule.

* From παράλλαξις, *differentia*.

To

secant of the apparent altitude, add the proportional logarithm of the horizontal parallax; the sum, rejecting radius, will be the proportional logarithm of the parallax in latitude.

EXAMPLE.

Let the apparent Altitude of the Moon's Center be 23° 32', and horizontal Parallax 58' 46". Required the Parallax in Altitude?

Apparent altitude,	23° 32'	Secant,	-	0.0377
Horizontal parallax,	58' 46"	P. Log.	-	0.4861
Parallax in altitude*,	53' 53"	P. Log.	-	0.5238

The correction of the Moon's altitude, contained in Table IX. is the difference between the parallax and refraction of that object in altitude: if, therefore, the refraction at the given altitude be taken from the parallax, the remainder will be the correction of the Moon in altitude. Thus, in the above example, the

Moon's parallax in altitude is,	53' 53"
——— refraction at alt. 23° 32', Table VI.	—2 11
Correction of Moon's altitude,	51 42

Upon account of the rotation of the Earth, its figure is that of an oblate spheroid; the equatoreal radius is, therefore, the greatest; the polar semi-axis is the least; and the other radii are of an intermediate length, depending on the latitude of the place: and since the Moon's horizontal parallax is the angle under which the Earth's semidiameter appears at the Moon, it hence follows, that the horizontal parallax of the Moon will vary with the latitude, being greatest at the equator, and least at the poles; but the horizontal parallax of the Moon, as given in the Nautical Almanac, is that answering to the equatoreal radius, and, therefore, in order to reduce it to any given latitude, it must be diminished by a certain quantity. Table XXXIII. contains this quantity, agreeable to Sir Isaac Newton's hypothesis.

Summary of the Corrections.

When the altitude of the lower limb of any object is observed, its semidiameter is to be added thereto, in order to obtain the central altitude; but if the upper limb be observed, the semidiameter is to be subtracted. If the altitude be taken by the back observation, the contrary rule is to be applied.

* The parallax in altitude may be found with sufficient accuracy, for several purposes, by entering a traverse table, with the Moon's altitude taken as a course; and the difference of latitude, answering to the horizontal parallax found in a distance column, will be the parallax in altitude.

 The

The dip is to be subtracted from, or added to, the observed altitude, according as the fore or back observation is used.

The refraction is always to be subtracted from, and the parallax added to, the observed altitude.

If the distance between the nearest limbs of any two objects be observed, that distance is to be increased by the sum of their semidiameters; but if the remote limbs be observed, the distance is to be diminished by the above sum. If the Moon's enlightened limb is to be compared with the nearest and remote limbs of the Sun alternately, half the sum of these distances will be the distance between the Sun's center and the Moon's enlightened limb. The distance between a star and the Moon's nearest limb is to be increased by the Moon's apparent semidiameter; and the distance between the Moon's remote limb and a star is to be diminished by the Moon's semidiameter. If, at the time of full Moon, both limbs of the Moon be compared with a star, half the sum of these distances will be the apparent central distance between these two objects In like manner, if the distance between the Moon's enlightened limb, and the nearest and remote limbs of a planet, apparently full, be observed, then half the sum of these distances will be the apparent distance between the Moon's enlightened limb and the center of the planet. If the planet be not full, and the distance between the enlightened limbs observed, allowance is to be made for the planet's semidiameter: this may, however, be avoided, by observing the distance between the Moon's limb and the planet's center. It may be observed, that the discs of the planets beyond the orbit of Mars, are, in general, observed to be circular, which arises from their great distance from the Sun, with respect to the Earth; Mars appears sometimes gibbous; and Venus, when brightest, has the form of a crescent, and, when having this aspect, the distance between the limbs of the Moon and this planet is to be observed. Mercury is too seldom seen, so as to be used with the Moon in ascertaining the longitude.

BOOK

BOOK III.

CONTAINING

The Method of finding the Longitude at Sea or Land by Lunar Observations.

CHAP. I.

Introduction.

IF the latitude and longitude of a ship at sea were accurately known, the course and distance therefrom to any given port might be easily deduced by charts or otherwise. The latitude is ascertained from observation; but the common method of deducing the ship's longitude, from the course and distance made good, is at best only an approximation to the truth, and, at the end of the voyage, the accumulated error is sometimes very considerable; having, in a run from Britain to the West Indies, been found to exceed eight or ten degrees. It has, therefore, been the wish of the practical navigator, and of every maritime nation, to have some certain method of ascertaining the longitude of a ship at sea. In order to obtain so useful a discovery, several very considerable rewards have been offered to any person who would give a method, by which the ship's longitude might be determined within proper limits, as often as may be necessary.

The first who offered a reward for the discovery of the longitude at sea, was Philip III. of Spain, in the year 1598, and soon after, the States of Holland followed his example. The reward offered by Philip was 1000 crowns, and that by the States 10,000 florins. In the year 1714, the British Parliament offered a premium of 20,000*l.* for any method by which the longitude could, at all times, be determined at sea within 30 miles; 15,000*l.* if the proposed method gave the longitude within 40 miles; 10,000*l.* if within one degree, or 60 geographical miles. This part, however, of the act was repealed, and a new one framed, to take place after 24th June 1774; of which there is an extract at the beginning of the Nautical Almanac; and in 1716, a reward of 100,000 livres was promised by the Duke of Orleans, who was then regent of France. In consequence of which many and

 various

various methods have been proposed to solve this important problem; of these which have hitherto appeared, that by observing the distance between the Moon and the Sun or a fixed star, seems to be the most proper for this purpose, both on account of the frequency of the observations, and of the shortness of the calculations.

John Werner of Nuremberg appears to be the first who proposed the method of finding the longitude, by observing the distance between the Moon and a star, in his annotations on the first book of Ptolemy's Geography, printed 1514. He recommends the cross staff as a very proper instrument for the purpose of observing the distance between these objects.

This method was recommended by Oronce Finé of Briançon, in his book *De Inveniendа Longitudine*; and again by Gemma Frisius, in his treatise, entitled, *Structura Radii Astronomici et Geometrici*, printed at Antwerp in 1545, which usually accompanies his edition of Appian's Cosmography. At folio 39th, he expresses himself as follows, "*Sed quotidie fere, si quis velit longitudinem loci alicujus perquirere, is diligenter consideret lunæ distantiam ab aliquo sidere, sydere firmamento per radium nostrum. Ita tamen ut illa stella fixa secundum rectum eclipticæ ductum, lunam, præcedit aut sequatur**." He then proceeds to show how the Moon's longitude is found from the above observation, and directs the apparent time of observation to be inferred from the altitude of the star observed at the same instant; the Moon's longitude at this time is also to be computed from the best astronomical tables. Hence the difference between the observed and computed longitudes of the Moon will be known; with which, and the Moon's horary motion, the difference of longitude, between the place of observation and that to which the tables are adapted, may be found.

The celebrated Kepler was fully persuaded of the utility of this method of finding the longitude at sea. In his Rudolphine tables, he gave directions for observing the distance between the Moon and a star, and for making the necessary computations, nearly the same as those given by Gemma Frisius. This method, or rather that by observing the difference of longitude between the Moon and a fixed star, is described by Christian Longomontanus, in his *Astronomia Danica*, printed at Amsterdam, 1622, in page 194, *pars altera*. He directs the observation to be taken when the Moon is in or near the nonagesimal; because, then, the parallax in longitude either vanishes, or is very little; and the Moon is in this position, when a line joining the cusps is perpendicular to the horizon. In the following page, he applies this method to an example of an observation of the difference of longitude between the Moon and Aldebaran. Longomontanus then gives a

* But almost every day, if any one wishes to find out the longitude of any place, let him carefully observe the distance of the Moon from any star in the heavens by means of our radius; yet in such manner as that star may either precede, or follow the Moon, in the line of the ecliptic.

catalogue

catalogue of those stars*, with which the Moon is to be compared. He next describes the cross staff, improved so as to observe the difference of longitude between the Moon and a star; and, in page 197, illustrates the same, by a figure of a person observing the difference of longitude between the Moon and Aldebaran.

Mr. Blundevil describes this method of ascertaining the longitude at sea, in his Exercises, page 391†, the invention of which he attributes to Appian. It is also taken notice of by Carpenter in his Geography, printed at Oxford in the year 1635, wherein he says, *This way was taught by Appian, illustrated by Gemma Frisius and Blundevil*‡. He then proceeds to describe the manner of observing the distance, from whence the longitude of the place of observation is to be inferred as formerly. He concludes as follows: *This way, though more difficult, may seeme better than all the rest, for as much as an eclipse of the Moone seldome happens, and a watch, clocke, or houre glass cannot so well bee preserved, or at least so well observed in so long a voyage: whereas every night may seeme to give occasion to this experiment: if so bee the ayre be freed from clouds, and the Moone shew her face above the horizon*§.

M. John Baptiste Morin, professor of mathematics at Paris, improved the above method; which, by order of Cardinal Richlieu, was examined in 1634. It was, however, judged to be incomplete, upon account of the imperfections of the lunar tables. Morin therefore wrote against those appointed to examine his method, particularly M. Pierre Herigone, who had proposed several methods of finding the longitude, in the fourth volume of his *Course of Mathematics*; which methods Morin endeavoured to confute. An answer was given thereto by Herigone, at the end of the fifth volume of the above work, printed at Paris in 1637. Although Morin did not obtain the reward he claimed for his improvement of the above method, yet, in 1645, Cardinal Mazarin procured him a pension of 2000 livres.

A method for the above purpose was now much desired in England. The Royal Observatory at Greenwich was founded by Charles II. in the year 1675, and Mr. Flamstead appointed Astronomer Royal. The words of his commission were, *To apply himself with the utmost care and diligence to the rectifying the tables of the motions of the heavens, and the places of the fixed stars, in order to find out the so much desired longitude at sea, for perfecting the art of navigation.*

* α Arietis, Aldebaran, ζ Tauri, Propus, Pollux, δ Cancri, Regulus, β Virginis, Spica Virginis, α Libræ, β Scorpionis, ξ Sagittarii, β Capricorni, δ Capricorni, and λ Aquarii.

† The 4th edition of this work was printed in London in 1613, and the 7th in 1636. The title of the chapter in which this method is described is, "Another way taught by Appian to find out the longitude of any place with the Crosse Staffe, by knowing the distance betwixt the Moone and some knowne Starre that is situated nigh unto the ecliptique line." This last word is omitted in the seventh edition.

‡ Geographie Delineated, &c. part I. page 242.

§ Do. page 246.

Among

Among all the celestial objects, the Moon, upon account of the quickness of its motion, appears to be the best adapted for the above purpose. Any method, however, depending on observations of the Moon, which were to be compared with lunar tables, was very uncertain, as long as those tables remained so inaccurate. *The longitude*, says Mr. Flamstead*, *might be also attained by observations of the Moon, if we had tables that would answer her motions exactly; but after 2000 years, we find the best tables extant erring sometimes 12 minutes or more in her apparent place, which would cause a fault of half an hour, or 7½ degrees in the longitude deduced, by comparing her place in the heavens with that given by the tables.*

In 1696, Mr. Edward Harrison, a lieutenant in the navy, published his *Idea Longitudinis*, in 98 pages 12mo, including the preface, &c. in which he mentions the methods of finding the longitude by the Moon's motion, by automata, magnetic variation, and by the eclipses of Jupiter's satellites.

That justly celebrated astronomer and navigator, Dr. Edmund Halley, recommended observations of the Moon, as the most certain method of ascertaining the longitude at sea, having, by his own experience, found the impracticability of all the other methods proposed for that purpose. He gave an excellent paper on that subject, in the Philosophical Transactions, No. 420, wherein he shows the defects of the lunar tables then extant. By comparing the place of the Moon, as given by the tables, with that deduced from observation, he found the errors of the tables recur with great regularity at the end of 18 years 11 days; so that whatever error was found in a former period, the same error was again repeated, under the like circumstances, of the same distance of the Moon from the Sun and apogeon. Being encouraged by this, he next examined what difference might arise from the period of nine years wanting nine days; in which time there are performed very nearly one hundred and eleven lunations; but the return of the Sun to the apogee in that time, differing above four times as much from an exact revolution, as in the period of eighteen years, a like agreement was not to be expected. Having, however, entered upon the 10th year of his observations of the Moon's transit, he compared his late observations of 1730 and 1731, with those he had made in 1721 and 1722, and very seldom found a difference of more than one single minute of motion; but most commonly this difference was wholly insensible; so that, by the help of what he observed in 1722, he presumed that he was able to compute the true place of the Moon with certainty within two minutes of motion, during the year 1731, and so on for the future. He finishes this paper, by recommending Hadley's quadrant as a proper instrument for taking the necessary observations at sea. This instrument had been described in the preceding number of the Transactions of the Royal Society.

Dr. Halley again treats on this subject in his Astronomical Tables,

* Phil. Transactions, Lowthorpe's abridgment, vol. 1. page 537.

in

in which are given two complete examples of finding the longitude from the observed distance between the Moon and the stars, ν Leonis and ζ Tauri, and concludes with observing, that *by a like method of computation, may the difference of the meridians be found from observations of the Moon's distance from the Sun, in her first and last quarter.*

M. Emanuel Swedenborg takes notice of the above method of finding the longitude, in his *Dædalus Hyperboreus*, printed in Swedish, at Stockholm, in the year 1716, and in other tracts. It is also briefly explained in a very small pamphlet, entitled, *An Introduction to a true Method for the Discovery of the Longitude at Sea, by Stephen Plank*, printed in London in 1720.

Mr. Robert Wright of Winwick in Lancashire, presented to the Commissioners of Longitude, a tract entitled *Viaticum Nautarum:* and in 1728, he published in 4to, his *Humble Address to the Right Honourable the Lords, and the rest of the Honourable Commissioners, appointed by act of Parliament to judge of all Performances relating to the Longitude*, in which he asserts, page 4, that Sir Isaac Newton's "Theory being freed from some errors of the press, and restored to its original exactness, the Moon's place will be so nearly given, that the *longitude* (if due care be taken) may be found thereby either precisely exact, or within very few miles of certainty." In 1732, he published in 4to, his *New and Correct Tables of the Lunar Motions, according to the Newtonian Theory.*

In 1755, Professor Mayer of Gottingen sent a manuscript copy of his lunar tables* to the British Admiralty, claiming at the same time some one of the rewards, promised by Parliament, which he might be thought to merit. In these tables the arguments are investigated on the Newtonian principle of universal gravitation, and the maxima of the equations are deduced from his own observations, and from those of Dr. Bradley and others. The above tables were delivered to Dr. Bradley to be examined; who, having compared them with a great number of his own observations, was convinced of their excellence. He then sent an account of them to Mr. Cleveland, secretary of the Admiralty, in a letter dated Greenwich, Feb. 10, 1765, from which the following is extracted.

"In obedience to their Lordships commands, I have examined the same, and carefully compared several observations that have been made (during the last five years) at the Royal Observatory at Greenwich, with the places of the Moon computed by the said tables. In more than 230 comparisons which I have already made, I did not find any difference so great as $1\frac{1}{2}'$ between the observed longitude of the Moon and that which I computed by the tables; and although the greatest difference which occurred is, in fact, but a small quantity, yet, as it ought to be considered as partly arising from the error of the observations, and partly from the error of the tables, it seems probable,

* Mayer's tables first appeared in the Memoirs of Gottingen for 1742.

that,

that, during this interval of time, the tables generally gave the Moon's place true within one minute of a degree.

"A more general comparison may perhaps discover larger errors; but those which I have hitherto met with being so small, that even the biggest could occasion an error of but little more than half a degree in longitude, it may be hoped that the tables of the Moon's motions are exact enough for the purpose of finding at sea the longitude of a ship, provided that the observations that are necessary to be made on ship-board can be taken with sufficient exactness."

In another letter to Mr. Cleveland, dated Greenwich April 14, 1760, Dr. Bradley shows the utility of these tables, from the comparison of observations made at sea by Capt. Campbell, in the years 1757, 1758, and 1759. By the mean of twelve different days or nights observations, taken at sea, and reduced to Ushant, he found the longitude of the middle of that island to be about 5° 23′ west of Greenwich, and the greatest difference between the mean result, and that of any particular day, amounted in one case only to 37′ in defect, and the greatest difference in excess was but 23 minutes. He also observes, that since his former account of the near agreement of Professor Mayer's lunar tables with the observations that had been made at the Royal Observatory, he had compared several others, which concurred to prove, that the difference between the observed and computed places no where amounted to more than one minute and a half; and he found that the difference, small as it is, may yet be diminished by making alterations in some of the equations, whose true quantity could not be determined without proper observations. After making the necessary corrections, it appeared by the comparison of above eleven hundred observations, taken since the new instruments were fixed up, that the difference did no where amount to more than one minute. It may, therefore, be reasonably concluded, that so far as it depends upon the lunar tables, the true longitude of a ship at sea may in all cases be found within half a degree, and generally much nearer.

Professor Mayer died in the beginning of the year 1762, and left a still more complete and accurate set of lunar tables. These were soon after transmitted to the Board of Longitude, by his widow, for which she received 3000l. Mr. Euler also received a reward of 300l. in consideration of Mayer having availed himself of Euler's lunar theory. These tables, together with a set of Mayer's solar tables, were printed in London in 1770, under the inspection of Dr. Maskelyne; from which all the articles in the Nautical Almanac relative to the Sun and Moon are computed. An edition of the lunar tables, improved by Mr Mason from Dr. Bradley's series of lunar observations, published in the Nautical Almanac for 1774, was printed in London in 1787, and reprinted at Paris in the *Connaissance des Tems* for 1790. "These new tables," says Dr. Maskelyne, "when compared with the abovementioned series of observations, seem to give always the Moon's longitude in the heavens, correctly within 30 seconds of a degree." Mayer adopted the method of distances in his *Methodus Longitudinum Promota*, prefixed to his tables, which he illustrates

illustrates by two examples of the distance between the Moon and Aldebaran. He then describes the circular instrument of reflexion, which he invented for observing the apparent distance between the Moon and a star, with a greater degree of accuracy than can be done by a quadrant or sextant.

Our present astronomer royal, the Rev. Dr. Nevil Maskelyne, very much deserves the thanks of every navigator, and of the public at large, for his great industry and exertions in bringing this method of ascertaining the longitude at sea into common practice. In his *British Mariner's Guide*, printed in London in 1763, he very strongly recommends it, and gives new precepts for making the necessary observations and computations. The rules for computing the effects of refraction and parallax, which he had formerly communicated to the Royal Society, and which were published in the British Mariner's Guide, were again communicated to that Society with their demonstrations, in 1764, and published in the LIV. vol. of the Phil. Tran. On the 9th of February, 1765, he presented a memorial to the Commissioners of Longitude, in which he shows, that in his voyage to St. Helena, and return thence, he made frequent observations of the distance of the Moon from the Sun and fixed stars, with Hadley's quadrant, from which, by the help of Mayer's printed tables, he computed the longitude of the ship from time to time; and from the near agreement of the observations, especially at making the land, when the ship's common reckoning was very erroneous, he inferred that the longitude thus determined would never err a whole degree; that the same method which he had explained in his British Mariner's Guide, had been practised with success by the commanders and mates of several of the East India ships. He then proceeds to point out the method he used to adjust his quadrant, in his voyage to Barbadoes, and return to England, which was, by measuring the Sun's diameter with the index placed alternately to the right and left of the beginning of the divisions; by which means, he found his observations to agree still nearer than in his former voyage to St. Helena, having made Barbadoes within half a degree of its true situation; and at his return home, he made the isle of Wight, by observations taken 24 hours before, within 16 minutes of its true situation. He concludes his memorial in the following words; "I am authorised by them* to say, they humbly apprehend that nothing is wanting to make this method generally practicable at sea, but a Nautical Ephemeris, an assistance which they, with many more, hope for from this Board."

The gentlemen mentioned in this memorial were examined separately, as to the utility and practicability of the above-mentioned observations; and all agreed in testifying, that they had determined the longitude of their respective ships from time to time, by observations of the Moon, taken in the manner directed in the British Mariner's Guide; and that the longitude resulting always agreed with the

* The gentlemen mentioned in this memorial, namely, the commanders and mates of the ships that had used the method he had described.

making of land, to one degree. In consequence of which, the Board came to the following resolution:

"That it is the opinion of this Board, upon the evidence given of the utility of the late Professor Mayer's Lunar Tables, that it is proper the said tables should be printed; and that application should be made to Parliament for power to give a sum not exceeding 5000*l.* to the widow of the said Professor, as a reward for the said tables, part of which have been communicated by her since her husband's decease; and also for power to give a reward for persons to compile a Nautical Ephemeris, and for authority to print the same when compiled, in order to make the said lunar tables of general utility."

This was immediately put into execution; and accordingly, since the year 1766, a work, entitled, *The Nautical Almanac and Astronomical Ephemeris,* has been published annually, by order of the Commissioners of Longitude, under the inspection of Dr. Maskelyne. This is, perhaps, the best and most accurate publication of the kind that has hitherto appeared*.

At the same time with the first Nautical Almanac, a set of tables, to facilitate the calculations, was published by Dr. Maskelyne; in which are given two excellent methods, with their demonstrations, for clearing the apparent distance between the Moon and the Sun or a fixed star, from the effects of refraction and parallax; the first by Mr. Lyons, and the other by Mr. Dunthorne: also two examples for ascertaining the longitude from observation. About twelve years afterwards, this edition being nearly all sold off, it became necessary to reprint it; previous to which it was new-modelled by Dr. Maskelyne, and the second edition appeared in 1781, and the third edition in 1802.

The publications on the longitude, by observations of the Moon, now became very numerous; almost every book on Navigation, published in Britain since the commencement of the Nautical Almanac, contained a few rules for that purpose. In Robertson's Navigation, there is a good tract on this subject. Besides those already mentioned, some of the other writers on lunar observations in Britain are, Messrs. Adams, Bishop, Clarke, Dunn, Elliot, Emerson, Gregory, Harrison, Heath, Keith, Kelly, Margetts, Martin, Nicholson, Vince, Waddington, Wilson, Witchell, Wright, &c.

This method of finding the longitude at sea is mentioned in a treatise, entitled, *Abrégé du Pilotage,* by M. le Monnier, printed at Paris 1766, page 225. In the same year, Pére Pezenas published his *Astronomie des Marins,* at Avignon; which, besides examples to the problems in the *Astronomie Nautique* of the celebrated M. de Maupertuis, and other articles, contains various methods of finding the longitude from observations of the Moon.

* "Le *Nautical Almanac* de Londres est l'Ephéméride la plus parfaite qu'il y ait jamais eu." M. la Lande's Astronomy, third edition, vol. i. page 145. Again, "Cette ouvrage calculé a grands frais, et avec une précision extrême sous la diction de M. Maskelyne, est extrêmement important pour les navigateurs." Montucla, Histoire des Mathématiques, vol. iv. page 321.

Upon

Upon the 15th November, 1766, M. de Charniéres presented to the Royal Academy of Sciences at Paris, his *Memoire sur les Longitudes en Mer*, which he published in February following; and his treatise, entitled, *Experiences sur les Longitudes, faites a la Mer en* 1767 *et* 1768, *publié par ordre du Roi*, appeared in 1768. In this tract, he has shown the utility of his instrument called the *Megameter*, in observing the distance between the Moon and a star. This instrument is constructed on the same principles as the object glass micrometer, and is only applicable to measure distances less than 10°. In 1772, M. de Charniéres published his *Theorie et Pratique des Longitudes en Mer*; which treatise contains an ample detail of the above-mentioned instrument, with plates; the methods of making and reducing the observations for the latitude and longitude, &c. In page 21, he says*, "Intimement persuadé, que la vrai solution du problème des longitudes en mer, tenoit à la mesure exact de la distance de la Lune aux etoiles, je trouvai que l'héliometre pourroit, en le modifiant, s'appliquer aux observations de mer."

The Abbé de la Caille recommends it as the only practical method at sea. He was, however, sensible, from his own experience, of the errors to which it was then liable. In his edition of Bouguer's Navigation, printed at Paris in 1781, he says, "La grande incertitude à laquelle nous avons dit qu'étoit sujette la méthode d'employer les observations de la Lune faites sur mer, ne doit pas décourager le marin, ni la lui rendre suspecte, puis que dans les voyages de long cours, où l'on a essuyé beaucoup de vents contraires, et de long coups de vents, il arrive souvent qu'aux atterrages on se trouve en erreur de sept ou huit degrés sur la longitude estimée selon les regles du pilotage†." In the same work, he gave a new and easy method of reducing the apparent to the true distance, by means of scales constructed expressly for that purpose. This subject is also very well treated in the sixth volume of Bezout's *Cours des Mathématiques*, Paris, 1781, page 249; and in M. Callet's edition of *Gardiner's Logarithmic Tables*, Paris 1783, page 54.

An excellent tract was published at Paris in 1787, entitled *Description et Usage du Cercle de Reflexion, par le chevalier de Borda*, in which that instrument is very particularly described, its use shown in nautical observations, and the manner of calculating these observations.

M. de la Lande treats very perspicuously on this subject, both in his *Exposition du Calcul*, and in his *Astronomie*, page 244, or third edition, printed at Paris in 1792, page 268. In the former of these

* Being intimately persuaded, that the true solution of the problem of the longitude at sea, depends upon the exact measure of the distance of the Moon from stars; I found that the heliometer, in modifying it, would be proper to apply to observations at sea.

† The great uncertainty, we have observed, to which the method of employing observations made at sea is subject, ought not to discourage the mariner, nor render it suspected, since in long voyages, where they encounter many contrary winds and of long duration, it often happens, that upon their return they find an error of 7 or 8 degrees of the longitude by account, according to the common rules of pilotage.

works, p. 168, he says, "Sans une méthode pour trouver ces longitudes, la navigation est toujours incertaine; l'estimé à laquelle on a recours, peut être fautive de 4 or 500 lieues après quelques mois de navigation, et jeter ceux qui naviguent dans les plus grands dangers*." He then recommends the lunar method of finding the longitude at sea, and shows the manner of constructing and using M. de la Caille's *Chassis de Reduction*, for clearing the apparent distance from the effects of refraction and parallax, and the method of deducing the ship's longitude from observation; and in page 155, he says, the distance between the limb of the Moon and a star may be both easily and accurately observed with the *Héliometre* of M. Bouguer. In his *Astronomie*†, he writes, "Cette méthode des distances à l'avantage de ne dependre essentiellement, que d'une seule observation de distance; elle ne suppose pas la hauteur connue avec une extreme précision; elle depend tres peu de la declinaison de la Lune, et de la hauteur du pole; elle n'exige pas qu'on ait un horizon *clair-fin*, c'est-a-dire, bien dégagê des vapeurs; elle ne suppose pas des calculs aussi longs que ceux de l'ascension droit de la Lune: enfin, la reduction de la distance apparente en distance vrai, à raison de la réfraction et de la parallax, se peut faire avec la regle et le compas, par une operation graphique" And the following is the last parapraph of that valuable work: "La Caille et M. Maskelyne ont éprouvé longs tems sur mer cette méthode des distances de la Lune au Soleil, et aux étoiles; ils l'ont trouvé la plus exacte; ils l'ont adopté de préference. M. de Charnières et M. de Verdun, officiers des vaisseaux du Roi, qui se sont exercé à ces observations dans des voyages de long cours, l'ont employé et l'on regardé comme la meilleure. M. d'Agelet, qui à fait, avec M. de Rosnevet, le voyage des Terres Australes en 1773, et le voyage autour du monde avec M. de la Peyrouse, à fait un usage continuel de la méthode des distances: il n'à jamais trouvé plus d'un demi-degré d'erreur dans tous les atterrages où il pouvoit verifier sa longitude. L'experience prouvé assez qu'on ne sauroit se dispenser de ces observations, pour peu qu'on ait de zele et de connoissance dans la navigation. Il ne nous reste donc qu'à inviter les navigateurs à en étudier les calculs, à en acquérir l'habitude, et à rendre cette pratique aussi gênerale, qu'elle est utile pour la navigation ‡."

In

* Without a method to find the longitude, navigation is always uncertain; the common method by account or dead reckoning, might be erroneous 4 or 500 leagues after several months navigation, and thereby put the navigator in the greatest danger.

† Prem. edit. tome II. art. 3224; ou sec. edit. tome III. art. 3977: ou troisi. edit. tome III. art. 4176.—"This method of distances has the advantage of not depending essentially, than only an observation of the distance; it doth not suppose the altitude to be known with extreme precision; it depends very little upon the declination of the Moon, and the latitude of the place; it does not require a very distinct horizon, or wholly free of vapours; it does not suppose calculations as long as those for the right ascension of the Moon; lastly, the reduction of the apparent to the true distance by reason of refraction and parallax, may be done with a rule and compass, by a graphical operation."

‡ La Caille and M. Maskelyne have proved a long time at sea, this method of the distances of the Moon from the Sun and stars; they have found it the most exact, and accordingly adopted it in preference to other methods. M. de Charnières and M. de Verdun

In the *Opuscules Mathématiques*, par M. L'Abbé de Rochon, printed at Paris in the year 1768, page 51, is a Memoir upon the improvement of the *Héliometre*, which he strongly recommends as the most proper of all instruments, for observing, with precision, the distance between the Moon and the Sun or a fixed star. In the following Memoir of the same work, he describes two other instruments for the same purpose, one of which he calls the *Astrometre*, see page 81, and the other, the *Mégameter of Reflexion*, see page 94. In page 112, he recommends the methods of finding the longitude by lunar observations, nearly in the same words as M. de la Lande, cited above.

In 1796, Mr. Kelly published his *Nautical Astronomy*, a work particularly adapted for those who wish to acquire a knowledge of Spherical Trigonometry. In this treatise, page 201, there is a very ingenious method of reducing the apparent to the true distance, by a graphical operation. The second edition of this work appeared in 1801, in which the investigation of the above method is given.

M. de la Lande published his *Abrégé de Navigation*, at Paris, in the year 1793. This work contains a brief history of navigation, a catalogue of nautical books, with various other particulars; and lastly, the method of finding the longitude by lunar observations; but the principal purpose for which this work was published, is the set of horary tables which it contains; and by which the apparent time at the ship may be readily deduced from the altitude of any heavenly body, whose declination is contained within the limits of the table, and the latitude of the place. In this work M. La Lande says, "the Moon furnishes the most general and certain method of finding the time under the first meridian; and it is the distance between the Moon and the Sun, or stars, that is employed for this purpose*."

A very good work, entitled *Verhandeling over het bepaalen der lengte op zee, door de afstanden van de maan tot de zon, of vaste sterren, &c.* by J. H. Van Swinden, was published at Amsterdam in the year 1796†, which was three years after the first edition of this book was printed. In the above-mentioned treatise, the methods given by Messrs. Borda, Dunthorne, Krafft, Lyons, Dr. Maskelyne, and by myself,

Verdun, officers in the royal navy of France, who have accustomed themselves to these observations in long voyages, have used, and regarded it as the best method. M. d'Agelet, who hath made, with M. de Rosvenet, the voyage to the Terræ Australis, in 1773, and the voyage round the world with M. de la Peyrouse, hath made constant use of the method of distances; he never found more than half a degree of error in all the places where he was able to verify this longitude. Experience sufficiently proves, that to dispense with these observations, will show little zeal and confidence in navigation. It remains, then, to invite navigators to study the calculations, and to acquire the habit, and to render this practice as general, as it is useful to navigation.

* La Lune fournit un moyen plus general & plus sûr pour trouver l'heure qu'il est sous le premier meridian, et c'est sa distance au soliel ou aux étoiles que l'on employé."

† Treatise concerning the determination of the longitude at sea, by the distance between the Moon and the Sun, or a fixed star, published by order of the Committee for Marine Affairs. In the preface to this work, page xi. M. Van Swinden expresses himself as follows: "When this part (meaning the third) was already at the press, the excellent work of Mackay fell into my hands; I have thought proper to make of use it, and regret it was not sooner known to me.

myself, as given in the first edition of this book, are explained and demonstrated; and beside other articles, he has also copied my table of natural versed sines; which, so far as I know, was the first of the kind ever published, but having acknowledged this, he has not incurred the disgraceful name of a plagiarist.

An excellent treatise on Navigation, in two volumes 4to, entitled *Tratado de Navegacion, por Don Josef de Mendoza y Rios,* was printed at Madrid in 1787; in which the method of finding the longitude by lunar observations is clearly explained. "Este métods (vol. ii. page 364) es el mas directo y el mejor de los que pueden proponerse, porque su exactitud depende poco del conocimiento de la latitud geografica, y de las observaciones de las alturas, y solo exige esencialmente medida de la distancia*." The investigations of five different methods of reducing the apparent to the true distance, are given in this work; the first, is that of the Chevalier de Borda; and the others of the methods giving in the Requisite Tables. There is a valuable paper by the same gentleman, entitled *Recherches sur l'Astronomie Nautique,* in the Philosophical Transactions for 1797; which, besides other things relating to navigation, contains the investigations of many different methods of reducing the apparent distance between the Moon and the Sun, or a fixed star, to the true distance. In the Philosophical Transactions for 1801, there is an account of an *Improved Reflexion Circle,* which is greatly superior to any former instrument of this kind, and on which he is still making further improvements. In the year 1800, a work in folio, by M. de Mendoza y Rios, entitled *Colleccion de Tables para varcos usos de la Navigacion,* was printed at Madrid. In the introduction he observes, that "La presente coleccion se dirige a facilitar los calculos de la observationes que se practican en el mar, para determinar la longitud y la latitud de la naves y algunos otros objetos de navegacion é hidrografia."† An explanation of the tables is prefixed, with their use exemplified in the solution of the various computations in Nautical Astronomy. In the year 1801, M. de Mendoza y Rios's collection of "*Tables for facilitating the Calculations of Nautical Astronomy,*" was printed at London, in one volume 4to. This work is nearly upon the plan as the above-mentioned. An appendix is annexed, containing tables by H. Cavendish, Esq. for reducing the apparent to the true distance between the Moon and the Sun or a fixed star, with precepts for the use of the same. Again, in 1805, Captain Mendoza, published his "*Complete Collection of Tables for Navigation and Nautical Astronomy,* in one vol. 4to, containing upwards of 700 pages, in which he has greatly simplified the method of reducing the

* This method is the most direct and the best of those which have been proposed, since its exactness depends but little on the knowledge of the latitude of the ship, and the observations of the altitudes; and only requires essentially the mean of the distances.

† The present collection is intended to facilitate the calculations from observations made at sea, to determine the longitude and the latitude of the ship, and other objects of navigation and hydrography.

apparent

apparent to the true distance, by means of new and extensive tables. To that work an explanation of the tables is annexed; and their use illustrated in a series of problems and examples.

From the mutual concurrence of the opinions of so many eminent men it appears, that the method of finding the longitude at sea, by observing the distance between the Moon and the Sun, or a fixed star, commonly called *Lunar Observations*, is the best in use at present; and, therefore, any improvement that may be made therein, either in the lunar tables, from which the articles in the Nautical Almanac relative to the Moon are computed, or in the construction of instruments for observing the distance, or in the methods of making and calculating the necessary observations, must certainly be of very great advantage to the practical navigator.

Before we conclude this chapter, it may be proper to remark, that the usefulness of any method of finding the longitude at sea, depends upon the frequency of the necessary observations, the accuracy with which they may be taken, and of the conclusions deduced therefrom. That this is eminently the case, with respect to observations of the Moon, will appear evident from the following considerations.

Of any of the heavenly bodies, which frequently present themselves for observation, there is none whose apparent velocity is so rapid as that of the Moon: the diurnal motion of that object being from 11° 52′ to 15° 21′*; and the mean diurnal motion 13° 10′ 34″.86 †. Hence, an error of 10″, in the distance between the Moon and a fixed star, will, at a mean rate, produce an error of only 4½′ in longitude‡.

In favourable weather, observations of the distance between the Moon and the Sun, or a fixed star, may be always had, except about the time of new Moon.

The distance may be observed to a very great degree of accuracy, with the sextant, in its present improved state, or with the circular instrument; and by taking the mean of several distances, observed at short, and nearly equal intervals of time, the apparent distance may be obtained within a few seconds of the truth; and the altitudes of the objects may be observed, with sufficient accuracy, with a Hadley's quadrant, for clearing the apparent distance from the effects of refraction and parallax.

In consequence of the great degree of exactness to which the lunar tables are brought, the Moon's place may be found within less than half a minute, at any given instant; and, by means of the Nautical Almanac; the calculations are rendered extremely short and easy.

* Some Comets excepted.

† For 27d. 7h. 43′ 11″51 : 24h. : : 360° : $\dfrac{360° \times 24\text{h.}}{27\text{d. } 7\text{h. } 43' \, 11''.5} = 13° \; 10' \; 34''.86$

‡ $\dfrac{360 \times 24\text{h.}}{27\text{d. } 7\text{h. } 43' 11''.5} : 360° :: 10'' : \dfrac{360° \times 10'' \times 27\text{d. } 7\text{h. } 43' \, 11''.5}{360° \times 24\text{h.}}$

$$= \frac{10'' \times 2360591''.5}{86400''} = 4' \; 33''.22.$$

It is hence evident, that, of all the methods which have been proposed to determine the longitude at sea, that by lunar observations will always claim the pre-eminence.

CHAP. II.

Of finding the Longitude at Sea by Lunar Observations.

THAT method of finding the longitude of any place, by observations of the distance between the Moon and the Sun, or a fixed star, is called *Lunar Observations.*

The observations necessary to determine the longitude at sea by this method are, the distance between the Moon and the Sun, or a fixed star, in or near the ecliptic, together with the altitude of each. The stars used in the Nautical Almanac, for this purpose, are mentioned in page 25. The instruments proper for taking these observations are, a sextant, or circular instrument, for observing the distance between the objects, or for the altitude, when the apparent time or error of the watch, is to be inferred from that observation: and the altitudes, taken at the same time with the distance, for the purpose of clearing it from the effects of refraction and parallax, are to be observed with Hadley's quadrant. These instruments, and the methods of making the above-mentioned observations, have been explained in Chapters I. II. and III. Book II. A good watch, having a second hand, or rather a good time-keeper, is necessary for connecting the observations for the apparent time at the ship, and for the distance with each other. The method also of taking a complete set of observations, and of deducing the mean time, distance, and altitudes therefrom, are explained in Chap. IV. Book II; and in the same chapter, the method is shown, by which one person, without any assistants, may take all the necessary observations. It now remains to show the manner of reducing the various observations; that is, of finding the apparent time and longitude: this will, therefore, be the subject of the following chapters of this book; and, for this purpose, it becomes requisite to premise the following problems.

PROBLEM I.

To convert Degrees, or Parts of the Equator, into Time.

RULE.

Multiply the given motion by 4, and the product will be the corresponding time.

REMARKS.

REMARK.

In this operation it is to be observed, that minutes multiplied by 4, produce seconds; degrees multiplied by 4, produce minutes, which, divided by 60, give hours.

EXAMPLE.

Let 26° 45′ be reduced to time.

$$\begin{array}{r} 26^\circ\ 45' \\ 4 \\ \hline 1\text{h. } 47'\ 0'' = \text{time required.} \end{array}$$

PROBLEM II.

To convert Time into Motion.

RULE.

Multiply the given time by 10, to which add half the product; the sum will be the corresponding degrees, &c.

EXAMPLE.

Let 3h. 4′ 28″ be converted into motion.

	3	4	28
			10
	30	44	40
Half =	15	22	20
Corresponding motion =	46	7	0

REMARK.

Time may be reduced to longitude, and conversely by Tables I. and II. See vol. ii. page 41.

PROBLEM III.

Given the Time, under any known Meridian, to find the corresponding Time at Greenwich.

RULE.

Let the given time be reckoned from the preceding noon, to which the longitude of the place in time is to be applied by addition or subtraction, according as it is west or east, and the sum or difference will be the corresponding time at Greenwich.

Examples.

I.

What is the time at Greenwich, when it is 6h. 15′ at a ship in longitude 76° 45′ W.?

Time at ship,	- - -	6h. 15′
Longitude in time,	- -	5 7 W.
Time at Greenwich,	- -	11 22

II.

Required the time at Greenwich, answering to 5h, 46′ 39″ of May 1st at Canton, whose longitude is 113° 2′ 15″ E.?

Time at Canton, May 1st,	-	5h. 46′ 39″
Longitude in time,	-	7 32 9 E.
Time at Greenwich, April 30,		22 14 30

Problem IV.

To reduce the Time at Greenwich to that under any given Meridian.

Rule.

Reckon the given time from the preceding noon, to which add the longitude in time if east, but subtract it if west; and the sum or remainder will be the corresponding time under the given meridian.

Examples.

I.

What is the expected time of the beginning of the lunar eclipse of October 22, 1809, at a ship in longitude 109° 45′ W.

Beginning of eclipse at Greenwich, per Naut. Alm.		19h. 41′
Ship's longitude in time,	- - - -	7 19 W.
Beginning of the eclipse at the ship,	- -	12 22

II.

When may the immersion of the first satellite of Jupiter be observed at Bombay, in long. 72° 54¼′ E. which, by the Nautical Almanac, happens at Greenwich, Dec. 26, 1810, at 5h. 16′ 50″

Apparent time of imm. at Greenwich,		5 16 50
Longitude of Bombay in time,	- -	4 51 38 W.
Apparent time of imm. at Bombay,		10 8 28

Problem

PROBLEM V.

To reduce the Declination of the Sun, as given in the Nautical Almanac, to any other Meridian, and to any given Time of the Day.

RULE.

Take the Sun's declination from the Ephemeris for the noon of the given day, and convert the seconds into tenths of a minute, by dividing by 6; then enter Table XIII. with the declination at the top, and take out the equations answering to the given longitude, and time from noon. These will be the change of declination, answering to a north increasing declination: when the Sun is in either of the other quarters of the ecliptic, a correction from one of the three following Tables accordingly, is to be applied to each of the above equations. Now, between the 20th of March and 21st of June, and from the 22d of September to the 21st of December, or while the Sun's declination is increasing, the equation answering to the ship's longitude is to be added to, or subtracted from, the declination, according as the longitude is west or east; and the equation answering to the time from noon, must be added or subtracted, according as the given time is after or before mid-day. But from the 21st of June to the 22d of September, and from the 21st of December to the 20th of March, or while the Sun's declination is decreasing, the equation answering to the longitude is additive or subtractive, according as the longitude is east or west; and that answering to the time from noon is to be applied to the declination by addition or subtraction, according as the given time is before or after noon*.

EXAMPLES.

I.

Required the Sun's declination at noon, November 4th, 1810, in longitude 101° 5′ E.?

Sun's declination at Greenwich, - - -	15° 16′.6
Equation to declination and longitude, -	— 5.1
Reduced declination, - - -	15 11.5

* Those acquainted with the first principles of algebra will prefer the following [illegible] Let the sign + be prefixed to the declination if increasing, to the equation [illegible] ing to the longitude if west, and to that answering to the time from noon, [illegible] time be in the afternoon; but if the declination is decreasing, the [illegible] time before noon, prefix the sign — to these quantities, and their sum, according [illegible] rules of algebraic addition, will be the reduced declination.

EXAMPLE.

Required the Sun's declination April 2, 1791, at 6h. 35′ A. M. in [illegible]

Sun's declination at noon, per Naut. Alm. -	[illegible]
Equation to longitude 38° 40′ W. - - -	[illegible]
——— to time from noon, 5h.25′ - -	[illegible]
Reduced declination, - - - -	[illegible]

II.

What is the Sun's declination 25th May, 1810, at 10h. 48′ Greenwich time?

Sun's declination at noon, - - - -	20° 53	
Equation to declination and time from noon, -	+ 5	
Declination at given time, - - - -	20 58	

III.

Required the Sun's declination, 15th August, 1810, at 5h. 46′ A. M. in longitude 143° 7′ W.?

Sun's declination at noon, - - -	14°	12′.7
Equation to declination and longitude, - -	—	7.4
- - - - and time from noon, -	+	4.8
Reduced declination, - - - -	14	10.1

IV.

What is the Sun's declination, 19th February 1810, at 2h. 15′ P. M. in longitude 164° 56′ E.?

Sun's declination at noon, per Naut. Almanac, -	11°	23′ 5
Equation to decl. and longitude 164° 56′ E. -	+	9. 8
- - - and time from noon 2h. 15′ P. M.	—	2. 0
Reduced declination, - - - - - -	11	11. 3

Problem VI.

To reduce the Sun's right Ascension, as given in the Nautical Almanac, to any given Time of the Day, under a known Meridian.

Rule.

Enter Table xviii. with the Sun's right ascension at the top, and take out the equations answering to the ship's longitude, and the interval between the given time and noon. Then, if the longitude be west, and if the given time be between noon and midnight, add the corresponding equations to the Sun's right ascension; otherwise subtract them; and the sum or remainder will be the reduced right ascension.

Examples.

I.

Required the Sun's right ascension at noon, 22d April, 1810, in longitude 76° 45′ W.?

Sun's right ascension at noon, p. Naut. Alm. -	1h.	58′	1″
Equation to right ascension and longitude, 76° 45′W. +			48
Reduced right ascension, - - - -	1	58	49

II. What

II.

What is the Sun's right ascension at Greenwich, October 18, 1810, at 8h. 40′ P. M.

Sun's right ascension at noon, per Naut. Alm. -	13h.	30′	33″
Equation to right ascension and time from noon,	+	1	21
Reduced right ascension, - - -	13	31	54

III.

What is the Sun's right ascension, June 6, 1804, at 1h. 48′ A. M. in longitude 63° 10′ E.?

Sun's right ascension at noon, per Naut. Alm. -	4h.	55′	54″
Equation to right ascen. and long. 63° 10′ E. -	—		48
- - - - - and time from noon, 10h. 12′	—	1	45
Reduced right ascension, - - - - -	4	52	36

Problem VII.

To reduce the Declination of the Moon, as given in the Nautical Almanac, to any given Time, under a known Meridian.

Rule.

Find the variation of the Moon's declination in twelve hours, with which enter Table xix. at the top, and take out the equations answering to the ship's longitude, and to the time from the preceding noon or midnight. Then, if the Moon's declination is increasing, the equation corresponding to the time is additive; and that answering to the longitude is to be added, if the longitude is west; but subtracted, if east. If the declination is decreasing, the contrary rule is to be applied.

Examples.

I.

Required the Moon's declination, May 19, 1810, at 5h. 13′ apparent time under the meridian of Greenwich?

Moon's declination 19th September at noon, -		17°	6 S.
Equation to variation 0° 47′, and time from noon 5h. 13′	+		20
Reduced declination, - - - - - -		17	26 S.

II.

What is the Moon's declination, 3d July, 1810, at 14h. 9′ apparent time, in longitude 65° 14′ W.?

Moon's declination, 3d July, at midnight, -		14°	34′ N.
Equation to variation 1° 13′ and longitude 65° 14′ W.	—		27
- - - - - and time from midnight 2h. 9′	—		14
Reduced declination, - - - - - -		13	53 N.

Problem

Problem VIII.

To reduce the Horizontal Parallax and Semidiameter of the Moon, as given in the Nautical Almanac, to any given Time, under a known Meridian.

Rule.

Reduce the given time to the meridian of Greenwich, by Prob. III. and find the variation of parallax in twelve hours; then take from Table XIX. the correction answering to the variation of parallax at the top, and the given time in the side column; which being added to, or subtracted from, the Moon's horizontal parallax at the preceding noon or midnight, according as it is increasing or decreasing, will give the required parallax. In like manner, the Moon's semidiameter may be found*.

Example.

Required the Moon's horizontal parallax and semidiameter, December 19, 1810, at 11h. 15′ in longitude 38° 40′ E.?

Given time, - - - - - -			11h.	15
Longitude in time, - - - - -			2	25′ E.
Reduced time, - - - - - -			8	40
Moon's hor. paral. noon -	55′ 24″	Semid.	15′	6″
Cor. Tab. XIX. to 8h. 40′ & var. 20′ =	+ 14	Corr.	+	3
Reduced parallax -	55 38	Red. sem.	15	9

Problem IX.

Given the observed Altitude of a fixed Star, to find the True Altitude.

Rule.

To the observed altitude of the star apply the index error, if any, and the dip of the horizon; from which subtract the refraction answering thereto, from Table VI. and the remainder will be the true altitude of the star.

* Three elevenths, or one fourth nearly, of the proportional part of parallax, will be that of semidiameter.

Examples.

Examples.

I.

The observed altitude of Arcturus is 38° 40′, and the height of the observer's eye 18 feet above the surface of the water. Required the true altitude?

Observed altitude of Arcturus, -	38° 40′.0
Dip. Table III. - - -	— 4.1
Apparent altitude, - -	38 35.9
Refraction, Table VI. - -	— 1.2
True altitude of Arcturus, -	38 34.7

II.

Let the observed altitude of Antares be 49° 52′, the error of the instrument 5′.3 subtractive, and height of the eye 14 feet. Required its true altitude?

Observed altitude of Antares,	49° 52′	
Index error, - -	— 5.3	
Dip, - - - - -	— 3.6	
Apparent altitude, - -	49 43.1 =	49° 43′ 6″
Refraction, Table VI. -		— 48
True altitude of Antares, - -		49 42 18

Problem X.

Given the observed Altitude of the lower or upper Limb of the Sun, to find the true Altitude of its Center.

Rule.

Correct the altitude of the Sun's limb by the index error, if any, the dip of the horizon, and semidiameter; hence the apparent altitude of the Sun's center will be obtained; from which the correction* in Table V. answering thereto, is to be subtracted, and the remainder will be the true altitude of the Sun's center.

* In some cases, it will be proper to apply the refraction and parallax from Tables VI. and VII. respectively, in place of the above correction.

Examples.

Examples.

I.

The observed altitude of the Sun's lower limb is 18° 41′, height of the eye above the surface of the sea 22 feet, and Sun's semidiameter 16′.1. Required the true central altitude?

Observed altitude of Sun's lower limb,		18° 41′.0
Sun's semidiameter, - - -	+	16 .1
Dip, - - - - - -	—	4 .5
Apparent altitude Sun's center, -		18 52 .6
Correction, - - - - -	—	2 .6
True altitude of Sun's center, -		18 50 .0

II.

The observed altitude of the Sun's upper limb is 28° 31½′, index error 40″ additive, height of the eye 12 feet, and Sun's semidiameter 15′ 18″. Required the true altitude?

Observed altitude of Sun's upper limb,		28°	31′	30″
Index error, - - -	+			40
Sun's semidiameter, - -	—		15	54
Dip of the horizon 3.3, or -	—		3	18
Apparent altitude of the Sun's center,		28	12	58
Refraction, Table VI. - -	—		1	46
Parallax, Table VII. - -	+			8
True altitude of the Sun's center, -		28	11	20

Problem XI.

Given the observed Altitude of the lower or upper Limb of the Moon, to find the correct central Altitude.

Rule.

Agreeable to the reduced time of observation, find the Moon's horizontal parallax and semidiameter, by means of the Nautical Almanac; and increase the semidiameter by the augmentation from Table XXXI. answering to the semidiameter and observed altitude.

To the observed altitude of the Moon's limb, apply the index error, the augmented semidiameter and dip, as formerly; the aggregate will be the apparent altitude of the Moon's center; to which, the correction from Table IX. answering to the Moon's horizontal parallax and apparent altitude being added, the sum will be the correct altitude of the Moon's center.

Examples.

Examples.

I.

Let the observed altitude of the Moon's lower limb be 31° 18'. horizontal parallax 58' 37", semidiameter 15' 58", and height of the eye 16 feet. Required the true altitude of the Moon's center?

Moon's semidiam.	15' 58"	Observed alt. Moon's lower limb	31° 18'.0
Augment. Tab. XXXI. +	8	Dip, - - - - -	— 3.8
Augm. semidiameter	16 6	- - - - - -	+ 16.1
Apparent alt. of Moon's center,		- - -	31 30.3
Cor. to alt. 31° 30' and hor. par. 58' 37" Tab. IX. = 48' 26"			= + 48.4
True altitude of the Moon's center,		- -	32 18.7

II.

The observed altitude of the Moon's upper limb is 41° 25', horizontal parallax 55' 40", semidiameter 15' 10", and height of the eye 15 feet. Required the true altitude of the Moon's center?

Moon's semidiameter,	15' 10"	Observed alt. Moon's up. limb,	41° 25' 0"
Augment. Tab. XXXI. +	10	Dip of the horizon = 3'.7	— 3 42
Augm. semidiameter,	15 20	- - - -	— 15 20
Apparent altitude of the Moon's center,		- - -	41 5 58
Cor. to alt. 41° 6' and hor. par. 55' 40" Tab. IX.			= + 40 51
True altitude of the Moon's center,		- - -	41 46 49

Altitudes used for the computation of the apparent time, or for the latitude, ought to be expressed in degrees, minutes, and tenths of a minute; but in some of the methods for correcting the apparent distance between the Moon and the Sun or a fixed star, it is necessary, that the true altitudes of the objects be expressed in degrees, minutes, and seconds. In the first example to each of the three last problems, the correct altitude is, therefore, found in degrees, minutes, and tenths; and in the second, it is expressed in degrees, minutes, and seconds.

The correction of a star in altitude, is the refraction answering to that altitude; and the correction of the altitude of the Sun or Moon, is the difference between the refraction and parallax answering to the given altitude. As the refraction in altitude is greater than the Sun's parallax at the same altitude, this correction is, therefore, subtractive; but the Moon's parallax in altitude being ever greater than the refraction answering to the same altitude, the Moon's correction is, therefore, to be added. This last correction may be found, independent of Table IX. by subtracting the refraction from the Moon's parallax in altitude, computed by the rule given in page 89.

It has already been observed, that in order to reduce the Moon's horizontal parallax, as given in the Nautical Almanac, to a given place, it must be diminished by an equation from Table XXXIII. depending on the latitude. The horizontal parallax in the Connaissance des Tems is adapted to the latitude of Paris; in order, therefore, to adapt it to a place whose latitude is less than that of Paris, it must be increased by the difference of the equations answering to those latitudes, and diminished by the above difference, when the latitude of the place exceeds that of Paris. Moreover, the altitude used for computing the parallax in altitude must be increased or diminished by a certain quantity, from Table XXVI.

Hence, in order to compute the altitude of the Moon's center above the true horizon, the observed altitude should be corrected by the error of the quadrant and the dip of the horizon, from which the corrected refraction, answering thereto, is to be subtracted. This last quantity being increased by the parallax in altitude, computed with the reduced horizontal parallax and altitude, will be the true altitude of the Moon's limb, to which the Moon's true semidiameter being applied, will give the true altitude of the Moon's center.

CHAP. III.

Of the Methods of ascertaining Time, and regulating a Chronometer, or Watch, at Sea or Land.

TIME, as inferred from observations of the Sun, is denominated *Apparent*, and *Mean Solar Time*.

Apparent time is that which is deduced immediately from altitudes, or other observations of the Sun or stars; and mean time arises from a supposed uniform motion of the Sun. Hence, a mean solar day is always of the same determinate length of absolute time, but the length of an apparent day is ever variable, being longer at one time, and shorter at another, than a mean day; the instant of apparent noon will, therefore, sometimes precede, and sometimes follow, that of mean noon. The interval between apparent and mean time, is called the *Equation of Time*.

The equation of time results from the unequal motion of the Sun in its orbit, combined with the obliquity of the ecliptic; and hence, consists

consists of two distinct parts. The first part of the equation of time is, therefore, the difference between the mean and true longitudes of the Sun, converted into time, at the rate of 15° to one hour: and the second part is, the difference between the true longitude and true right ascension of the Sun, converted also into time, at the same rate. Hence, the absolute equation of time is equal to the difference between the mean longitude, and the true right ascension of the Sun, converted into time, at the rate of 15° to one hour. When the true right ascencension of the Sun is equal to the mean longitude, the length of an apparent day is equal to that of a mean solar day; and hence, the equation of time is then nothing. This happens on the 14th of April, the 15th of June, the 30th of August, and the 23d of December. But when the difference between the apparent right ascension and the mean longitude is greatest, the equation of time is then greatest, and this happens on the 10th of February; and the greatest equation of time, which happens at this season, is about 14′ 40″: again, on the 14th of May, the greatest equation is about 4′ 0″; on the 26th of July, when the greatest equation is about 6′ 4″; and lastly, on the 2d of November, when the greatest equation of time is about 16′ 15″.

On the 10th of February, and the 26th of July, the Sun's right ascension exceeds the mean longitude, and, therefore, the mean time exceeds the apparent time; but, on the 14th of May, and the 2d of November, the true right ascension being less than the mean longitude, the mean time is, therefore, less than the apparent time.

Since, at these times, the diurnal motion in right ascension is equal to the mean motion in longitude, the length of an apparent day is, therefore, equal to that of a mean day. But, when the diurnal motion in right ascension differs most from that of the mean longitude, or 59′ 8″, then the difference between the lengths of an apparent and a mean day is greatest. This accordingly happens on the 25th of March, the 18th of June, the 15th of September, and the 23d of December. On the 25th of March, the true diurnal motion in right ascension is so much less than the mean motion in longitude, that the length of the apparent day is 18¾″ less than that of a mean day. On the 18th of June, the length of the apparent day exceeds that of the mean day by 13″. The apparent day is 21¼″ shorter than the mean day on the 15th of September, and it is 30¼″ longer on the 23d of December.

Tables have been constructed, to exhibit the parts of the equation of time answering to the mean anomaly, and to the true longitude of the Sun; and these parts being taken out, with their proper arguments, and connected according to their respective signs, will give the compound equation of time. Both these parts have, by some, been blended together, and put into one table; the argument of which being the Sun's true longitude. In order, however, to save these computations, a table of the absolute equation of time is given in the Nautical Almanac, page II. of the month, for the instant of apparent noon by the meridian of Greenwich.

As the principle upon which a watch, or time-keeper, is constructed, is that of an equable or uniform motion; it is, therefore, evident, that

 when

when a series of successive and repeated observations are to be made, the watch ought to be regulated according to mean solar time, or its rate established on that time. But as it is apparent time that is immediately deduced from observations of the Sun, the equation of time becomes, therefore, an essential article, and must be applied thereto, in order to reduce it to mean solar time. Indeed, when a watch is used for no other purpose than that of connecting a few observations, taken within short intervals of time of each other, the reduction of apparent to mean time is not necessary.

Problem I.

To find the Equation of equal Altitudes.

Rule.

Enter Table xxiii. with the interval of time between the observations at the top, and the latitude of the place of observation in the side column, and take out the correspondent number; take out the number from Table xxiv. answering to the interval of time and the Sun's declination; subtract it from the former, if the latitude and declination are of the same name, otherwise, add them, and find the log. corresponding to the remainder or sum, which subtracted from the P. Log. of the daily variation of the Sun's declination, increased by 5, the remainder* will be the P. Log. of the equation of equal altitudes.

Example.

Let the latitude of the place of observation be 57° 9′ N. the interval of time between the observations 5h. 17′; Sun's declination 17° 48′ S. and change of declination 16′ 19½″. Required the equation of corresponding altitudes?

No. from Table xxiii. to interval and lat. =	1782		
No. from Table xxiv. to inter. and decl. =	284		
Sum, - - - -	2066	- log. -	3.3151
Daily variation of declination, -	16′ 19″.5	P. log. + 5 =	6.0424
Equation of equal altitudes, -	20″.2	Prop. log.	2.7273

Problem II.

To find the Error of a Watch by equal Altitudes of the Sun.

Rule.

In the morning, when the Sun is more than two hours distant from

* Or, from this remainder subtract the constant log. 1.7782, and the remainder will be the P. Log. of a certain number of minutes and seconds; which, being esteemed seconds and thirds, will be the equation of equal altitudes. The thirds may be reduced to a decimal by dividing by 6.

the

the meridian, in these latitudes, let a set of observations be taken, consisting, for the sake of greater accuracy, of at least three altitudes; which, together with the corresponding times per watch, are to be wrote down regularly, the time of each observation being previously increased by 12 hours. In the afternoon, observe the instants when the Sun comes to the same altitudes, and write down each opposite to its respective altitude. Now, half the sum of any two times, answering to the same altitude, will be the time of noon per watch uncorrect; find the mean of all the times of noon, thus deduced from each corresponding pair of observations, to which the equation of equal altitudes is to be applied, by addition or subtraction, according as the Sun is receding from, or approaching to, the elevated pole; the sum or difference will be the time per watch of apparent noon, or the instant when the Sun's center was on the meridian; the difference between which and noon is the error of the watch for apparent time, and the watch will be fast or slow, according as the time of noon thereby is more or less than twelve hours.

If the watch be regulated to mean solar time, it is obvious the time of noon, found as above, should agree with that found by applying the equation of time to noon, according to its sign in the Nautical Almanac. If these times do not agree, their difference will be the error of the watch for mean solar time. Instead of applying the equation of time to twelve hours, it, perhaps, will be found to be more convenient to apply it with a contrary sign to the time per watch of apparent noon; and the difference between this time and 12 hours will be the error of the watch as formerly.

Examples.

I.

January 29, 1786, in lat. 57° 9′ N. the following equal altitudes of the Sun were observed. Required the error of the watch?

Alt. = 8° 5′ Time		21h. 35′ 8″	A. M. -	2h. 55′ 43″ P. M.
8 10	-	- 36 8	- -	54 42
8 20	-	- 38 9	- -	52 41.2
8 25	-	- 39 12½	- -	51 38
21h35′ 8	-	21h.36 8	21h.38′ 9″	21h39 12.5
2 55 43	-	2 54 42	2 52 41.2	2 51 38
Sum 24 30 51	-	24 30 50	24 30 50.2	21 30 50.5
Mean 12 15 25.5	-	12 15 25	12 15 25.1	12 15 25.2
				25.1
				25.0
				25.5
Sum, - - - - - -				.8
Time of noon per watch uncorrected, -				12 15 25.2

Time

Time of noon per watch uncorrected,	-	12	15	25.2
Equation of equal altitudes,	-	—		20.2
Time per watch of apparent noon,	-	12	15	5.0
Watch fast for apparent time,	-		15	5
Time per watch of apparent noon,	-	12	15	5.0
Equation of time,	-	—	13	29.8
Time per watch of mean noon,	-	12	1	35.2
Watch fast for mean time,	-		1	35.2

Remark.

In observing equal altitudes, it will be found convenient to put the index of the quadrant to a certain division, and to wait till either limb of the Sun attains that altitude. If the successive altitudes of the same set are equidistant from each other, the mean of the morning observations may be compared with the mean of those observed in the afternoon, in order to find the time of noon.

II.

April 20th, 1786, in latitude 57° 9′ N. the following observations were made, in order to ascertain the error of the clock.

Alt. = 35° 40′ Time p. clock	21h. 20′ 27″.5	- -	2h. 37′ 29″.5
35 45	21 16 .0	- -	36 41 .0
35 50	22 4 .5	- -	35 52 .5
35 55	22 53 .0	- -	35 4 .0
36 0	23 41 .3	- -	34 15 .6
	10 22 .3	- -	29 22 .6
Mean,	21 22 4 .46	- -	2 35 52 .52
	2 35 52 .52		
	23 57 56 .98		
Time p. clock of noon uncorr.	11 58 58 .49		
Equation of equal altitudes,	— 19 .53		
Time p. clock of app. noon,	11 58 38 .96	-	11 58 38 .96
Clock slow for apparent time,	1 21 .04	eq. t. +	1 16 .20
Time p. clock of mean noon,	- - -		11 59 55 .16
Clock slow for mean time,	- - - -		4 .84

Hence, the observations of the two preceding examples being supposed to be made at the times specified, by the same watch or clock,

clock, its daily rate may be established, upon the supposition of an uniform motion, as follows:

January 29th.	Clock fast at noon =	1′ 35″.3
April 20th.	Clock slow at noon =	4 .8
Interval 81 days.	Difference,	1 40 .1

Now 1′ 40″.1, div. by 81, gives 1″.236 for the daily rate of the clock.

In observations made at land, with a good instrument, in favourable weather, the time may be easily ascertained within the fifth part of a second, by observing the position of the Sun's limb with respect to the wire in the focus of the telescope, at the instants immediately preceding and following the transit of the limb; and though the time of each observation at sea be only ascertained within a few seconds, yet the error may be so far diminished, by taking the mean of repeated sets, as to be rendered almost insensible.

Observations for this purpose should not be taken when the Sun is within less than two hours of the meridian, unless in case of absolute necessity, because the motion of the Sun in altitude, at places distant from the equator, would then be too slow; neither should the altitude be under 4 or 5°; otherwise the refraction may be considerably altered between the times of the morning and afternoon observations, and thereby sensibly affect those times. If the rate of the watch is not uniform during the interval between these observations, an error in the time of noon may thence arise, which it is more than probable will augment with the observed interval. Moreover, if the rate of the watch differs considerably from mean solar time, the interval between the means of the morning and afternoon sets of observations should be reduced to that time, with which the equation of equal altitudes is to be taken from the tables.

It very often happens, in places distant from the equator, that in the afternoon, by the interposition of clouds, a correspondent set to that observed in the forenoon cannot be obtained; it is, therefore, proper to take several sets in the morning, and though the afternoon be a little cloudy, yet a correspondent set to some of those observed in the morning will perhaps be obtained.

We have hitherto supposed the observations to have been made in the same civil day; the interval may, however, be considerably extended, provided the rate of the watch is regular. Thus, equal altitudes may be observed in the afternoon, and in the following morning, and the mean of the times will be the time of the intermediate midnight, or instant per watch uncorrected, when the opposite meridian was directed to the Sun; to which the equation of equal altitudes is to be applied with a contrary sign. In case of absolute necessity, the time between the observations may be still farther extended, by taking equal altitudes in the forenoon, and in the afternoon of the following day;

day; in this case the error of the watch will be found at the intermediate midnight, the equation of equal altitudes being applied as usual*.

As the method of ascertaining time by equal altitudes depends neither on the accuracy of the latitude of the place of observation, nor on that of the declination of the observed object, since these elements are only necessary in taking out the equation of equal altitudes, and as any probable error therein will not sensibly affect that equation; neither does it depend on the exact quantity of the altitude, provided only it is the same at both observations; it is, therefore, universally used by the practical astronomer, especially when not provided with a transit instrument, or before a meridian mark is placed for that instrument.

Problem III.

To find the Error of the Watch by equal Altitudes of the Sun, the Ship being under way.

Rule.

Let several sets of equal altitudes be observed in the morning and afternoon, and from thence find the corrected time of noon, as formerly; also let the Sun's azimuth be observed, to which the variation of the compass being applied, the true azimuth at the time of observation will be obtained†.

Now, to the constant log. .9.2219, add the proportional log. of the interval of time between the equal altitudes, the hours and minutes being considered as minutes and seconds, the prop. log. of the hourly rate of sailing, the log co-sine of the ship's latitude, the log secant of the course, and the log. tangent of the Sun's azimuth; the sum, rejecting tens in the index, will be the prop. log. of the correction answering to the change of latitude; and to the sum of the first four logs. add the log. co-secant of the course; the sum, rejecting tens in the index, will be the prop. log. of the change of longitude. The first correction is to be added to, or subtracted from, the time of noon formerly found, according as the ship's latitude is increasing or diminishing, and the second correction is additive or subtractive, according as the ship's course has been in the eastern or western hemisphere. The result thus deduced will be the time per watch of apparent noon, under the meridian of the first place of observation.

If the two last corrections be applied with a contrary sign, the time of apparent noon, under the meridian of the second place of observation, will be obtained.

* These methods have been successfully practised by the Author.

† Or if the Sun's magnetic azimuth be taken at the times when the morning and afternoon sets were observed, then half the sum of the eastern and western azimuths, reckoned from the meridian, will be Sun's true azimuth nearly. Or, if the observation of the azimuth had been neglected, it may be computed from the known latitude of the ship, the Sun's altitude, and declination.

The

The first correction vanishes, if the course made good between the observations is either due east or west; and the second, if the ship sails on a meridian.

Example.

August 7th, 1810, equal altitudes of the Sun's lower limb were observed, whereof the means were 9h. 14′ 52″ A. M. and 2h. 48′ 18″ P. M. respectively, the corrected azimuth of the Sun from the south was 69¾°, the ship's course during the elapsed time S. W. by W. at the rate of 8.6 knots per hour, and the ship's latitude and longitude at noon were 39° 18′ N. and 31° 24′ W. respectively. Required the error of the watch for apparent noon, under the meridian of the place where the first set of observations was made?

Constant logarithm, - -	9.2219		
Int. time = 5h.33′ or 5′ 33″ P. Log. =	1.5110		
Hourly rate of sailing 8h. 6′ or 8′ 36″ - - P. Log. =	1.3208		
Latitude 39° 18′ - co-sine, -	9.8886		
	1.9423	- -	1.9423
Course 3 points, - secant,	0,0801	co-secant, -	0.2553
Azimuth 69° 45, - tangent,	0.4331	Sec.Cor. 1′9″ P.L.	2.1976
First correction 0′ 38″ P. Log: -	2.4555		

	h.	′	″
Mean of morning set, - - - - - -	9	14	52
—— afternoon set, - - - - - -	2	48	18
Sum, - - - -	12	3	10
Uncorrected time of noon, - -	12	1	35
Equation of equal altitudes, - -	+		7
Equation of latitude, - -	−		38
Equation of longitude, - -	−	1	9
Time per watch of apparent noon, under the meridian of the first place of observation,	11	59	55
Watch slow for apparent time, - -			5
Equation of time, - - -		5	23
Watch slow for mean time, - -		5	28

This problem may otherwise be performed, by estimating how many minutes the Sun is higher or lower, in consequence of the change of latitude in the elapsed time, at the instant it will attain the corresponding altitude in the afternoon, and setting the index of the quadrant accordingly. This quantity may be found with sufficient accuracy from a traverse table.

Or, with the latitude of the ship at the time of the first observation, the

the declination and correct altitude of the Sun's center, compute its distance from the meridian by Prob. VII. which being added to the mean of the first set, will give the time per watch of apparent noon. In like manner, with the ship's latitude at the second observation, the Sun's declination and altitude, compute the meridian distance of the Sun, which subtracted from the mean of the afternoon set, the remainder will be the time by the watch of apparent noon, under the meridian of the place where that observation was made. Now half the difference of longitude made good between these observations, being applied as formerly to the mean of the computed noons, will give the time per watch of apparent noon, under the meridian of the first place of observation, and hence the error of the watch for that meridian will be known If the interval between the computed times of noon is not the same as the difference of longitude, expressed in time, there is an error either in the difference of longitude, in the rate of the watch during the observed interval, or in the actual observation or computation. In this method it is not necessary, that the altitudes observed be correspondent.

Problem IV.

To find the Error of a Watch by equal Altitudes of a fixed Star.

Rule.

Let several altitudes, and the corresponding times per watch, of a known star, he observed when in the eastern hemisphere, and when the star is in the western hemisphere, observe the instants when it comes to each of the former altitudes.

Take the mean of each corresponding pair of times, and the mean of these will be the apparent time per watch of the star's transit over the meridian.

From the apparent right ascension of the star, subtract the Sun's right ascension, and the remainder will be the approximate time of the star's transit; from which subtract the equation corresponding thereto, and to the Sun's right ascension, from Table XVIII. and from the same table, take the equation answering to the ship's longitude, which must be added, if the longitude is east, but subtracted if west. Hence the apparent time of the passage of the star over the meridian will be obtained.

Now the difference between the observed and computed times of the star's transit, will be the error of the watch for apparent time, and which is fast or slow, according as the time by observation is later or earlier than the computed time of the star's transit.

Example.

July 24, 1810, in latitude 35′ 48″ S. and longitude 23° 26′ E. the following

following equal altitudes of Altair were observed. Required the error of the watch for apparent time?

Time per watch.		Altitude.		Time per watch.
8h. 17′ 0″	-	27°. 23′	-	14h. 35′ 57″
19 16	-	27 40	-	33 42
20 12	-	27 55	-	32 44
21 54	-	28 12	-	31 5
23 16	-	28 30	-	29 41
8 25 55	-	28 52	-	14 27 1
Sum 7 33	-	-	-	10 10
Mean 8 21 15.5	-	-	-	14 31 41.6
				8 21 15.5
				22 52 57.1
Time of transit per watch,	-	-	-	11 26 28.6

Altair's right ascension,	-	19 41 31	
Sun's right ascension at noon, p. N. Alm.		8 12 16	
Approximate time of Altair's transit,	-	11 29 15	
Equation to 8h. 14′ and 11h. 27′ Table XVIII.		− 1 53	
Equation to 8h. 14′ and 23° 26′ Table XVIII.		+ 16	
Apparent time of star's transit,	-	11 27 38	
Apparent time of transit per watch,	-	11 26 29	
Watch slow,	-	1 9	

Problem V.

To find the Error of a Watch by equal Altitudes of a fixed Star, the Ship being under Way.

Rule.

Let the time per watch of the star's transit be inferred from the equal altitudes as usual, and let it be reduced to the meridian of either place of observation, by applying thereto the equations arising from the ship's run, as directed in Problem III.

Then compute the apparent time of the transit of the observed star, as in the last problem, the difference between which, and the time per watch, inferred from equal altitudes, will be the error of the watch for apparent time.

Example.

November 29, 1810, the time per watch of the transit of Bellatrix, as inferred from the mean of several equal altitudes, was 12h. 52′ 18″, and the corrected azimuth of its center 59° 40′, from the south; the distance run in the elapsed time was 34 miles E. N. E. and the latitude and longitude of the ship at the time of the star's transit were 41° 33′ N. and 51° 18′ W. respectively. Required the error of the watch for apparent time, under the meridian of the westernmost place of observation?

Constant logarithm,	1.4771 *			
Dist. run 34′ P. Log.	0.7238			
Lat. 41° 33′ co-sine,	9.8741			
Sum, - -	2.0750	- - -		2.0750
Course 6 points, sec.	0.4171	- co-secant, -		0.0344
Azim. 59° 40′ tan.	0.2327	- - -		
		Second cor.	1′ 24″ P. Log.	2.1094
First cor. 20″ P. Log.	2.7248			

Time of transit of Bellatrix per equal altitudes, -	12h.	52′	18″
First correction, - - -	+		20
Second correction, - - -	+	1	24
Time of transit reduced to the first place of observation,	12	54	2
Right ascension of Bellatrix, - -	5	15	0
Sun's right ascension, - - -	16	19	16
Approximate time of transit, - -	12	55	44
Equation to 16h. 19′ and 12h. 56′ p. Table XVIII.	—	2	20
Equation to 16h. 19′ and 51° 18′ † Table XVIII.	—		36
Approximate time of transit, - - -	12	52	48
Time per watch of transit, - - -	12	54	2
Watch fast, - - - - - -		1	14

Remark.

In observing equal altitudes of a fixed star, it will be proper to use one whose declination is a little greater than the latitude of the ship, and of the same name. Hence, in places distant from the equator, the motion of the star in altitude will be a maximum, when near the

* When the course and distance made good between the observations, are given in place of the observed interval, and hourly rate of sailing, as in the above example, the constant log. 1.4771 (=9.2219+ P. L. of one min.) is to be used instead of the former.

† In strictness, the argument of this equation should be the longitude of that meridian to which the error of the watch is to be found.

meridian

meridian; and its altitude when in this position may be found by the following formula: Sine alt. = sine lat. co-secant declination, to radius unity. In places considerably distant from the equator, the interval between the observations will be much contracted, and, therefore, any probable irregularity in the rate of the watch will be diminished. In these latitudes, the refraction near the horizon is very variable; but as the altitude of a star, when in a proper position for observation, increases with the latitude, therefore, in this case, no material error can arise from the change of refraction between the observations.

Problem VI.

Given the Latitude of a Place, the Altitude and Declination of the Sun, to find the Apparent Time of Observation, and the Error of the Watch.

Rule.

Correct the observed altitude of the Sun's limb, and reduce the declination to the time and place of observation, which subtracted from, or added to, 90°, according as the declination and latitude are of the same, or of a contrary name, the remainder or sum will be the Sun's *polar distance.*

Now, add together the Sun's corrected altitude and polar distance, and the latitude of the place of observation, and call the difference between half the sum of these and the altitude, the *remainder.*

Then, to the log. co-secant of the polar distance, add the log. secant of the latitude, the log. co-sine of the half sum, and the log. sine of the remainder; half the sum of these will be the log. sine of an arch; which, being multiplied by 8, will be the Sun's distance from the meridian in apparent time. Hence, the apparent time of observation, and the error of the watch, will be known.

Remark.

In practice, it will be necessary to take several altitudes, and the corresponding times per watch, within one or two minutes of time of each other; and in making the computations, it will be found very convenient, and at the same time sufficiently accurate, to estimate the altitude, declination, and latitude, in degrees, minutes, and tenths of a minute, instead of degrees, minutes, and seconds. Moreover, in this and the following computations, 10 is always rejected from the index of the log. secant or tangent, when it exceeds that quantity.

Examples.

I.

March 4, 1810, in latitude 45° 36′ N. and longitude 19° 19′ W. the following altitudes of the Sun's lower limb were observed, the height

height of the eye being 16 feet above the surface of the sea. Required the apparent time of observation, and the error of the watch?

Time per watch.	Alt. $\odot$'s l. limb		
2h. 53' 32" -	24° 59'	$\odot$'s dec. at noon, p. N. A. =	6° 34'
54 30 -	52	Eq. Tab. XIII. to dec. and 2h. 55' P. M. } =	— 3
55 36 -	44		
56 47 -	35	— to dec. and 19° 19' W. =	— 1
20 25	190	Reduced declination, -	6 30
Mean, 2 55 6	24 47.5	Polar distance, -	96 30
Semidiameter, -	+ 16.2		
Dip, - -	— 3.8		
Correction, -	— 1.9		

Cor. alt. $\odot$'s center	= 24 58.0		
Sun's polar dist.	= 96 30	co-secant, - -	0.00280
Ship's latitude,	= 45 36	secant, -	0.15511
Sum, -	167 14		
Half, -	83 32	co-sine, - -	9.05164
Remainder,	58 34	sine, - -	9.93108
			19.14063
Arch, -	21° 49' $\frac{5}{8}$" 8	sine - -	9.57081
Apparent time, -	2 54 37		
Time per watch, -	2 55 6		
Watch fast,	29		

Remark.

If to the sum of the four logarithms 5.30103 be added, and 20 rejected from the index; then this logarithm being found in the Table of Rising, will give the apparent time from noon.

Thus, the sum of the four logs. in the above example,	19.14063
Constant logarithm, - -	5.30103
Time from noon 2h. 54' 37" rising, - -	4.44166

II.

April 20, 1786, in latitude 57° 9'N. and nearly under the meridian of Greenwich, the following altitudes of the Sun's lower limb were observed; height of the eye 10 feet. Required the error of the watch for apparent time?

Time

Time per watch.		Alt. ☉'s l. limb.				
9h. 20′ 27″ A.M.		35° 40′		Dec. at noon, -	11°	40′ N.
21 16	-	35 45		Correction, -	-	3
22 4	-	35 50				
22 53	-	35 55		Reduced dec. -	11	37 N.
23 41	-	36 0		Polar distance, -	78	23
21	Mean,	35 50				
Mean, 9 22 4	Semid.	+ 16				
	Dif. & ref.	— 4				
True altitude,		36 2				
Polar distance,	-	78 23	-	co-secant,	-	0.00899
Latitude,	- -	57 9	-	secant,	-	0.26565
Sum,	- -	171 34				
Half,	- -	85 47	-	co-sine,	-	8.86645
Remainder,	- -	49 45	-	sine,	-	9.88266
						19.02375
Arch,	- -	18° 58′	-	sine,	-	9.51187
		8				
Time from noon,	-	2 31 44				
		12				
Apparent time,	-	9 28 16				
Time per watch,	-	9 22 44				
Watch slow,	- -	6 12				

III.

At noon, October 9, 1810, the latitude of the ship by observation was 43° 23′ N. and longitude by account 158° 20′ W. In the afternoon several altitudes of the Sun's lower limb, and the corresponding times by the watch, were observed, in order to find the error of the watch. The mean of the altitudes was 22° 51′, and that of the times of observation 3h. 16′ 47″, and height of the eye 18 feet. Since noon the course per compass was N. E. by E. 7 knots per hour, and variation 1¾ points westerly. Required the error of the watch?

Distance run = 3h. 17′ × 7k. = 23 miles; and 1¾ points allowed to the left of N. E. by E gives N. E. ¾ N. the true course; to which and the distance 23 miles, the difference of latitude is 18.5′, and departure 13.7′. Now to the latitude 43°, taken as a course, in a traverse table, and departure 13.7′ in a diff. lat. column, the distance is 19′, which is, therefore, the difference of longitude.

Latitude

Latitude at noon,	43° 23′ N.	Longitude at noon,	158° 20′ W.
Difference of latitude,	18 N.	Diff. of longitude,	19 E.
Present latitude,	43 41 N.	Long at time of observ.	158 1 W.
Alt. Sun's lower limb,	22 51	Sun's decl. at noon,	6 7 S.
Semidiameter, -	+ 16	Eq. to dec. and 3h. 17′ P.M.	+ 3
Dip and refract. -	— 6	- - and 158° 20′ W.	+ 10
Corrected altitude,	23 1	Reduced declination,	6 20 S.
Polar distance, -	96 20	- - co-secant,	0.00266
Latitude, -	43 41	- - secant,	0.14076
Sum, -	163 2		
Half, -	81 31	- - co-sine, -	9.16886
Remainder, -	58 30	- - sine, -	9.93077
			19.24305
Arch, -	24 43¼	- - sine	9.62152
	8		
Apparent time,	3 17 50		
Time per watch,	3 16 47		
Watch slow,	1 3		

Many different methods might be given for computing from the same data, the horary distance of an object from the meridian. It is, however, thought proper to subjoin only the following; the second of which is performed by means of a set of tables calculated expressly for that purpose. Vide Tables XXVII. XXVIII. XXIX.

Method Second.

For computing the Horary Distance of an Object from the Meridian.

Rule.

If the latitude and declination are of different names, let their sum be taken, otherwise their difference; subtract the natural versed sine of this sum or difference, from the natural coversed-sine of the corrected altitude, and find the logarithm of the remainder; to which add the log. secants of the latitude and declination, the sum will be the log. rising of the horary distance of the object from the meridian; and if this object is the Sun, the apparent time is known as before.

Examples.

I.

August 10, 1810, in latitude 51° 31′ N. and longitude by account 50° W. at 4h. 5′ 46″ P. M. per watch, the altitude of the Sun's lower limb

limb was 29° 4′, and height of the eye 22 feet. Required the error of the watch for apparent time?

Alt. ☉'s l. limb,	-	29° 4′	☉'s declin.	-	15° 43′ N.
Sun's semidiameter,	-	+ 16	Equation to 50° W.	—	2
Dip and refrac.	-	— 6			
			Reduced declin.		15 41′ N.
True altitude,	-	29 14			

Latitude,	-	51° 31	-	secant,	0.20601
Declination,	-	15 41	-	secant,	0.01648
Difference,	-	35 50	N. v. sine, 18928		
True altitude,	-	29 14	N. co. v. s. 51163		
Difference,	-	-	-	32235 log.	4.50833
Apparent time,	-	4h. 9′ 57″	-	rising,	4.73082
Time per watch,	-	4 5 46			
Watch slow,	-	4 11	for apparent time.		

II.

September 16th, 1810, in latitude 33° 56′ S. and longitude 55° 22′ E. the mean of the times per watch was 8h. 12′ 10″ A. M. and that of the altitudes of the Sun's lower limb 24° 48′; height of the eye 24 feet. Required the error of the watch?

Obs. alt. ☉'s l. limb,	24° 48′	☉'s dec. at noon, p. N. A.		2° 48′.8 N.
Semidiameter,	-	+ 16.0	Equat. to 3h. 48′ A. M.	+ 3 .7
Dip,	- -	— 4.7	—— to 55° 22′ E. -	+ 3 .5
Correction,	-	— 1.9		
			Reduced declination, -	2 56.0 N.
Cor. alt. ☉'s center,	24	57.4		

Latitude,	-	33 56	- -	secant, -	0.08109
Declination,	-	2 56	- -	secant, -	0.00057
Sum,	- -	36 52	N. ver. sine, 19997		
Sun's altitude,	-	24 57.4	N. co. v. s. 57807		
Difference,	-	- - -	37810 log.		4.57761
Sun's mer. dist:	3h. 48′ 16″	-	rising,	-	4.65927
Apparent time,	8 11 44 A. M.				
Time per watch,	8 12 10 A. M.				
Watch fast,	0 26				

Method Third.

For computing the Meridian Distance of an Object.

Rule.

Enter Tab. xxvii. with the declination of the object at the top, and the latitude of the place of observation in the side column; take out the corresponding number, to which prefix the index 4, and add to it the log. sine of the corrected altitude; find the natural number answering thereto, to which apply the number from Table xxviii. by subtraction or addition, according as the latitude and declination are of the same or of contrary names. Now, find the above difference or sum in Table xxix. and the corresponding time will be the distance of the object from the meridian.

Example.

May 7, 1803, in latitude 56° 4′ N. and longitude 7° 30′ W. at 4h. 37′ 4″ P.M. per watch, the altitude of the Sun's lower limb was 25° 6′.1, and height of the eye 18 feet. Required the error of the watch for the apparent time?

Alt. ☉'s lower limb,	25° 6′.1	☉'s dec. p. N. A. =	16° 37′.5 N.
Semidiameter, -	+ 15.9	Eq. to 4h. 37′ P.M. +	3.2
Dip, - -	— 4.1	- to 7° 30′ W. +	.3
Correction, - -	— 1.9		
		Reduced declination,	16 41.0 N.
Cor. alt. ☉'s center,	25 16.0		

To latitude 56° 4′, and declination 16° 41′, the number from

Table xxvii.	=	4.2719	Table xxviii.	=	4451
Alt. 25° 16′	sine	9.6303			
Sum, - -		3.9022	Natural number,	-	7984 N.
Apparent time, -		4h. 37′ 20″	per Table xxix.	-	3529
Time per watch, -		4 37 4			
Watch slow, -		16			

Method Fourth.

For computing the Horary Distance of an Object from the Meridian.

Rule.

Add the declination of the object to the latitude of the place, if they are of the same name, but take their difference if of contrary names: then find half the sum of this sum or difference, and the true altitude; also, half their difference, which reduce to time. Now, to

to the secant of the latitude, add the secant of the declination, the co-sine of the half sum, and the log. of the half difference in time from Table XLIX.; then the sum of these four log.'s, rejecting tens from the index, being found in Table L. will give the apparent time from noon.

EXAMPLE.

In latitude 38° 30′ S. and longitude 28° 56′, May 18, 1810, the true altitude of the Sun's center will be 18° 47′, at 9h. 8′ 52″, A. M. Required the error of the watch?

Declination at noon at Greenwich,				19° 28′ N.
Equation to 2h. 53 from noon,				— 2
to 28 36..				— 1
Reduced declination,				19 25 N.
Co-latitude,	51° 30″		co-secant,	0.10646
Declination,	19 25		secant,	0.02547
Difference,	32 5			
Altitude,	18 47			
Sum,	50 52	half 25° 26′	co-sine,	9.95573
Difference,	13 18	half 6 39 = 26′ 36″	mid. t.	4.36475
Time from noon,		2h. 56′ 54″	Rising,	4.45237
Apparent time,		9 3 6		
Time per watch,		9 8 52		
Watch fast,		5 46		

METHOD FIFTH.

For computing the Apparent Time from the observed Altitude of a known Object.

RULE.

Find the difference between the natural sine of the true altitude, and the natural number answering to the sum of the log. sines of the latitude and declination, if of the same name; but, if of contrary names, the sum of these quantities is to be taken; then, to the log. of this difference or sum, add the log. secants of the latitude and declination; the sum will be the log. co-sine of the hour angle, in degrees.

EXAMPLE.

Let the latitude be 43° 39′ N. the Sun's altitude 39° 28′, and declination 16° 37′ N. Required the apparent time?

 Lat.

Lat. -	43° 39′	- sine,	- 9.83901	- secant,	-	0.14052
Declin.	16 37	.. sine,	- 9.45632	- secant,	-	0.01853
Sum,	- -	-	- 9.29533	- 19739		
Altitude, 39° 28′	-	-	nat. sine,	- .63563		
Difference,		-	- -	43824	log.	9.64171
Time from noon,	-	-	50° 48′	- co-sine,	-	9.80076
			4			
Apparent time,		-	3 23 12	P. M.		
Or,	- -	-	8 36 48	A. M.		

Method Sixth.

For computing the Apparent Time from the observed Altitude of a known Object.

Rule.

If the latitude and declination be of the same name, take their difference, and then their sum; but if of a contrary name, their sum is to be taken, and then their difference. Now, find the difference between the natural versed sine of this sum or difference, and the natural co-versed sine of the true altitude: also the difference between the natural co-versed sine of altitude, and the natural versed sine of the supplement of the sum or difference of the latitude and declination, according as they are of the same, or of a contrary name. Then, from the logarithm of the first of these differences, its index being increased by 20, subtract the logarithm of the second, and half the remainder will be the logarithmic tangent of half the hour angle in degrees *.

Example.

Let the latitude be 53° 24′ N. the declination 5° 44′ N. and altitude 19° 51′. Required the apparent time?

Lat. - 53° 24′ N.
Decl. - 5 44 N.

Diff. -	47 40	n. v. sine 326557	Dif. 1st & 2d 333884	log. 5.523596
Alt. -	19 51	n. co-v. sin. 660441	Dif. 2d & 3d 852601	log. 5.930751
Sum, -	59 8	n. v. s. sup. 1.513042		
				19.592845
Half hour angle, - -	32° 2′ 15″	- tangent, -		9.796422
Hour angle, - - -	64 4 30 = 4h. 16′ 18″			

* If the log. of the difference between the second and third nat. ver. sines, be subtracted from that of the difference of the first and third, its index being increased by 20, then half the remainder will be the log. co-sine of half the hour angle. Or, to the remainder, add the constant log. 0.301030, and the nat. number answering to this sum, rejecting tens in the index, will be the nat. versed sine of the supplement of the horary distance of the object from the meridian, in degrees, &c.

Remark.

Remark.

From the same data, the apparent time may be found by inspection in a traverse table, or by the sliding gunter. [See the Author's treatise upon the description and use of that instrument in navigation.]

Problem VII.

Given the Latitude of a Place, the Altitude of a known fixed Star, and the Sun's Right Ascension, to find the Apparent Time, and Error of the Watch.

Rule.

Correct the observed altitude of the star, and let its declination and right ascension be reduced to the time of observation.

With the latitude of the place, the true altitude, and apparent declination of the star, compute its horary distance from the meridian, by any of the methods given in the last problem; which being added to, or subtracted from, its right ascension, according as it was observed in the western or eastern hemisphere, the sum or remainder will be the right ascension of the meridian.

From the right ascension of the meridian, increased by 24 hours, if necessary, subtract the Sun's right ascension, as given in the Nautical Almanac, for the noon of the proposed day; the remainder will be the *approximate time* of observation; from which subtract the equation answering thereto, and the Sun's right ascension, from Table XVIII. and let the equation from the same table, corresponding to the longitude, be added or subtracted, according as the ship is to the east or west of Greenwich, and the result will be the apparent time of observation. Hence the error of the watch will be known.

Example.

December 13, 1812, in latitude 37° 46′ N. longitude 21° 15′ E. a certain phenomenon was observed, and at the same instant the altitude of Arcturus, east of the meridian, was observed to be 34° 6′, the height of the eye 10 feet. Required the apparent time of observation?

Observed alt. of Arcturus =	34° 6′		
Dip and refraction, -	— 4		
True altitude of Arcturus =	34 2		
Polar distance - -	69 50	- co-secant, -	0.02748
Latitude, - - -	37 46	- secant, -	0.10209
Sum, - - -	141 38		
Half, - -	70 49	- co-sine, -	9.51666
Remainder, - -	36 47	- sine, -	9.77727
			19.42350
Arch, - -	30 59½	- sine, -	9.71175
			Arch

Arch - - - -		30	59½
			8
Arcturus east of meridian, =	4	7	56
Arcturus's right ascension, —	14	7	4
Right ascension of merid. —	9	59	8
Sun's right ascension, -	17	22	38
Approximate time, -	16	36	30
Eq. to long. Tab. XVIII.	+		16
Eq. to approx. time, -	—	3	2
App. time of observation,	16	33	44

Remark.

In order to attain the greatest accuracy from observations of this kind, several stars should be observed, and the error of the watch deduced from each star separately. If an equal number of stars be observed on each side of the meridian, and nearly equidistant therefrom, those errors which arise from the instrument, the spheroidal figure of the Earth, &c. will, by this means, be rendered almost insensible. If the ship is under way, during the interval between the observations of the different stars, and if that interval is considerable, it will be necessary to reduce the error to the same meridian, by allowing for the difference of longitude made good between the observations.

Example.

January 30, 1804, in latitude 53° 24′ N. and longitude 25° 18′ W. by account, the following altitudes of Procyon to the west, and Alphacca to the east of the meridian, were observed by two separate persons at the same instant. The elevation of the person's eye, who observed the altitude of Procyon, was 20 feet above the surface of the sea, and the height of his eye, who observed the altitude of Alphacca, was 22 feet. Required the error of the watch?

	Time per watch.			Alt. Procyon.		Alt. Alphacca.	
	14h.	53′	0″	20°	46′	41°	23′
	14	57	0	20	12	42	2
	15	0	32	19	42	42	29
	15	4	0	19	13	43	0
		234	32		233		54
Mean,	14	58	38	19	58.2	42	13.5
	Dip,			—	4.3	—	4.5
	Refraction,			—	2.6	—	1.0
	Cor. altitude,			19	51.3	42	8.0

Computation

Computation of the apparent time from the corrected altitude of Procyon.

Procyon's altitude =	19°	51'.3		
polar distance -	84	16.3	- co-secant, -	0.00217
Ship's latitude, - -	53	24.0	- secant. -	0.22459
Sum, - -	157	31.6		
Half, - -	78	45.8	- co-sine, -	9.28973
Difference - -	58	54.5	- sine, -	9.93265
Sum, - - - - - - -				19.44914
Arch, - -	32	1.8	- sine, -	9.72457
		8		
Procyon's mer. distance,	4 16	14		
—— right ascen.	7 29	2		
Right asc. meridian,	11 45	16		
Sun's right ascension,	20 47	42		
Approximate time,	14 57	34		
Equation to long.	—	17		
Eq. to approx. time,	— 2	34		
Apparent time, -	14 54	43		

Computation of the apparent time from the corrected altitude of Alphacca.

Alphacca's altitude =	42°	8'		
—— polar dist. —	62	37	- co-secant, -	0.05161
Ship's latitude, - -	53	24	- secant, -	0.22459
Sum, - -	158	9		
Half, - -	79	4.5	- co-sine, -	9.27766
Difference, - -	36	56.5	- sine, - -	9.77888
Sum, - - - - - - -				19.33274
Arch, - -	27	38.$\frac{1}{8}$	- sine, -	9.66637
		8		
Alphacca's mer. dist.	3 41	5		
—— right ascen.	15 26	23		
Right ascension mer.	11 45	18		
Sun's right ascension.	20 47	42		
Approximate time,	14 57	36		
Equat. to time and lon.	— 2	51		
Apparent time, -	14 54	45		

App.

App. time by ob. Alph.	14h	54′	45″
App. time by ob. Proc.	14	54	43
Mean, - -	14	54	44
Time per watch, -	14	58	38
Watch fast, - -		3	54

Problem VIII.

Given the Altitude of a Planet to find the Apparent Time of Observation.

Rule.

Let the mean of several altitudes of the planet's center be corrected by the dip of the horizon, and the refraction as usual*.

Reduce the declination of the planet, as given in the Nautical Almanac, to the time and place of observation, and from thence find its polar distance; with which, its corrected altitude, and the latitude of the ship, compute its meridian distance by Problem VI.

Find the proportional part answering to the interval between the reduced time of observation, and the time of the planet's preceding passage over the meridian; which added to, or subtracted from, the time of the preceding transit, according as the following transit happens at a later or an earlier hour, and the sum or difference will be the reduced time of transit; to which the planet's meridian distance being applied by addition or subtraction, according as it was observed in the western or eastern hemisphere, and the sum or difference will be the apparent time of observation: the difference between which, and that shown by the watch, will be its error.

Example.

February 14, 1792, in latitude 43° 26′ N. and longitude 54° 16′ W. the following altitudes of Jupiter were observed, the height of the eye being 14 feet. - Required the error of the watch?

Time p. watch =	12h.	21′	10″	Alt. ♃. =		15°	56′
		22	50	-		16	8
		25	15	-		16	25
		9	15	-	-		29
Mean, - -	12	23	5	-		16	9.7
Long. in time, -	3	37	4	Dip,	—		3.5
				Refract.	—		3.2
Reduced time,	16	0	9				
True altitude of Jupiter.				-		16	3

* If great accuracy is required, the parallax of the planet in altitude must be computed from astronomical tables, and added thereto.

True

True altitude of Jupiter, -	16° 3′		
Polar distance, - -	100 41	co-secant,	0.00759
Latitude, - - -	43 26	secant, -	0.13896
Sum, - - -	160 10		
Half, - - - -	80 5	co-sine, -	9.23607
Remainder, - - -	64 2	sine, -	9.95378
Sum, - - - - -			19.33640
Arch, - - -	27 45½	sine, -	9.66820
Multiply by - - -	8		

Jupiter's meridian distance,	3 42 6		
Time of preced. tran. 13d. 16h. 9′		13d. 16h. 9′	
—— of follow. tran. 19 15 46	red. time	14 16 0	
5 23 37 : 23′ : :		23 51 : 3′ 49″	

Preceding transit, - - - -	16 9 0
Reduced time of transit, - - -	16 5 11
Jupiter's meridian distance, - -	3 42 6
Apparent time, - - - -	12 23 5
Time per watch, - - - -	12 23 5
Watch exact, - - -	0

Problem IX.

Given the Altitude of the Moon, the Latitude and Longitude of a Place, and the estimate Time of Observation, to find the Apparent Time.

Rule.

Correct the altitude of the Moon's limb by Prob. XI. page 112, and reduce its declination to the time and place of observation, by Problem VII. page 109. Now, with the latitude of the place, the altitude and declination of the Moon, compute its distance from the meridian.

To the time of the Moon's passage over the meridian of Greenwich, as given in the Nautical Almanac, apply the equation from Table XX. answering to the daily variation of the Moon's passage and meridian distance, and also that corresponding to the daily variation of transit, and the longitude of the place of observation. The first of these equations is additive, if the Moon be in the western hemisphere at the time of observation, but subtractive, if in the eastern; and the second equation is to be added, if the ship's longitude is west, otherwise subtracted. Hence, the position of the Moon with respect to the meridian of the place of observation will be known; to which the Moon's meridian distance being applied by addition or subtraction according

according as the Moon was observed in the western or eastern hemisphere, and the sum or remainder will be the apparent time of observation.

Example.

March 3, 1792, the following altitudes of the Moon's lower limb were observed, the height of the eye being 10 feet, and the ship's latitude and longitude 51° 38′ N. and 32° 18′ W. respectively. Required the apparent time of observation, and the error of the watch?

Time per watch =	11h. 24′ 16″		Alt. ☽'s lower l. =	38° 12′
	27 0		- - -	37 55
	29 10		- - -	37 31
	31 0		- - -	37 13
	34 9		- - -	36 46
	45 35		- - -	157
Mean, - -	11 29 7		- - -	37 31.4
			Augm. semidiam.	+ 15.1
☽'s dec. at midnight,	17 6 N.		Dip, - -	— 3.0
Eq. to var. dec. and 0h. 31′,	+ 2			
and to long. 32° 18′ W.	— 8		App. alt. ☽'s center	37 43.5
			Correction, Tab. IX.	+ 42.0
Reduced declination, -	17 0 N.			
			Cor. alt. ☽'s center,	38 25.5

To latitude 51° 38′ N. and declination 17° 0′ N. the number

from Table XXVII. =	4.2265	from Table XXVIII. =	3862
☽'s alt. 38° 25′.5 sine,	9.7934		
Sum, - -	4.0199	- Natural number, -	10470
Meridian distance of the Moon, -		3h. 14′ 33″ Table XXIX.	6608

☽'s passage over mer. Gr.	8h. 10′			
Eq. to daily var. and 3h. 14′	+ 6			
and long. 32° 18′ W.	+ 4			
Reduced time, - - - -		8	20	0
Apparent time, - - -		11	34	33
Time per watch, - - -		11	29	7
Watch slow, - - - - -			5	26

By this method of operation, the time can scarcely be found nearer than one minute. If, however, a greater degree of precision be required, let the right ascensions of the Sun and Moon be reduced to the place of observation, and to the apparent time before found. Then to the Moon's reduced right ascension, apply its meridian distance by addition or subtraction, according as the Moon is observed in the western

western or eastern hemisphere, and the right ascension of the meridian will be obtained; from which subtract the Sun's right ascension, and the remainder will be the apparent time of observation, more correct than that found by the former method. Thus in the preceding example.

☉'s R. A. at noon is =	22h. 59′ 52″	☽'s R. A. at midnight,	109° 50′
Eq. to 11h. 34′ T. xviii. +	1 48	Eq. to red. time 1h. 44′ +	0 54
to long. 32° 18′ W. +	20		
			110 44
Reduced right ascens.	23 2 0		4

Moon's right ascension in time, - -	7 22 56
Moon's meridian distance, - - -	3 14 33
Right ascension of the meridian, - -	10 37 29
Sun's right ascension, - - - -	23 2 0
Apparent time, - - -	11 35 29
Time per watch, - - -	11 29 7
Watch slow, - - - -	6 22

If the error of the watch, and the change of the Moon's declination be considerable, the computation of the Moon's distance from the meridian must be repeated.

If the apparent time at Greenwich be known, the time at the ship may be very accurately found, by taking out the right ascensions of the Sun and Moon, and the Moon's declination agreeable to that time. Then, with the ship's latitude, the Moon's correct altitude and declination, its meridian distance is to be computed; which being compared with its right ascension, will give the right ascension of the meridian, from which the Sun's right ascension being taken, the remainder will be the apparent time of observation.

Since the right ascension and declination of the Moon cannot be accurately reduced to any given time and meridian by even proportion, this method of ascertaining the time is, therefore, still liable to some uncertainty, unless the above data be previously corrected by the equation of second difference from Table xxxvii. or computed from astronomical tables for the given time reduced to the meridian of Greenwich.

The position of a celestial object, most favourable for determining the apparent time with accuracy is, when it is in the prime vertical; or, if it does not set, when in that part of its diurnal path which is in contact with an azimuth circle; as then, the change of altitude is quickest, and the latitude is not an essential element. The nearer an object is to either of these positions, the less will be the error in the time, arising from given unavoidable errors in the observations and

and latitude. Table xxv. contains the altitude most proper for this purpose.

When the declination and latitude are of contrary names; the object is past the prime verticle before it rises; and is set before it comes to that circle: in this case, and upon account of the irregularity to which the refraction near the horizon is liable, the altitude of the object should not be under three degrees, nor its meridian distance less than two hours, at the time of observation. Otherwise, it is not proper to use that object for the purpose of ascertaining the apparent time.

It may be also observed, that, the less the latitude and declination, and the greater the altitude of the object, the less is the error in the apparent time, answering to a given error in the altitude, declination, or latitude.

The error in the apparent time answering to a given error in any of these quantities may be found by computation, for which purpose the three following problems are inserted.

Problem X.

Given the Latitude, Azimuth, and Error in Altitude, to find the Error in the apparent Time answering thereto.

Rule.

To the constant log. 1.1761 add the log. co-sine of the latitude, the log. sine of the azimuth, and the proportional log. of the error in altitude; the sum* will be the prop. log. of the error of apparent time; which is additive to the meridian distance, if the error in altitude be in excess, otherwise subtractive.

Example.

Let the latitude of the place of observation be 51° 31′ N. azimuth of the observed object S. 48° 10′ E. and error of altitude 10′. Required the corresponding error of the apparent time?

Constant log.				1.1761
Latitude,	51° 31′		co-sine,	9.7940
Azimuth,	48 10		sine,	9.8722
Error in altitude,	10		p. log.	1.2553
Error in the apparent time,		1′ 26″	p. log.	2.0976

* It is always to be observed, that in order to simplify the operation, tens are constantly to be rejected from the indices, and the sums of indices.

Problem

PROBLEM XI.

Given the Error in Declination, to find the corresponding Error in the Apparent Time.

RULE.

To the constant log. 1.1761 add the log. co-sine of the declination, the log. tangent of the angle of position *, and the prop. log. of the error in declination; the sum will be the prop. log. of the error in the apparent time, which is additive to the meridian distance, if the correction of declination is to be subtracted from the elevated polar distance; otherwise, this correction is subtractive.

EXAMPLE.

Let the declination be 18° 37′, angle of position 37° 29′, and error in declination 10′. Required the resulting error in the apparent time?

Constant log.	- -		- - - -	1.1761
Declination, - -	18° 37′	- -	co-sine, -	9.9767
Angle of position, -	37 29	- -	tangent, -	9.8847
Error in declination, -	10	- -	p. log. -	1.2553
Error in the app. time,	- 0.55″	- -	p. log. -	2.2928

PROBLEM XII.

Given the Error in Latitude, to find the Error in the Apparent Time answering thereto.

RULE.

To the constant log. 1.1761, add the log. co-sine of the latitude, the log. tangent of the azimuth, and the prop. log. of the error in latitude; the sum will be the prop. log. of the error in the apparent time, which is additive if the estimate latitude is greater than the true latitude, otherwise subtractive, the azimuth being less than 90°; if greater, the contrary rule is to be applied.

EXAMPLE.

Let the latitude be 54° 42′, the azimuth 26° 17′, and the error in latitude 10′. Required the error in the apparent time?

Constant log.	- -		- - - -	1.1761
Latitude, - -	54° 42′	- -	co-sine, -	9.7618
Azimuth, - -	26 17	- -	tangent, -	9.6936
Error in latitude, -	10	- -	p. log. -	1.2553
Error in apparent time,	2′ 20″	-	p. log. -	2.8868

* The angle of position may be computed by the following formula. Sine angle of position = co-s. lat. sec. alt. sine mer. dist. = co-s. lat. sec. decl. sine azimuth, to radius unity.

In

In the preceding methods of computing the apparent time at sea, the altitude and declination of the observed object, and the latitude of the ship, were the necessary data. It is, however, almost impossible to obtain these elements sufficiently correct at sea; we have, therefore, shown how to make an allowance for any probable error that may be in either of these quantities; but if those errors are not known, a correction cannot be applied. It is, therefore, thought proper to subjoin the three following methods of finding the apparent time; in the first of which the latitude is not required as a necessary element; in the second, neither the latitude nor altitude of the observed object are necessary; and in the third method, neither the latitude nor declination of the objects are requisite.

PROBLEM XIII.

Given the Latitude of the Place of Observation, to find the Apparent Time when two known fixed Stars were observed in the same Vertical.

RULE.

To the log. co-tangent of the declination of the star nearest the elevated pole, add the log. tangent of the other star's declination, and the log. co-secant of the difference of their right ascensions; and the sum will be the log. of a natural number, the difference between which, and the natural co-tangent of the difference of right ascension*, will be the natural tangent of arch *first*, if the declinations of the stars are of the same name; but if of contrary names, the sum of these quantities will be arch *first*.

Now to the log. co-sine of arch first, add the log. tangent of the latitude, and the log. co-tangent of the declination of the star nearest the meridian; the sum will be the log. co-sine of arch *second*. The difference between arches first and second, when the common azimuth is less than a quadrant, otherwise their sum, will be the meridian distance of the star nearest the meridian. Hence the apparent time is found, as in Prob. VII.

EXAMPLE.

July 2, 1810, in latitude 57° 9′ N. and longitude 2° 8′ W. the stars Vega and Altair were observed in the same vertical at 10h. 9′ per watch. Required the apparent time of observation, and the error of the watch?

* The difference of the right ascensions of the two stars is supposed to be less than six hours, since in practice it will scarcely ever exceed that quantity.

App.

App. R. A. of Vega =	277°	37′	50″			
of Altair =	295	22	59			
Diff. of app. right as. =	17	45	9	-	co-secant, -	0.515834
App. dec. of Vega =	38	36	41	-	co-tangent,	0.097662
— — of Altair =	8	22	27	-	tangent, -	9.167927
					60314 -	9.780421
Difference of right asc.	17	45	9	N. co-tan.	312353	
Arch first, - -	68	21	31	N. tan.	25203 co-s.	9.566786
Latitude, - -	57	9	0	- - -	tangent, -	0.189975
Appar. decl. of Vega, -	38	36	41	- - -	co-tangent,	0.097662
Arch second, -	44	20	28	- - -	co-sine, -	9.854423
Arch first, - -	68	21	31			

	h.	′	″
Mer. distance of Vega, 24 1 3 =	1h.	36′	4″
Apparent right ascension of Vega, -	18	30	31
Right ascension of the meridian, -	16	54	27
Sun's right ascension, - -	6	42	55
Approximate time, - -	10	11	32
Equation to approx. time, -	—	1	45
to longitude, -	—		1
Apparent time, - - -	10	9	46
Time per watch, - - -	10	9	0
Watch slow, - - - -			46

Remarks.

I.

If the right ascensions are equal, the vertical will be the meridian; and hence the apparent time is easily found, by subtracting the Sun's right ascension from that of the star, and applying the equations as formerly.

II.

If the stars be observed on the same vertical at different times, the difference between the observed interval reduced to sidereal time, and the difference of right ascension, is to be used in place of this last quantity.

Problem XIV.

Given the Interval of Time between the Rising of two known fixed Stars, to find the Apparent Time of either Observation.

Rule.

Reduce the observed interval to sidereal time, which may be done with sufficient accuracy, by adding to the interval in mean solar time, the

the proportional part answering to 24 hours, the above interval, and 3′ 56″, which is the mean daily increase of the Sun's right ascension; and let the difference between the reduced interval, and the difference of the right ascensions of the stars, be called arch first.

Now, to the log. co secant of arch first, add the log. co-tangent of the declination of the first observed star, and the log. tangent of the declination of the other star; the sum will be the log. of a natural number, the sum or difference of which, and the natural co-tangent of arch first, according as the declination of the stars are of different, or of the same affection, will be the natural tangent of the meridian distance of the first observed star, if the declination of that star, and the latitude, are of contrary names; otherwise the supplement of the above quantity will be the meridian distance of that star, and hence the apparent time is found as formerly.

EXAMPLE.

October 25, 1792, in longitude 21° E. by account, the interval in mean time between the rising of Aldebaran and Rigel was 3h. 17′ 30″. Required the apparent time of the rising of Aldebaran: and suppose the watch to have pointed out 6h. 39′, at that time, its error is required?

App. R. A. of Rigel = 5h. 4′ 36″   Observed interval, 3h. 17′ 30″
of Aldebaran = 4 24 3   Proportional part, + 32

Difference of R. A. - 0 40 33   Reduced interval, 3 18 2
Reduced interval, - 3 18 2

Arch first, - - 2 37 29 = 39°22′15″ co-s. 0.197680
App. dec. Aldebaran, 16 4 55 - - co-t. 0.540165 0.540165
App. dec. Rigel, - 8 27 9 - - tan. 9.172029

81260 9.909874
Arch first, - - - 39 22 15 N. co-tan. 121868

63 47 20 N. tang. 203128 co-sine, 9.645108

Mer. dist. Aldeba. 116 12 40   Latitude, 56° 52′ tang. 10.185273

In time, - 7h. 44′ 51″
R. A. Aldebaran, 4 24 3

R. A. of the mer. 20 39 12
Sun's right ascen 4 2 18

Approx. time, - 6 36 54
Eq. to appro. time, — 1 3
to long. 21° E. + 13

App t. of ris. Ald. 6 36 4

App.

Appt. t. of ris. Ald.	6h.	36′	4″
Time per watch,	6	39	0
Watch fast, -		2	56

Remarks.

I.

The truth of the operation may be verified by computing the apparent time of the rising of the other star; and if the interval between the times of the rising of the two stars by computation agree with that by observation; or, if the errors of the watch be the same in both cases, the operation is right.

Arch first, - 39° 22′ 15″ - co-secant, - 0.197680
Appt. dec. Rigel, 8 27 9 - co-tangent, 0.827971 0.827971
Appt. dec. Ald. 16 4 55 - tangent, - 9.459835

305834 10.485486
Arch first, - 39 22 15N.co-tang. 121868

Mer. dist. Rigel, 76 50 23N. tangent, 427702 co-sine, 9.356317

In time, - 5h 7′ 22″ Latitude, 56° 52′ tangent, 10 184288
Rt. asc. Rigel. - 5 4 36

R. A. meridian, 23 57 14
Sun's right. asc. 14 2 18

Approx. time, 9 54 56
Eq. to app. time — 1 35
to long. 21° E. + 13

Ap.t. of ris.of Rig. 9 53 34
——— of Ald. 6 36 4

Interval, 3 17 30, which agrees with the observed interval; and hence the operation is right.

II.

If two known stars are observed to rise, or to set, at the same instant, or, if a given star is observed to rise at the time another star is setting, the difference between the right ascensions of the two stars will be arch first; and, the operation being performed by the preceding rules, the apparent time of observation, and latitude of the place, will be obtained.

Problem XV.

Given three Altitudes of the Sun, or a fixed Star, with the Intervals of Time between the Observations, to find the Apparent Time of either Observation.

Rule.

Let half the interval between the observations of the two greatest altitudes be reduced to degrees, and called arch *first*; call half the interval between the instants when the two least altitudes were observed, arch *second*; and let the sum of arches first and second be arch *third*.

From the natural sine of the greatest altitude subtract the natural sines of the other two altitudes respectively; call the greatest remainder arch *fourth*, and the least arch *fifth*.

Now, to the ar-co-log. of arch fourth, add the log. of arch fifth, and the log. sine of arch third; the sum, rejecting radius, will be the log. of arch sixth. To the log. sine of arch first add the log. cosine of arch second; the sum, rejecting radius, will be the log. of arch seventh. To the ar-co-log. of the difference of arches sixth and seventh, add the log. sines of arches first and second; the sum, rejecting radius, will be the log. tangent of arch eighth, the difference between which and arch third, is the apparent time from noon, when the greatest altitude was observed.

Remark.

If the object is a star, the observed intervals must be reduced to sideral time.

Call the index of the log. of the difference of the natural sines 9, when that difference is 1 less than the radius of the table, 8 when it is 2 less, 7 when 3 less, and son on.

Example.

At 9h. 51′ 58″ A. M. per watch, the correct altitude of the Sun's center was 21° 11′ ; at 10h. 48′ 54″ the altitude was 24° 40′; and at 11h. 29′ 42″ the altitude was 26° 0′. Required the apparent time when the greater altitude was observed?

Time per watch.

9h 51′ 58″	Dif.=56′56″ Half=28′28″ In deg. 7° 7′ arch 2d	12° 13′
10 48 54	Dif.=40 48 Half=20 24 In deg. 5 6 arch 1st	=arch 3d.
11 29 42		

Alt. = 21° 11′ Nat. sine, 36135		
24 40 Nat. sine, 41734	dif. = 7702 = arch fourth.	
26 0 Nat sine, 43837	dif. 2103 = arch fifth.	

Arch

Arch fourth	= 7702 ar-co-log. 1.11340	arch first	5° 6′ sine,	8.94887
Arch fifth	= 2103 - log. 8.32284	arch sec.	7 7 co-s.	9.99664
Arch third	= 12° 13′ sine, 9.32553	arch sev.	8821 log.	8.94551
Arch sixth	= - - - - 8.76177	- -	5778	

Difference, - - -	3043 ar-co-log.	1.51670
Arch first, - - -	5° 6′ sine, -	8.94887
Arch second, - - -	7 7 sine,	9.09304
Arch eighth, - - -	19 54 tang.	9.55861
Arch third, - - -	12 13	
Merid. distance, - - -	7 41 in time =	30′ 44″
Apparent time, - - - - - -		11 29 16
Time per watch, - - - - - -		11 29 42
Watch fast, - - - - - - -		26

Remark.

In the preceding problems for computing the apparent time from an observation of the altitude of a celestial object, the Earth was supposed to be spherical, and the computations were performed agreeable to that hypothesis. But because the figure of the Earth is that of an oblate spheroid, the latitude of a place, as inferred directly from observation, is, therefore, greater than the angle at the Earth's center, contained between the equatoreal radius, and a line joining the center of the Earth and the place of observation. This reduction of latitude, according to Sir Isaac Newton's hypothesis, is contained in Table xxxvi.

As the altitude of a celestial object is observed from the visible horizon, whose pole is the apparent zenith; therefore, in order to make an allowance for the spheroidal figure of the Earth, a correction must be applied to the observed altitude, to refer it to the reduced horizon. Table xxvi. contains this reduction. Since the reduced zenith is always more distant from the elevated pole than the apparent zenith, the above correction is, therefore, additive, when the azimuth reckoned from the meridian is less than 90°, but substractive, when it exceeds that quantity.

In order to illustrate the above, the following example of the computation of the apparent time, from an observation of the Sun's altitude, is inserted.

June 24, 1802, in latitude 57° 8′.9 N. and long. 2° 8′ W. about 3h. P. M. the altitude of the Sun's lower limb was 42° 24′, and height of the eye 13 feet. Required the apparent time of observation?

 Altitude

Altitude, - -	42° 24′	Latitude, -	57° 8′.9
Semidiameter, -	+ 15.8	Reduction, -	— 13.7
Dip, - - - -	— 3.4		
Correction, - -	— .9	Reduced lat. -	56 55.2
Corrected alt. -	42 35.5		
Red. Tab. XXXVI. -	+ 5.7		
Reduced altitude, -	42 41.2		
Polar distance, - -	66 33.2	- co-secant, -	0.037427
Latitude, - -	56 55.2	- secant, -	0.262959
Sum, - -	166 9.6		
Half, - -	83 4.8	- co-sine, -	9.080927
Remainder, - -	40 23.6	- sine, -	9.811596
			19.192909
Half hour angle -	23 15.5	- sine, -	9.596454
Multiply by - -	8		
Apparent time, -	3 6 4		

CHAP. IV.

Of the Methods of clearing the Apparent Distance between the Moon and the Sun, or a fixed Star, from the Effects of Refraction and Parallax.

SINCE the observed altitude of a celestial object is affected by two physical causes, the refraction and parallax, whose effects are produced in a vertical direction; it is, therefore, obvious, that the observed distance between any two objects will be also affected by these causes. Indeed, with regard to the fixed stars, the parallax vanishes; and, therefore, these objects are affected by refraction only. But in observations of the Moon particularly, the effect of parallax is very sensible, upon account of its proximity to the Earth. Therefore, by reason of the above causes, the true distance between the Moon and any celestial object is, for the most part, considerably different from that observed.

Let Z (fig. 24) represent the zenith, *s* the apparent place of the Sun,

Sun, and *m* that of the Moon; the arch sm will, therefore, be the apparent distance between these objects. Also ZS, ZM, being vertical circles, passing through the centers of the Sun and Moon, the true and apparent places of these objects will be found therein.

Now, since the refraction is ever greater than the parallax of the Sun at the same altitude, the true place of the Sun will, therefore, be lower than the apparent place, which let be S; and because the Moon's parallax, at any given altitude, is greater than the refraction at that altitude, its true place will, therefore, be higher than the apparent place, which let be M; hence, SM will be the true distance.

The method of reducing the apparent to the true distance, or, in other words, that of clearing the apparent distance from the effects of refraction and parallax, being the most tedious part of the calculus for ascertaining the longitude, when the calculation is performed by the common spherical analogies; many eminent astronomers and mathematicians have, therefore, given compendiums to facilitate the solution of this problem; among which are those by the Chevalier de Borda, the Abbé de la Caille*, Messrs. Delambre, Dunthorne, Elliot, Emerson, Jeaurat, Krafft, De la Lande, Legendre, Lyons, Maskelyne, Robertson, Romme, Witchel, Vince, &c.; but the largest and most elaborate work, that has hitherto appeared for the purpose of correcting the apparent distance, is the *Cambridge Tables* †. These tables were calculated by Messrs. Lyons, Parkinson, and Williams, under the inspection of Dr. Shepherd, Plumian Professor of Astronomy at Cambridge, by the rule formerly given by Mr. Lyons, in the first edition of the *Requisite Tables*.

All the methods that have hitherto been given, for the purpose of reducing the apparent to the true distance, depend on one or other of the two following principles; of which, the first appears to be the most simple and accurate.

First General Principle.

With the apparent zenith distances Zm, Zs, (fig. 24) and the apparent distance between the objects ms, compute the vertical angle mZs: with which, and the true zenith distances, ZS, ZM, the true distance Zm, may be found.

Second General Principle.

Let Z (fig. 25) be the zenith; Zm, Zs, the apparent zenith distances of the Moon and star, and sm the apparent distance. Let Ss, mn, be the refractions in altitude of these objects respectively.

* Mr. James Ferguson's Parallactic Rotula is constructed on the same principles as the Abbe de la Caille's *Chassis de Reduction*.

† These tables are adapted to give the correction answering to the distance between the Moon and a star only; therefore, when the true distance between the Sun and the Moon are required, it becomes necessary to apply another correction depending on the Sun's parallax. This correction may be taken from Tables v. and vi. supplemental of the first edition of the Requisite Tables. Mr. Margett's Longitude Tables are deduced from the Cambridge Tables.

Join

Join Sn, and draw sa, mb perpendicular thereto; and Sa, nb will be the effects of refraction; which, therefore, being applied to the apparent distance, sm will give Sn, the distance corrected by refraction.

Again, let nM be the parallax of the Moon in altitude; then SM being joined, will be the true distance. From M draw Mc perpendicular to Sn, and cn will be the principal effect of parallax in distance; which, being applied to the distance corrected by refraction, will give the arch Sc. Now, in the right-angled spherical triangle ScM, Sc and Mc being given, SM may be found, or rather the difference between SM and Sc may be computed, which, being applied to Sc, will give the true distance SM.

If the object with which the Moon is compared be the Sun, another correction depending on the parallax of that object is necessary.—This correction may be computed on the same principles as the effect of the Moon's parallax.

The first of the following methods is, perhaps, as easy a solution of this problem as has hitherto appeared, especially when the table of natural versed sines is used, whieh accompanies this work. The second is another solution, which will be found extremely easy, by using the table of log. sines, now inserted in the second volume of this edition; and the third and fourth methods are given, merely because they may be performed entirely by natural versed sines. These methods depend on the first general principle. The other methods are deduced from the second general principle.

PROBLEM I.

The Apparent Distance between the Moon and the Sun, or a fixed Star, together with the Altitude of each being given, to find the True Distance.

METHOD I

Of reducing the Apparent to the True Distance.

RULE.

Take the correction of the Moon's altitude from Table IX. to which add the correction* of the Sun's altitude, or the refraction of the star. Now, this sum added to, or subtracted from, the difference of the apparent altitudes, according as the Moon is higher or lower than the Sun or star, will give the difference of the true altitudes.

From the natural versed sine of the observed distance, subtract the natural versed sine of the difference of the apparent altitudes, and to the log. of the remainder add the log. from Table XLII. answering to the Moon's apparent altitude and horizontal parallax, corrected by the number from Table XLIII. or XLIV. according as the distance between

* The difference between the refraction and parallax of the Sun in altitude. The refraction is contained in Table VI. and the Sun's parallax in Table VII.

the

the Moon and the Sun, or a fixed star, is observed. Now, the natural number answering to the sum of these two logs. being added to the natural versed sine of the difference of the two altitudes, will give the natural versed sine of the true distance*.

Examples.

I.

Let the apparent distance between the centers of the Sun and Moon be 81° 23′ 38″, the apparent altitude of the Sun 27° 43′, the apparent altitude of the Moon 48° 22′, and the Moon's horizontal parallax 58′ 45″. Required the true distance?

App. distance	= 81° 23′ 38″	N.V.S. = 850359	
Dif. app. alt.	= 20 39 0	N.V.S. = 64248	
			log.dif.t.XLII. 9.994623
Cor. ☽'s alt.	= + 38 12	Dif. = 786111	- log. - 5.895484
☉'s alt.	= + 1 40	N.No. = 776438	- 5.890107
Dif. true alt.	= 21 18 52	N.V.S. = 68401	
True distance,	81 4 26	N.V.S. = 844839	

II.

Let the apparent distance between the centers of the Sun and Moon be 72° 21′ 40″, the apparent altitude of the Moon 19° 19′, that of the Sun 25° 16′, and the Moon's horizontal parallax 56′ 32″. Required the true central distance?

App. distance	= 72° 21′ 40″	N.V.S. = 696983	
Diff. app. alt.	= 5 57 0	N.V.S. = 5387	
			log. dif.t.XLII. 9.997814
Cor. ☽'s alt.	= — 50 39	Dif. = 691596	- log. - 5.839152
☉'s alt.	= — 1 52	N.No. = 688123	- - 5.837666
Dif. true alt.	= 5 4 29	N.V.S. = 3920	
True distance	= 72 3 50	N.V.S. = 692043	

III.

Let the apparent distance between the centers of the Sun and Moon be 96° 19′ 25″, the apparent altitude of the Sun's center 8° 37′, that of the Moon's 5° 30′, and the horizontal parallax 56′ 20″. Required the true distance?

*Mr. Keith, in his Trigonometry, page 297, says, "Dr. Mackay's first method, page 112 of his Treatise, (first edition), is the simplest I ever met with, where his Tables are used."

App. distance	96° 19′ 25″	N.V.S.	1110144		
Dif. app. alt.	3 7 0″	N.V.S.	001479		
				Log. dif.	9.099501
Cor. ☽'s alt.	— 46 58	Dif.	1108665	- log.	6.044800
☉'s alt.	— 5 53″	N No.	1107390	- - -	6.044301
Dif. true alt.	2 14 9″	N.V.S.	000761		
True distance	96° 12′ 31″	N.V.S.	1108151		

Remarks.

When the objects are in the same vertical, their true distance is greater than the apparent distance, by the difference of the refractions answering to the altitudes of the objects; and, when in the same almacantar, the effect of refraction is nearly equal to 114″ multiplied by the tangent of half the ditance, which quantity, is additive to the apparent distance.

Again, when the objects are in the same vertical, the true distance is greater or less than the observed distance, by the difference of their parallaxes in altitude, according as the altitude of the Moon is greater or less than that of the other object; and, when the objects are in the same almacantar, the true distance is less than the apparent, by a quantity equal to the product of the sum of the horizontal parallaxes, by the tangent of half the distance, and the sine of the common altitude.

The correction of distance is very little, when the angle at the Moon is a right angle; that is, when the sine Moon's alt. = secant dist. × sine alt. Sun or star. Now, the distance and altitude of the star remaining the same, the true distance will be greater or less than that observed, according as the Moon's altitude is greater or less than that found by the above formula. Or, sine alt. star = co-sine dist. × sine alt. Moon. when the correction nearly vanishes, And the true distance will be greater or less than the apparent, according as the altitude of the star is less or greater than the above quantity.

When the observed distance is greater than 90°, it is almost always greater than the true distance..

When the Moon is in the nonagesimal, that is, when a line joining the cusps of the Moon is perpendicular to the horizon, the correction of distance is very little; the objects being nearly at equal distances from the ecliptic, and on the same side. If the object, from which the Moon's distance is observed, be nearer the elevated pole of the ecliptic than the Moon, the true distance will be less than the observed distance; but if the object be more distant from that pole, the true distance will be greater than the apparent.

Method

METHOD II.

Of reducing the Apparent to the True Distance.

RULE.

To the sum of the apparent altitudes of the objects add the correction of the Moon's altitude, and subtract that of the Sun or star, and half the aggregate will be half the sum of the true altitudes.

To the apparent distance add the apparent altitudes of the centers of the Sun and Moon; find the difference between half the sum and the apparent distance.

Take the log. from Table XLII. answering to the Moon's apparent altitude and horizontal parallax, to which add the log. co-sines of the above half sum and difference, reject 10 from the sum of the index; these three logarithms, and half the remainder, will be the log. sine of an arch.

Now add together the log. co-sines of the sum and difference of this arch, and half the sum of the true altitudes; then will half the sum of these two logarithms be the log. sine of half the true distance.

EXAMPLES.

I.

Let the apparent distance of the Sun and Moon's centers be 38° 45′ 40″, the apparent altitude of the Moon's center 29° 31′, that of the Sun's center 35° 43′, and the Moon's horizontal parallax 57′ 43″? Required the true distance?

		Sum of apparent altitudes	65° 14′ 0″
		Correction ☽'s altitude	+ 48 33
		Correction ☉'s altitude	— 1 12
App. distance	= 38° 45′ 40″		
App. altitude ☽	= 29 31 0	Sum of true altitudes -	66 1 21
App. altitude ☉	= 35 43 0	Half sum true altitudes	33 0 40½
Sum - - -	103 59 40	Log. dif. tab. XLII. XLIII.	9.996578
Half - - -	51 59 50	- - - co-sine - -	9.789369
Difference -	13 14 10	- - - co-sine - -	9.988307
Half sum true alt.	33 0 40½	- - - - -	19.774254
Arch - - -	50 27 19½	- - - sine - - -	9.887127
Sum - - - -	83 28 0	- - - co-sine - -	9.056071
Difference - -	17 26 39	- - - co-sine - -	9.979553
			19.035624
	19 14 11	- - - sine - - -	9.517812
True distance -	38 28 22		

Remark.

In place of taking the sine of half the sum of the three logs. its co-sine may be found; then half the sum of the log. sines of the sum and difference of this arch, and half the sum of the true altitudes, will be the log. sine, and half the true distance. By this transformation, the first three logs. will be co-sines, and the last three, sines; which, probably, may assist the memory.

II.

The apparent distance between the Moon's center and α Arietis, is 64° 36′ 40″, their apparent altitudes 44° 33′ and 11° 51′ respectively, and the Moon's horizontal parallax 61′ 10″. Required the true distance?

				Sum of apparent altitudes =	56° 24′ 0″
				Correction of ☽'s altitude	+ 42 38
				Refraction of ✱'s altitude	— 4 27
Apparent distance	64°	36′	40″		
Apparent alt. ☽	44	33	0	Sum of true altitudes -	57 2 11
Apparent alt. ✱	11	51	0	Half sum of true altitudes	28 31 5½
Sum - -	121	0	40	Log. dif. Tab. XLII. and XLIV.	9.994750
Half - -	60	30	20	- - co-sine - -	9.692264
Difference - -	4	6	20	- - co-sine - -	9.998884
Half sum true alt.	28	31	5½	- - - -	19.685898
Arch - -	45	50	58½	- - co-sine - -	9.842949
Sum - -	74	22	4	- - sine - - -	9.983631
Difference -	17	19	53	- - sine - - -	9.474067
					19.457698
	32	23	7	- - sine - - -	9.728849
True distance -	64	46	14		

Remarks.

I.

The remaining part of this operation may be performed as follows: from the natural versed sine of the supplement of the sum of the true altitudes, subtract twice the natural number answering to the sum of the three logs. and the remainder will be the natural versed sine of the true distance.

Sum of four logs. -	9.685898 - -	Nat. No. -	485174½
		Double -	970349
Sum true altitudes -	57° 2′ 11″ -	Nat. ver. sine sup.	1544106
True distance - -	64 46 14 -	Nat. ver. sine -	573757

Or,

Or, from half the sum of the three logarithms, its index being increased by 10, subtract the log. co-sine of half the sum of the true altitudes, the remainder will be the log. sine of an arch; the log. co-sine of which being added to the log. co-sine of half the sum of the true altitudes, the sum, rejecting 10 from the index, will be the log. sine of half the true distance.

Half sum logs.	+ 10	= - -	19.842949		
Half sum true alt.	28° 31′ $5\frac{1}{2}''$	co-sine	9.943824	- -	9.943824
Arch - -	52 26 29	sine -	9.899125	co-sine	9.785025
Half true distance	- - -		32 23 7	sine -	9.728849
True distance	- - - -		64 46 14		

Many other transformations might be given.

II.

When the distance is given in degrees, minutes, and seconds, and the computation performed by means of the common log. tables, it will facilitate the practice to omit the seconds till the operation is finished, and then add them to the computed distance. It may also be observed, that if the unit figure in the minutes of the sum of the apparent altitudes and distance be an odd digit, it will be proper to increase that sum by one minute; and in this case, the complement of the omitted seconds to 60″ must be deducted from the computed distance.

III.

Let the apparent altitude of the Sun's center be 27° 43′, that of the Moon's 46° 18′, the apparent distance 54° 57′ 35″, and the Moon's horizontal parallax 59′ 33″. Required the true distance?

		Sum of apparent altitudes	= 74° 1′ 0″
		Correction of ☽'s altitude	= + 40 14
		Correction of ☉'s altitude	= — 1 40
App. distance — 35″ =	54° 57′		
Apparent altitude ☽	46 18	Sum of true altitudes -	74 39 34
Apparent altitude ☉	27 43	Half sum true altitudes -	37 19 47
Sum - - -	128 58	Log. dif. tab. XLII. XLIII.	= 9.994729
Half - - -	64 29	- - co-sine - -	9.634249
Difference - -	9 32	- - co-sine - -	9.993960
Half sum true altitudes	37 19 47		19.622938
Arch - - -	49 37 15	- - sine - -	9.811469
Sum - - -	86 57 2	- - sine - -	9.999384

 Difference

Difference -	12 17 28	- - sine - -	9.328131
			19.327515
	27 27 20	- - sine - -	9.663757
Computed distance -	54 54 40		
Seconds omitted -	+ 35		
True distance -	54 55 15		

METHOD III.

Of reducing the Apparent to the True Distance.

RULE.

To the natural number answering to the sum of the logarithmic sines of the apparent altitudes, add the natural versed sine of the apparent distance, and reject the left hand unit; then, to the log. of the remainder, add the log. secants * of the apparent altitudes, and the log. co-sines of the true altitudes; and from the natural number answering to the sum of these five logarithms, unity being prefixed, subtract the natural number corresponding to the sum of the logarithmic sines of the true altitudes, the remainder will be the natural versed sine of the true distance.

EXAMPLE.

Let the apparent distance between the centers of the Sun and Moon be 108° 14′ 34″, the apparent altitude of the Sun 36° 25′, that of the Moon 24° 50′, and horizontal parallax 59′ 10″. Required the true distance?

Apparent alt. of the Sun	36° 25′ 0″	Apparent alt. Moon	24° 50′ 0″
Correction of Sun's alt.	— 1 10	Cor. of Moon's alt.	+ 51 39
True alt. of the Sun	36 23 50	True alt. of Moon	25 41 39

App. alt. ☉ 36° 25′ -	sine 9.773533	- secant - -	0.094355	
App. alt. ☽ 24 50 -	sine 9.623229	- secant - -	0.042137	
	9.396762	Nat. N. 249323		
App. dist. 108 14 34 -		Nat. ver. sine - 1.313044		
		562367	log. 9.750020	
True alt. ☉ 36 23 50	sine 9.773333	- co-sine -	9.905754	
True alt. ☽ 25 41 39	sine 9.637057	- co-sine -	9.954783	
	9.410390	Nat. N. 1.558534	9.747049	
		Nat. N. 257270		
True dist. 107 32 1 -		Nat. ver. sine - 1301264		

* Or the arithmetical complements of the logarithmic co-sines.

REMARK.

Remark.

From this method many others may be deduced for ascertaining the true distance between the Moon and the Sun, or a fixed star.

Method IV.

Of reducing the Apparent to the True Distance, by Natural Versed Sines only.

Rule.

From the natural versed sine of the supplement of the sum of the apparent altitudes, subtract the nat. vers. sine of the difference of the apparent altitudes, and call the remainder *arch first.* In like manner proceed with the true altitudes, and call the remainder *aroh second;* and, from the nat. ver. sine of the supplement of the sum of the apparent altitudes, subtract the nat. ver. sine of the apparent distance, and call the remainder *arch third.*

Now, divide the product of arches second and third by arch first; and the quotient, subtracted from the nat. ver. sine of the supplement of the sum of the true altitudes, the remainder will be the nat. versed sine of the true distance.

It will facilitate the operation, to perform this last part by logarithms.

Example.

Let the apparent distance between the Sun and Moon be 59° 28′ 20″, the apparent altitude of the Sun 56° 52′, and that of the Moon 38° 24′, and the Moon's horizontal parallax 61′ 16″. Required the true distance?

App. alt. ☉ 56° 52′ 0″ — 0′ 32″ (corr. ☉) = true alt.		56° 51′ 28″	
☽ 38 24 0 + 46 50 (corr. ☽) = - -		39 10 50	
Sum - 95 16 0 n.v.s. sup.	908209—908209	96 2 18 n.v.s.sup.	894807--894807
Difference 18 28 0 n.v.s.	051492	17 40 38 n.v.s.	047218
Arch first - - -	856717	Arch second -	847589
Appt. distance 59° 28′ 20″ nat. ver. sine	492044		
Arch third - - - - -	416165;	Now, $\frac{416165 \times 847589}{856717}$ =	411731
True distance - - -	58° 52′ 26′	Nat. Ver. Sine - - -	483076

Method V.

Of reducing the Apparent to the True Distance.

Rule.

From the logarithmic difference answering to the Moon's apparent altitude, and horizontal parallax, subtract the constant log. 0.301030, the remainder will be the log. co-sine of an arch. To the natural versed sine of the supplement of the sum, and the difference of the apparent

apparent altitudes of the objects, add the natural versed sines of the sum, and difference of the above arch, and the apparent distance, and the natural versed sine of the difference of the true altitudes, the sum, rejecting 4 in the left hand place, will be the natural versed sine of the apparent.

Example.

Let the apparent distance between the Sun and Moon be 50° 9′ 39″, apparent altitude of the Sun 44° 1′, that of the Moon 35° 44′, and horizontal parallax 57′ 25″. Required the true distance ?

Log dif. to ☽'s alt.	35° 44′			and hor. par. 57′ 25		-	9.995936
Dif. appt. alt. -	8° 17′		0	-	const. log.	-	0.301030
Arch - -	60	18	28	- -	co-sine	-	9.694906
Appt. dist. - -	50	9	39				
Sum of two first -	68	35	28	-	Nat. ver. sine sup.	-	1.365021
Difference - -	52	1	28	-	Nat. ver. sine sup.	-	1.615325
Sum of two last -	110	28	7	-	Nat. ver. sine	-	1 349694
Difference - -	10	8	49	-	Nat. ver. sine	-	015641
Dif. true alt. -	7	30	50	-	Nat. ver. sine	-	008587
True distance -	49	46	19	-	Nat. ver. sine	-	4.354168

Remarks.

I.

If the arithmetical complement of the two first quantities be used, instead of the natural versed sines of their supplements, then 2 is to be omitted in the left hand figure; or, from the sum of the nat. versed sines of the three last numbers, subtract the sum of the versed sines of the two first, and the remainder will be the natural versed sine of the distance.

II.

This method is the same as that given by M. Krafft, of Petersburgh*, the only difference being in the mode of arrangement, and he employs the last method of computation mentioned in the preceding remark, namely, that of subtracting the sum of the two first natural versed sines from that of the three last, to obtain the natural versed sine of the true distance: and in order to simplify this operation, M. Van Swinden calculated a table which he calls *p*, but which has since been named a Table of *Auxiliary Angles*, which may be easily deduced from the table of logarithmic difference, as shewn in the rule.

* Nova Acta Petripolitanæ, 1791.

Upon

Upon this subject M. Van Swinden expresses himself as follows: "De beroemde *Krafft* heeft in het jaar 1791, eene nieuwe en ongemeen eenvouwige manier voorgedraagen om den schynbaaren afstand tot den waaren te herleiden. De bewerking geschiedt zonder *logarithmen*, en bestaat enkel in het optellen van drie, en aftrekken van twee *sinus versus*. Er behooren dan tot deeze handel wyze Tafels van *sinus versus*, ten minsten van 10″ tot 10″: deeze zyn door *Mackay* bereekend geworden, en wy hebben ze van denzelven overgenomen, en in onze *Verzameling*, als de XXIV. Tafel geplaastst. Maar in de manier van *Krafft* wordt een zeekere hoek gebruickt, dien wy *p* zullen noemen: en waarvan de grootte afhangt van de schynbaare hoogte van het middelpunt en van het horizontaal verschilzicht der Maan: Wy hebben eene Tafel van die hoeken *p* bereekend: het is de XXI; waar by de XXII. en XXIII. behooren, om de zelfde reeden als de XVIII. en XXIX. tot. de XVII. Die Tafel is dus in het gebruik volkomen gelik aan de XVII. Tafel*."

Remark.

The preceding methods being derived from the first general principle, are strictly accurate; the rules are also easily remembered, upon account of the uniformity of the mode of calculation. In order to illustrate the second general principle, the following methods are subjoined, which, though only an approximation to the truth, will generally give the true distance within a few seconds.

Method VI.

Of reducing the Apparent to the true Distance.

Rule.

To the log. co-tangent of half the sum of the apparent altitudes, add the log. tangent of half their difference, and the log. co-tangent of half the apparent distance; the sum will be the log. tangent of arch *first*.

If the Moon's altitude is less than that of the star, the sum of arch first, and half the apparent distance, will be arch *second*, and the difference arch third; but if greater, their difference is arch *second*, and sum arch third.

* The celebrated Krafft proposed in the year 1791, a new and uncommonly simple method to find the true from the apparent distance. The operation is without logarithms, and consists only in adding three and subtracting two versed sines. Tables of versed sines are necessary for this method from 10″ to 10″. These have been constructed by Mackay, and we have adopted them in our collection, and placed them as the XXIV. Table. In Krafft's method, a certain *angle* is used which we call *p*, the measure of which depends upon the apparent altitude of the center, and upon the parallax and refraction of the Moon. We have constructed a table of these angles *p*, it is the XXI. to which the XXII. and XXIII. belong, for the same reason that Tables XVIII. and XIX. belong to Table XVII.

To

To the log. co-tangent of arch third, add the log. co secant of the Moon's altitude corrected by refraction, and the prop. log. of the Moon's horizontal parallax*, the sum will be the prop. log. of the first correction, which is to be added to the apparent distance, when arch first is greater than half the distance, the Moon's altitude, at the same time, being greater than that of the Sun. In every other case, this correction is to be subtracted from the apparent distance.

Find the refraction answering to the complements of arches second and third, and their sum, if half the apparent distance, is greater than arch first, otherwise their difference will be the second correction, which is always to be added to the apparent distance.

Enter Table LXXII. with the distance at the top or bottom, and take out the numbers opposite to the Moon's parallax in altitude, and to the first correction, and their difference will be the third correction † of distance, which is to be added if the distance is less than 90°, and subtracted when greater. Hence the true distance between the Moon and the star will be obtained.

But if the object with which the Moon is compared be the Sun, another correction, depending on the parallax of that object, is necessary. This may be found by an operation similar to that employed for computing the second correction, by using arch third, the Sun's altitude, and horizontal parallax. Or find the parallax answering to the complement of the Sun's altitude; now, in the traverse table, the departure, corresponding to the above parallax found in a latitude column, arch third being taken as a course, will be the effect of the Sun's parallax in distance, which is to be applied with a contrary sign to part first of the second correction.

Example.

Let the apparent distance between the Moon's center and a star be 73° 46′ 14″, the apparent altitude of the star 53° 48′, that of the Moon's center 20° 16′, and horizontal parallax 59′ 19″. Required the true distance?

* The proportional logarithm of the Moon's horizontal parallax, may be taken directly from the Nautical Almanac, page VII. of the month.

† The parallax of the Moon in altitude may be found by the rule given in page 84. Or it may be found with sufficient accuracy for this purpose, by entering a traverse table, with the Moon's altitude taken as a course; and the difference of latitude, answering to the horizontal parallax found in a distance column, will be the parallax in altitude. This last correction may be found by means of Table XXII. as directed in the former editions of this work, as follows: Take from Table XXII. the numbers answering to the supplement of twice the distance, and the parallaxes in altitude and distance; half the difference of these numbers being added to, or subtracted from the corrected distance, according as it is less or greater than 90°, will give the true distance between the Moon and a star. Or it may be found, independent of the table, by the rule given in the next method, using the parallax in altitude, (which is to be found as directed in page 84), in place of the correction of the Moon's altitude.

Moon's ap. alt. = 20° 10′
Star's ap. alt. = 53 48

Sum -	74 4	Half	37° 2′	- co-tangent - -	0.1224
Difference -	33 32	Half	16 46	- tangent - -	9.4790
Apparent dist.	73 46 14	Half	36 53	- co-tangent - -	0.1247
First correc.	— 43 48				
Second correc.	+ 2 10	Arch 1st	28 1	- tangent - -	9.7261
No. to dist. 73°					
& p. in alt. 56 = 8″		Arch 2d	64 54	- co-tangent - -	9.6706
first cor. 44 = 5		☽'s alt. ref.	20 14	- co-secant - -	0.4611
		☽'s hor. p.	59 19	- prop. log. - -	0.4821
Third correc.	+ 8				
		First cor. -	43 48	- prop. log. - - -	0.6138
True distance	73 4 39	Arch 2d -	4 54	- complt. 25° 6′ refract.	2′ 1″
		Arch 3d -	8 52	- complt. - 81 8 refract.	9
				Second correction -	2 10

Since, in this example, the altitude of the Moon is less than that of the star, the sum of half the apparent distance and arch first, is arch second, and their difference is arch third. And although half the distance is greater than arch first, yet the Moon's altitude being less than that of the star, the first correction is, therefore, subtractive : also, half the distance being greater than arch first, the sum of the refractions answering to the complements of arches second and third will be the second correction of distance, which is always additive. The third correction is additive, because the distance is less than 90°.

If the Moon had been compared with the Sun, the effect of the parallax of that object in distance should be found, and applied to the distance previously corrected. Thus, in the preceding example, the Sun's altitude corrected by refraction is 53° 47′; hence its complement is 36° 13′, and the parallax of the Sun in altitude answering thereto is 7″; which being found in a latitude column, under 9°, the third arch, the corresponding departure 1″, is the effect of the Sun's parallax in distance; which is subtractive, because part first of the first correction is additive. Hence, in this case, the true distance would be 73° 4′ 38″

The first correction is liable to a small error, which arises from the assumption of the refraction being as the co-tangent of the altitude. This error, however, decreases as the altitude increases; and nearly vanishes, when the altitude exceeds nine or ten degrees.

Method VII.

Of reducing the Apparent to the True Distance *.

Rule.

To the log. co-secant of the apparent distance, add the log. secant of the Moon's apparent altitude, and the log. sine of the Sun's; the sum will

* Another method of reducing the apparent to the true distance, commonly called *Witchel's*, but which was given by *Mr. Emerson*, (see his Astronomy, page 318,) and deduced from Lord Napier's Theorems, is inserted in the Requisite Tables. See also a paper

will be the log. of a natural number, which call *arch first.* And the sum of the log. co-tangent of the distance, and log. tangent of the Moon's altitude, will be the log. of a natural number, which call *arch second.* The difference between arches first and second will be *arch third,* the distance being less than 90°; but if greater, their sum is arch third. Subtract the log. of arch third from the proportional log. of the correction of the Moon's altitude, and the remainder will be the prop. log. of the first correction of distance; which is additive, when arch first is less than arch second, otherwise, it is subtractive.

paper upon this subject, by the Rev. Thomas Elliot, minister at Cavers, in the first volume of the Transactions of the Royal Society of Edinburgh, in which the above method of Witchel's is explained and investigated.

The first part of the operation is the same as in the preceding method; but the remaining part is performed as follows; observing, however, that arch first, is called arch A by Witchel.

When the Sun or star's altitude is greater than the Moon's, take the difference between A and half the apparent distance; but if the Moon's altitude be greatest, take their sum: and to the co-tangent of this sum or difference, add the co-tangent of the Sun or star's apparent altitude, and the proportional logarithm of the correction of the Sun or star's apparent altitude; their sum, rejecting 20 from the index, will be the proportional logarithm of the first correction, which must always be added to the apparent distance, unless the arc A be greater than half the apparent distance, and the Sun or star's altitude, at the same time, greater than the Moon's, in which case it must be subtracted from it.

If the difference between arc A and half the apparent distance was taken in the preceding article, take now their sum; but if their sum was then taken, take now their difference: and to the co-tangent of this sum or difference, add the co-tangent of the Moon's apparent altitude, and the proportional logarithm of the Moon's altitude, their sum, rejecting 20 from the index, will be the proportional logarithm of the second correction, which must always be subtracted from the distance once corrected, unless the arc A be greater than half the apparent distance, and the Moon's altitude, at the same time, greater than the Sun's, in which case it must be added to it, and the sum or difference will be the *corrected distance.*

The third correction is found by Table LXXII. as formerly. The operation is as follows:

Moon's app. alt.	20° 16′				
Star's app. alt. -	53 48				
Sum -	74 4 -	Half - -	37° 2′ -	co-tangent -	0.1224
Difference -	33 33 -	Half - -	16 46 -	tangent - -	9.4790
Apparent dist.	73 46 14 -	Half - -	36 53 -	co-tangent -	0.1247
First correction	+ 9				
Second correction	— 41 51 -	Arch A -	28 1 -	tangent - -	9.7261
Corrected distance	73 4 32	Differ. -	8 52 -	co-tangent -	0.8069
Third cor. found as formerly	3 -	Star's app.	53 48 -	co-tangent -	9.8644
	-	Ref. star -	0 41 -	prop. log. -	2.4206
True distance -	73 4 95				
		First corr. -	0 9 -	prop. log. -	3.0919
		Sum -	64 54 -	co-tangent -	9.6706
		Moon's alt. -	20 16 -	co-tangent -	0.4327
		Corr. ☽'s alt	38 5 -	prop-log. -	0.3303
		Second correction	41 51 -	prop. log. -	0.6336

A fourth correction is necessary in this method, which is always additive, but, being small, may be neglected.

The

The second correction of distance is found in the same manner as the first; the Sun and Moon's altitudes being transposed, and the correction of the Sun's altitude being used in place of that of the Moon. This is additive to the apparent distance, when arch first is greater than arch second; otherwise, it is subtractive.

The third correction may be found as directed in the last method, or as follows:

To the proportional logarithm of the sum of the correction of the Moon's altitude, and first correction of distance, add the prop. log. of their difference, the log. tangent of the distance, and the constant log. 1.5824, and the sum, rejecting radius, will be the prop. log. of the third correction; which is additive or subtractive, according as the distance is less or greater than 90°.

Example.

Let the apparent distance between the Sun and Moon be 107° 44′ 12″, the apparent altitude of the Sun 15° 51′, that of the Moon 6° 36′, and the Moon's horizontal parallax 54′ 30″. Required the true distance?

Ap.dist. 107° 44′	co-sec.	0.02114	cot.	9.50481
alt. ☽ 6 36	sec.	0.00289	tan.	9.06335
☉ 15 51	sine	9.43635 370		8.56816
		9.46038 2886		
Sum - - -		3256 lg		9.5127
Corr. Moon's alt.	46′ 23″	P. Log.		0.5889
First correction -	15 6	- P. L. -		1.0762
Sum - -	61 29	- P. L. -		4655
Difference -	31 17	- P. L. -		7599
Distance -	107 44	- tangent		0.4952
Constant log. - - -				1.5824
Third correction -	5″	- P. L. -		3.3040

Ap.dis. 107 44	co-sec.	0.02114	co-t.	9.50481
alt. ☉ 15 51	sec.	0.01683	tan.	9.45319
☽ 6 36	sine	9.06046 906		8.95800
		9.09843 1254		
Sum - -		2162 L.		9.3349
Correction ☉'s alt.	3′ 11″	P. Log.		1.7524
Second correction	0 41	P. Log.		2.4175
Apparent distance -				107° 44′ 12″
First correction - -				— 15 6
Second correction -				+ 41
Third correction - -				— 5
True distance - -				107 29 41

Method VIII.

Of reducing the Apparent to the True Distance.

This is performed by means of Plate v. * and a circular piece of paper, whose diameter is equal to that of the interior circle, (fig. 26,) in which a line is drawn from its center to the circumference, called the *vertical* or *perpendicular*. In the circle, fig. 26, the altitudes and distance of the objects are to be found. The scale, No. 1, gives the principal effect of the Moon's parallax in distance, and the scales No. 4.

* This plate is a miniature sketch of a large plate drawn with the utmost accuracy, by which the true distance may be found within one or two seconds. It is intended to publish this plate, with its description, and use in finding the longitude, and an investigation of its principles, in one vol. 4to.

and

and 5, exhibit the second correction of the Moon's parallax. The effect of refraction in distance is found by means of scale No. 2, and that of the Sun's parallax by scale No. 5. By this plate the whole operation of finding the longitude of a ship may be performed. This will be explained afterwards.

RULE.

Set the vertical on the moveable circle to the zenith, or 90°; lay a ruler over the given degree of the Moon's altitude, and draw a line which will represent the Moon's parallel of altitude; at the intersection of this line with the vertical, put the character ☽.

Then, set the vertical to the apparent distance of the objects, and draw a line from the center to the zenith; draw also the parallel of altitude of the Sun or star. At the intersection of these lines put the character ✳, and at that of the two parallels, let the character ☉ be marked.

Now the interval ☉, ☽ being applied to the line representing the Moon's horizontal parallax in scale No. 1, will give the principal effect of the Moon's parallax in distance; and to the scale No. 2, on the line expressing the Moon's altitude, will give the effect of the Moon's refraction in distance. The first of these corrections is additive, when ☉ is to the right of ☽, and the second subtractive. The contrary rule is to be applied. when ☉ is to the left of ☽.

Apply the interval ☉ ✳ to the line representing the altitude of the star, scale No. 2, and the correction depending on the star's refraction will be known. If the object with which the Moon's is compared be the Sun, then the above extent being applied to the scale No. 3, will give the effect of the Sun's parallax. The first of these corrections is additive or subtractive, according as ☉ is to the right or left of ✳. The other correction is to be applied with a contrary sign.

With the Moon's apparent altitude and horizontal parallax, find the Parallax in altitude *. Now from the scale No. 4, take the interval between the parallaxes in altitude and distance, on the line representing the distance, which being applied to the scale No. 5, will give the final correction of distance, which is additive when the distance is less than 90°, otherwise subtractive.

EXAMPLE.

Let the apparent distance between the Sun and Moon be 68° 35′ 40″, the apparent altitude of the Moon 30° 44′, that of the Sun 33° 31′, and the Moon's horizontal parallax 59° 17′. Required the true distance?

The parallel of altitude and the Sun's vertical being drawn, and the points of intersection marked, as directed, the measures from the several scales will be as follow:

* See note, page 160.

Apparent

Apparent distance - - - - - - - -		68° 35′ 40″
Interval ☉ ☽ applied to scale No. 1, gives -	—	23 17
- - - - - - - No. 2, - -	+	44
Interval ☉ ✳ applied to scale No. 2, gives -	+	34
- - - - - - No. 3, - -	—	3
Interval from scale No. 4, between 51′ and 23′, the parallaxes in alt. and dist. applied to scale No. 5, gives - - - - - -	+	7
True distance - - - - - - - -		68 13 45

Method IX.

Of reducing the Apparent to the True Distance by the Cambridge, or, as they are sometimes called, Shepherd's Tables *.

Rule.

Take out the reduction and correcting log. answering to the next less degrees of degree of distance, and of the altitudes of the Moon and Sun or star.

Then find the proportional parts of the reduction and correcting log. answering to the next less and next greater degrees of the star's altitude, and to the excess, in minutes, of the given, above the next less degree of altitude; which place under the reduction and correcting log. observing to prefix the signs +, or —, according as these quantities are increasing or decreasing. In like manner, find the prop. parts of reductive and correcting log. to minutes of excess of complete degrees of the Moon's altitude, and also to minutes and seconds above the degrees of distance, which being applied according to their respective signs, the aggregate will be the reduction and correcting logarithm answering to the given distance and altitudes, and to 53 min. of hor. parallax.

* In 1772, a large work was published by order of the Commissioners of Longitude, entitled, *Tables for correcting the Apparent Distance of the Moon and a Star from the Effects of Refraction and Parallax*. These are called by some the *Cambridge Tables*, in consequence of having been printed at Cambridge, and by others *Shepherd's Tables*, being completed under the direction of Professor Shepherd of Cambridge. These tables contain at the top, all distances from 10 to 120 degrees inclusive; each page contains five principal vertical divisions, and each of these is subdivided into other five The first vertical column of each division contains the Moon's altitude, and the second that of a star to whole degrees, within probable limits. The third column contains the joint effects of parallax and refraction, the horizontal parallax being 53. The fourth column contains the correcting logarithm of the change in the effect of parallax, by increasing the horizontal parallax, 9 being the difference between 53 and 62, the least and greatest horizontal parallaxes of the Moon. This table of parallactic logarithms at the beginning of the work, is constructed by subtracting the log. of any number of minutes and seconds less than 9′, from the log. of 9′ reduced to seconds, that is, from the log. of 541. And the fifth column, entitled *Variation*, contains a small correction answering to a difference of 20° from 55, the mean height of the mercury in the thermometer, and to a difference of 1½ inches between 30 inches, and the observed height of the mercury in the barometer. When the mercury in the thermometer is lower than 55°, and that in barometer higher than 30 inches, the variation answering to each is to be added, otherwise subtracted.

From

From the given horizontal parallax, subtract 53′, and add the parallactic log. of the remainder to the correcting log. the sum will be the parallactic log. of a quantity, which being added to the reduction, and the sum applied to the apparent distance, according to its sign, will give the true distance.

Example.

Let the apparent distance between the Moon's center and Regulus be 47° 28′ 40″, the Moon's apparent altitude 50° 46′, that of Regulus 42° 15′, and the Moon's horizontal parallax 58′ 38″. Required the true distance?

To dist. 47° 0′ 0′ alt. ☽ 50′ 0′ alt. * 42′ 0′ the reduction is -	9′ 38″	& cor. log.	707
Prop. part to the diff. between the Nos. to 42° and 43° of * alt.	+ 14	-	− 10
Prop. part to diff. bet. the Nos. to 50° and 51° of ☽'s alt. and 46′	26	-	+ 16
Prop. part to diff. bet. Nos. to 47° and 48 of distance, and 28′ 40″	+ 15	-	− 11
Therefore, to dis. 47° 28′ 40″, ☽'s alt. 50° 46′, & * 42° 15′ the red. is	9 41	cor. log.	707
Moon's hor. parallax 58′ 38″ − 53′ 0′ = 5′ 38″, the par. log. is	-	-	204
Number to the parallactic logarithm - - -	1 7	-	908
Correction of the apparent distance - -	− 10 48		
Apparent distance - - - - -	47 28 40		
True distance - - - - -	47 17 52		

General Remarks.

Upon the different Methods of reducing the Apparent to the True Distance.

Of all the various methods which have been proposed to reduce the apparent distance between any two objects to the true distance, and in which the strictest accuracy is to be prserved, the first of the preceding methods is, perhaps, the shortest; and the second, the most elegant. As the table of natural versed sines is not extended farther than 90°, the first method is, therefore, recommended when the distance is less than 90°; and the second, when it exceeds that quantity.

The several examples to the different methods are purposely varied, that the student may exercise himself in performing them all, by any of the methods he may choose; and the agreement of the final results of the same example, by two or more different methods, will be a pleasing proof of the accuracy of his operations.

To those acquainted with trigonometry, it will be an easy matter to deduce many other methods for reducing the apparent to the true distance, from those we have given for that purpose. But, because it would be an endless task to show all the various transformations that may be made on these methods, we shall, therefore, confine ourselves to a few of the most obvious changes that may be made on the first method;

method; and illustrate the same by the second example given to that method.

I.

In place of taking the difference of the apparent and of the true altitudes, their sum may be used; and, in that case, the natural versed sine of the supplement of the sum is to be employed in the operation, which will be as follows:

Apparent dist.	72° 21′ 40″	N.V.S.	696983		
Sum app. alt.	44 35 0	N.V.S. sup.	1712230	log. diff.	9.997814
Corr. ☽'s alt.	+ 50 39	Difference	1015247	log. -	6.006572
☉'s alt.	— 1 52	Nat. Numb.	1010150	-	6.004386
Sum true alt.	45 23 47	N. V. S. sup.	1702198		
True distance,	72 3 50	N. V. S.	692048		

When the sum of the altitudes exceeds 90°, this will be found to be a very simple method.

II.

Since the rectangle under half the radius, and the difference of the versed sines of any two arches, is equal to the rectangle contained by the sines of the sum and difference of these arches, hence the example will be as follows:

Apparent dist.	72° 21′ 40″	Half	36° 10′ 50″	Const. log.	0.301030
Diff. app. alt.	5 58 30	Half	2 58 30	Log. diff.	9.997814
Corr. ☽'s alt.	— 50 39	Sum	39 19 20	Sine -	9.800324
☉'s alt.	— 1 52	Diff.	33 12 20	Sine -	9.738499
Diff. true alt.	5 4 29	N.V S.	003920		
			688128	- -	9.837667
True distance	73 3 50	N.V.S.	692048		

III.

If the sum of the true altitudes be used in place of their difference, the co-sines of the half sum and half difference are to be employed, and the natural number answering to the sum of the four logarithms, being subtracted from the Nat. versed sine of the supplement of the sum of the true altitudes, will give the natural versed sine of the true distance.

Apparent

Apparent dist.	72° 21′ 40″	Half	-	36° 10′ 50″	Con. log.	0.301030
Sum app. alt	44 35 0	Half	-	22 17 30	Log. diff.	9.997814
Corr. ☽'s alt.	+ 50 39	Sum	-	58 28 20	Co-sine	9.718428
☉'s alt.	— 1 52	Diff.	-	13 53 20	Co-sine	9.977113
Sum true alt.	45 23 47	N.V.S.sup.		1702198		
				1010150		9.004385
True distance	72 3 50	N.V.S.		692048		

IV.

Find the natural number answering to the sum of the two logs. as directed in the first method; to the half of which add the square of the nat. sine of half the diff. of the true altitudes; then, half the corresponding log. will be the sine of half the true distance.

Natural number, (see page 151.)			688126		
Half dif. true alt. 2° 32′ 14½″	sine	8.646118	344063		
Square		7.292236	No. 001960		
Sum			346023	log.	9.539105
Half true distance	36° 1′ 55″		sine		9.976952
True distance	72 3 50				

V.

If to half the sum of the two logarithms in the first method of transformation, page 167, the square of the sine of half the sum of the true altitudes be added, then half the log. of this sum will be the log. co-sine of half the true distance.

Natural number (see page 167,)			1010155		
Half sum true alt. 22° 41′ 53½″	sine	9.586449	505077		
Square		.172898	9148901		
Sum			.653978	log.	9.815563
Half true distance	36° 1′ 55″		co-sine		9.907781
True distance	72 3 50				

VI.

From the sum of the two logs. in the first method, subtract the constant log. 0.301030; and from half the remainder, subtract the log. co-sine of half the difference of the true altitudes, the remainder will

will be the log. sine of an arch; the log. co-sine of which, being subtracted from the log. co-sine of the difference of the true altitudes, will give the log. co-sine of half the true distance.

Sum of two first logarithms		-	9.837668		
Constant log.		-	0.301030		
Remainder		-	9.536638		
Half		-	9.768319		
Half dif. true alt.	2° 32′ $14\frac{1}{2}''$	co-sine	9.999574	- -	9.999574
Arch	35 57 17	sine	9.768745	co-sine	9.908207
Half true distance	36 1 55	-		co-sine	9.907781
True distance	72 3 50				

VII.

If the tangent of the arch, whose sine was found, be subtracted from half the above remainder, the difference will be the log. co-sine of half the true distance.

Half		-	9.768319	- -	9.768319
Half diff. true alt.	2° 32′ $14\frac{1}{2}''$	co-sine	9.999574		
Arch	35 57 17	sine	9.768745	tangent	9.860538
Half true distance	36 1 55	-		co-sine	9.907781
True distance,	72 3 50				

VIII.

From half the above remainder, subtract the log. sine of half the diff. of the true altitudes, the remainder will be the log. tangent of an arch; the log. sine of which, being subtracted from half the remainder, will be the sine of half the true distance.

Half remainder,		-	9.768319	- -	9.768319
Half d. tr. alt.	2° 32′ $14\frac{1}{2}''$	sine	8.646118		
Arch	85 41 $2\frac{1}{4}$	tangent	1.122201	sine	9.998766
Half true distance	36 1 55			sine	9.769553
True distance	72 3 50				

IX.

If the log. co-sine of the arch, whose tangent was found, be subtracted from the log. sine of half the difference of the true altitudes, the remainder will be the log. sine of half the true distance.

Half remainder	- - -		9.768319		
Half d. tr. alt.	2° 32′ 14½″	- sine	8.646118	-	8.646118
Arch	- 85 41 2$\frac{1}{14}$	tangent	1.122201	co-sine	8.876565
Half true distance	- -	36 1 55		sine -	9.769553
True distance	- -	72 3 50			

X.

Find the arch, of which half the remainder is the sine; then, half the sum of the log co-sines of the sum and difference of this arch, and half the difference of the true altitudes, will be the co sine of half the true distance.

Arch - -	35° 54′ 50″	-	sine, -	9.768319
Half diff. true alt.	2 32 14½			
Sum - -	38 27 4½	-	co-sine -	9.893838
Difference -	33 22 35½	-	co-sine -	9.921724
				19.815562
Half true distance	36 1 55	-	co-sine -	9.907781
True distance -	72 3 50			

Various other transformations might be given, such as, by using the sum in place of the difference of the apparent and true altitudes, &c.; however, as the above will serve as a specimen of the variations, or changes, that may be made on the first method of reducing the apparent to the true distance, it, therefore, seems unnecessary to add any more. It may be observed, that each of the other methods which we have given for resolving this problem, are also equally capable of transformation; but these we will not insist upon at present, as not materially tending to illustrate the object we have in view.

Of ascertaining the True Distance, in the Spheroidal Hypothesis.

In the methods hitherto given for correcting the apparent distance between the Moon and the Sun, or a fixed star, the figure of the Earth has been supposed spherical; but it has been formerly mentioned, that the figure of the Earth is that of an oblate spheroid; and since the necessary observations are taken on its surface, it is hence evident, that a farther correction is necessary, upon account of the deviation of the Earth's figure from a sphere. This correction may be computed by a fluxional equation, and applied to the true distance found as formerly; but the most natural method for effecting this is, that of previously reducing the altitudes to those which would have been observed, if the

the figure of the Earth had been spherical, and adapting the Moon's horizontal parallax to the latitude of the place of observation.

Therefore, in order to obtain the true distance, the apparent altitude of each object's center must be corrected by the quantity from Table XXVI. answering to the azimuth of the object, and latitude of the place of observation. The equatorial horizontal parallax of the Moon, as given in the Nautical Almanac, must be diminished by the equation from Table XXXIII. answering to the latitude. Now, with these quantities thus adapted to a spherical Earth, and the apparent distance, the true distance is to be found by any of the methods given for that purpose.

Example.

Let the apparent distance between the centers of the Sun and Moon be 72° 21′ 40″: the apparent altitudes of the Sun and Moon 25° 16′ and 19° 19′ respectively. Azimuths 16¼° and 95½°; the latitude of the place 57° 9′, and the Moon's horizontal parallax per Naut. Almanac, 56′ 32″. Required the true distance?

☉'s ap. alt. =	25° 16′.0	☽'s app. alt. =	19° 19′.0	☽'s hor. par =	56′ 32″
Red.tab.XXVI. +	13 .1	Red.tab.XXVI. —	1 .2	Re.tab.XXXIII. —	10
☉'s red. alt.	25 29.1	☽'s red. alt.	19 17.8	☽'s red. par.	56 22
☽' red. alt.	19 17.8				

Difference	6	11.3		= 6° 11′ 18″			
App. dist.	72°	21′	40″	N. V. S. =	696983		
Diff. app alt.	6	11	18	N. V. S. =	5827	Log. diff.	9.997824
Corr. ☽'s alt. —		50	30	Diff. - -	691156	- log	5.839576
Corr. ☉'s alt. —		1	51				
				Nat. Numb.	687702	- log.	5.837400
Diff. true alts.	5	18	57	N. V. S. =	4301		
True distance	72	3	41	N. V. S. =	692003		

The distance computed in the spherical hypothesis is 72° 3′ 50″. See page 151, which exceeds that found above by 9″.

When the greatest precision is required, the mean refraction, as given in Table VI. must be reduced to the true refraction, by applying the change answering to the state of the barometer and thermometer, from Table VIII. The true distance between the observed limbs of the objects should be computed; for this purpose, the altitudes of those points of the limbs which were observed in contact, and the corrected refractions answering thereto, ought to be employed. To

 which

which distance the apparent semidiameters of the objects being applied, will give the true central distance.

The difference of altitude between the center of the object and the point of contact, may easily be estimated, or it may be found by first computing the angle at its center, contained between the distance and vertical diameter; and the co-sine of this angle will be the difference of altitude, the semi-diameter of the object being radius.

The contraction of semi-diameter is contained in Table XXXII. the arguments of which are, the apparent altitude of the object, and the angle at its center contained between the distance and vertical diameter.

Example.

Let the apparent distance between the centers of the Sun and Moon be 39° 27′ 46″, the apparent altitude of the Sun 20° 10′, that of the Moon 6° 30′; and horizontal parallax 60′ 26″: also, the height of the thermometer 36°, and that of the barometer 30.9 inches. Required the true distance?

Mean ref. at ☽'s alt.	-	7′ 51″	-	At ☉'s alt.	-	2′ 34″
Corr. for thermometer	-	+ 17	-	- -	-	+ 6
barometer	-	+ 20	-	- -	-	+ 6
Corrected refraction	-	8 28	-	Corrected ref.		2 46

Apparent alt. of the Moon	6° 30′ 0″	- secant	-	0.0028
Moon's horizontal parallax	- 60 20	- prop. log.	-	0.4740
Moon's parallax in altitude	- 60 3	- prop. log.	-	0.4768
Corrected refraction	- 8 28			
Correction of Moon's altitude	- 51 35			

App. distance	39° 27′ 46″	Half	19° 43′ 53′		
Diff. app. alt.	13 40 0	Half	6 50 0′	const. log.	0.301030
Corr. ☽s alt.	— 51 35			log. diff.	9.999297
		Sum	26 33 53	- sine -	9.650510
Corr. ☉'s alt.	— 2 46	Diff.	12 53 53	- sine -	9.348727
		Nat. Numb.	- 199326 -		9.299564
Diff. true alt.	12 45 39	N. V. S.	- 024699		
True distance	39 6 23	N. V. S.	- 224025		

Problem

PROBLEM II.

To find the Apparent Time at Greenwich, answering to a given Distance between the Moon and the Sun, or one of the Stars used in the Nautical Almanac.

RULE.

If the given distance is found in the Nautical Almanac in either of pages VIII. IX. X. XI. of the month, opposite to the given day, or to that which immediately precedes or follows it, the time is found at the top of the page. But if this distance is not found exactly in the ephemeris, then, subtract the prop. log. of the difference between the two contiguous distances, one of which being greater, and the other less than the given distance, from the prop. log of the difference between the given and preceding distances, the remainder will be the prop. log. of the excess of the time, above that answering to the preceding distance*; hence the apparent time is known.

EXAMPLE.

♀ December 7, 1804, the true distance between the centers of the Sun and Moon was 59° 43′ 8″. Required the apparent time at Greenwich?

Given distance =59° 43′ 8″		
	Differ.=0° 28′ 22″	P. Log.=8024
Dist. at III. hours=59 14 46		
	Differ.=1 25 27	P. Log.=3236
Dist. at VI. hours=60 40 13		
Excess - - - - -	0 59 46	P. Log. 4788
Preceding time - -	3 0 0	
Apparent time at Greenwich	3 59 46	

* In strictness, this proportional part ought to be corrected by the equation of second difference.

CHAP.

CHAP. V.

Of finding the Longitude at Sea or Land by Lunar Observations.

PROBLEM I.

The Latitude of a Place, and its Longitude by Account, being given, together with the Distance between, and the Altitudes of the Moon and the Sun, or one of the Stars in the Nautical Almanac, to find the correct Longitude of the Place of Observation.

RULE.

REDUCE the estimate time of observation to the meridian of Greenwich, by Prob. III. page 105. To this time take from the Nautical Almanac, page VII. of the month, the Moon's horizontal parallax and semidiameter. Increase the semidiameter by the augmentation from Table XXXI. answering to the Moon's altitude.

Find the apparent and true altitudes of each object's center, and the apparent central distance, with which let the true distance be found, by any of the methods given for that purpose; and find the apparent time at Greenwich answering thereto, by the last problem.

If the Sun or star be at a proper distance from the meridian, when the distance was observed, compute the apparent time at the ship by Prob. VI. Chap. III. If not, the error of the watch may be found from observations of the altitudes of the Sun or stars, taken either before or after that of the distance. Or, the apparent time may be inferred from the Moon's altitude, taken at the same time with the distance, by Prob. IX. page 137.

The difference between the apparent times of observation at the ship and Greenwich will be the longitude of the ship in time, which is east or west, according as the time at the ship is later or earlier than the Greenwich time.

REMARK.

The only purpose to which the longitude by account is applied, is to reduce the time at the ship to the meridian of Greenwich, in order to take the Moon's semidiameter and horizontal parallax from the Nautical Almanac, agreeable to this time; and it is evident that, in most cases, an error of a few degrees in the intimated longitude will not produce any sensible error in these quantities. But in order to ascertain the apparent time at the ship from the Sun's altitude, the declination of that object ought to be taken from the Nautical Almanac,

Almanac, answering to the apparent time at Greenwich deduced from the distance.

EXAMPLES.

I.

♃ November 8, 1804, in latitude 34° 53′ N, and longitude 24° W. by account, about 3h. 50′ P. M. the observed distance between the nearest limbs of the Sun and Moon was 67° 48′ 29″; the observed altitude of the Moon's lower limb 31° 10′: and that of the Sun's 14° 46′; height of the eye 12 feet. Required the true longitude?

Time at ship -	3h 50′ P. M.	Ob. dist. ☉ & ☽'s nearest l.	67° 48′ 29″
Long. in time	1 36	Sun's semidiameter - -	+ 16 13
		Moon's semidiameter -	+ 15 1
Reduced time	5 26 P. M.	Augmentation -	+ 7
		Apparent central distance	68 19 50
Alt. ☽'s l. limb	31 10		
Semidiam.	+ 15	Alt. Sun's lower limb -	14 46
Dip -	— 3	Semidiameter - -	+ 16
		Dip - - - - -	— 3
Ap. alt. ☽'s cen.	31 22		
Ap. alt. ☉'s cen.	14 59	Apparent alt. ☉'s center	14 59
Diff. app. alt.	16 23	Moon's horizontal parallax	55′ 6″

App. distance	68° 19′ 50″	N V. S.	630749		
Diff. app. alt.	16 23 0	N. V. S.	040604		
				log. diff.	9.996556
Corr. ☽'s alt.	+ 45 29	Diff.	590145.	log. -	5.770959
Corr. ☉'s alt.	+ 3 22	Nat. No.	585484	- - -	5.767515
Diff. true alt.	17 11 51	N. V. S.	044708		
True distance -	68 17 46	N. V. S.	630192		
Dist. at III. hours	67 9 17	Differ.	1° 8′ 29″	P. Log.	4197
at VI. hours	68 32 30	Differ.	1 23 13	P. Log.	3351
Excess - - -			2 28 . 8	P. Log.	0846
Preceding time - -			3		
Apparent time at Greenwich			5 28 8		

Sun's obs. altitude	14 46.0	Sun's dec. Nov. 8, at noon	16° 37′.2S.
Semidiameter -	+ 16 2	Equation to 5h. 28′ -	+ 3.9
Dip - - - -	— 3.3		
Correction -	— 3.4	Reduced declination -	16 41.1S.
Sun's true altitude	14 55.5		

Sun's

Sun's true altitude	14 55.5				
- polar distance	106 41.1	-	co-secant	-	0.01868
Ship's latitude -	34 53.0	-	secant	- -	0.08602
Sum - -	156 29.6				
Half - -	78 14.8	-	co-sine	- -	9.30899
Difference - -	63 19.3	-	sine	- -	9.95112
					19.36481
	28 46.2	-	sine	- -	9.68240
	8				

App. time at ship 3 50 10
App. t. at Green 5 28 8

Long. in time - 1 37 58 = 24° 29½′ W.

II.

♀ December 21, 1804, in latitude 42° 24′ S. and longitude 149° 18′ W. by account, about twenty minutes past eight, A. M. the following observations were made; the height of the eye being 17 feet. Required the ship's true longitude?

	Distance nearest limbs ☉ & ☽.		Alt. ☽'s up. l.	Alt. ☉'s l. lb.
Estimate time, 8h 20′ A.M.	114° 55′ 20″		17° 24′	39° 40′
Long. in time 9 57	54 30	- -	17 4	40 2
	53 10	- -	16 39	40 30
Reduced time 6 17 P. M.	51 40	- -	16 13	41 0
	50 0	- -	15 44	41 32
	14 40	- -	184	164
Mean - - -	114 52 56	- -	16 36.8	40 32.8
Sun's semidiameter -	+ 16 19	Semid.	— 15.8	+ 16.3
Moon's semidiameter -	+ 15 45	Dip	— 4.0	— 4.0
Augmentation - -	+ 4			
		App. alt. ☽	16 17.0	40 45.0
Apparent central distance,	115 25 4			

Moon's horizontal parallax	57 47	Apparent alt. ☽	16 17.0
		Sum app. alt. -	57 2 0
		Corr. ☽'s alt.	+ 52 14
		Corr. ☉'s alt.	— 1 0
Apparent distance - -	115° 25′ 4″	Sum true altitudes,	57 53 14
Sum of apparent altitudes	57 2 0	Half sum - -	28 56 37
Sum - - -	172 27 4		

Sum

Sum	172 27 4	Log. difference		9.998123
Half	86 13 32	co-sine		8.818416
Difference	29 11 32	co-sine		9.941008
Half sum true alt.	28 56 37			18.757547
Arch	76 9 37	co-sine		9.378773
Sum	105 6 14	sine		9.984732
Difference	47 13 0	sine		9.865653
				19.850385
Half true distance	57 19 38	sine		9.925192
True distance	114 39 16			
Distance at VI. hours	114 48 6	Diff. 0° 8′ 50″	P. L.	1.3091
Distance at IX. hours	113 15 42	Diff. 1 32 24	P. L.	0.2896
Proportional part		0 17 12	P. L.	1.0195
Preceding time		6		
Apparent time at Greenwich		6 17 12		

App. alt. ☉'s center	40 45	☉'s dec. at n. p. N. A.	23° 27′.8 S.
Correction	— 1	Equation	0.0
True alt. ☉'s center	40 44	Reduced declination	23 27.8 S.
Sun's polar distance	66 32.2	co-secant	0.03748
Ship's latitude	42 24	secant	0.13168
Sum	149 40.2		
Half	74 50.1	co-sine	9.41764
Difference	34 6.1	sine	9.74870
			19.33550
Half horary angle	27 43.8	sine	9.66775
Multiply by	8		
Time from noon	3 41 50		
Apparent time	8 18 10	A. M.	
App. time at Greenw.	6 17 12	P. M.	
Longitude in time	9 59 2 = 149° 45½′ W.		

III.

☿ October 10, 1804, in latitude 15° 15′ N. and longitude 68° E. by account; the following observations were made, the height of the eye being 14 feet. Required the ship's true longitude?

Time per watch.	Dist. ☽'s remote l. fr. Fom.	Alt. ☽'s up. l.	Alt. Fomalhaut.
6h 0′ 4″ P. M.	60° 39′ 45″	46° 57′	20° 34′
2 16	38 40	46 45	20 56
4 40	37 40	46 31	21 22
7 25	36 30	46 16	21 50
6 10 5	60 35 20	46 1	22 18
270	175	150	120
6 4 54	60 37 35	46 30	21 24
		semid. & dip. — 18.6	dip. — 3.6
Moon's semidiam.	— 14 53		
Augmentation	— 9	app. alt. 46 11.4	ap. al. 21 20.4

App. central dist.	60 22 33	N.V.S. 505692	Hor. par. 54′ 39″
Diff. app. altitudes	24 51 0	N.V.S. 092589	log. diff. 9.995204
Correction ☽'s alt.	+ 36 55	Diff. 413103	log. 5.616058
Refraction ✱'s alt.	+ 2 25	N. No. 408566	5.611262
Diff. true altitudes	25 30 20	N.V.S. 097456	
True distance	60 23 51	N.V.S. 506022	

Dist. at noon	61 7 44	diff. 0° 43′ 53″	P. L. 6130
Dist. at III. hours	59 48 5	diff. 1 19 39	P. L. 3541
Apparent time at Greenwich		1 39 10	P. L. 2589

App. alt. Fomalhaut	21° 20′.4		
Refraction	— 2.4		
True altitude	21 18		
Polar distance	120 39	co-secant	0.06535
Latitude	15 15	secant	0.01557
Sum	157 12		
Half	78 36	co-sine	9.29591
Difference	57 18	sine	9.92506
			19.30189
Half hour angle in deg.	26 35.6	sine	9.65094
Multiply by	8		
Mer. dist. Fomalh.	3 32 45		
Right asc. Fomalh.	22 46 50		
Right ascen. mer.	19 14 5		

Sun's

Right ascen. mer.	19° 14′ 5″
Sun's right ascen.	13 2 48
Ap. time at ship	6 11 17
— at Green.	1 39 10
Long. in time	4 32 7 = 68° 1′ E.

As a small error in the altitude of an object near the meridian, may produce a considerable error in the apparent time; and, because the altitude of a star cannot always be so accurately observed as is necessary for establishing the apparent time; nor can the time, inferred from the Moon's altitude, be depended on, unless its declination and right ascension are reduced to the time of observation, by applying the equation of second difference—it, therefore, seems most eligible to use the Sun for this purpose. Hence, if this object is too near the meridian at the time of observation of the distance, it becomes necessary to observe its altitude, when in a more proper situation, either previous, or subsequent thereto. Hence, the error of the watch will be known, which being applied to the time of observation of the distance, will give the apparent time of observation, agreeable to the meridian for which this error was determined. Or, if the change of longitude in time, between the meridians of the places where the observations were made for ascertaining the time and longitude, be applied to the above found apparent time, by addition or subtraction, according as the ship's course made good, has been in the eastern or western quarter, the apparent time will hence be known, for the meridian of the place where the observations were made for determining the longitude. If the watch gains or loses, considerably, allowance must be made for its rate in the above interval.

III.

♀ September 14, 1804, in latitude 17° 40′ S. and longitude 85° E. by account, intending to find the ship's true longitude by an observation of the distance between the Moon and a fixed star: and in order to ascertain the apparent time at the ship, and consequently the error of the watch, several altitudes of the Sun's lower limb were observed, together with the corresponding times per watch The mean of the altitudes was 26° 13′, and that of the times per watch 4h. 2′ 15″: and height of the eye 18 feet. Again, having observed several distances between the Moon's enlightened limb and Antares, with the corresponding altitudes of each, the mean of the times per watch was 10h. 1′ 24″ P. M.; that of the distances 42° 5′ 50″; the mean of the altitudes of the Moon's lower limb was 61° 40.7, and that of the altitudes of Antares 19° 54′.1. Required the true longitude of the ship?

Time per watch 4h 2′ P. M. Obs. alt. ☉'s lower limb 26° 13′
Longitude in time 5 40 E. Semidiameter - - + 16
Dip and refrac. - - — 6

Reduced time 10 22 A. M.
True alt. Sun's center 26 23

Sun's decl. at noon 3° 24′.4
Eq. to 1h. 38′ fr. noon + 1.6

Reduced declinat. 3 26 N. - secant - 0.00078
Latitude - - 17 40 S. - secant - 0.02098

Sum - - - 21 6 Nat. ver. sine 06705
Sun's true altitude 26 23 Nat. co-ver. s. 55563

Diff. - 48858 - - 4.68894

Apparent time 4h. 3′ 36″ - - - rising - - - 4.71070
Time per watch 4 2 15

Watch slow - 1 21
Time p. w. ob. dis. 10 1 24

Ap. time of ob. dis. 10 2 45
Longitude in time 5 40

Ap. time at Green. 4 22 45 Moon's horizontal parallax 55′ 58″

Obs. dist. ☽ & ✳ 42° 5′ 50″ Alt. Ant. 19° 54′.1 Alt. ☽'s l. limb 61° 40′.7
Moon's semid. + 15 13 Dip — 4.1 Semidiam. + 15.4
Augmentation + 13 Dip — 4.1
Ap. al. Ant. 19 50

Apparent dist. 42 21 16 N.V.S. 261009 App. alt. ☽ 61 52.
Diff. app. alts. 42 2 N.V.S. 257245 Log. diff. - 9.993957

Corr. ☽'s alt. + 25 50 Diff. 3764 log. - - - 3.575650
✳'s alt. + 2 37
3712 3.569607

Diff. true alts. - 42 30 27 N.V.S. 262811

True distance - 42 49 17 N.V.S. 266523
Diff. 0° 44′ 40″ - Prop. Log. - 6053
Dist. at III. hours 42 4 37
Diff. 1° 33′ 57 - Prop. Log. - 2824
Dist. at VI. hours 43 38 34

Proportional part - - 1 25 35 - Prop. Log. - 3229
Preceding time - - 3

Apparent time at Greenwich 4 25 35

Apparent

Apparent time at Greenw.	4	25	35
Apparent time at ship	10	2	45

Longitude in time - - 5 37 10 = 84° 17½′ E. the longitude of the ship at the time when the observations were taken to find the error of the watch.

Since, in this example, the Moon and star are nearly in the same vertical, the true distance will, therefore, be nearly equal to the sum of the apparent distance, and the corrections of the altitudes of the Moon and star, the Moon being the most elevated object. If the Moon's altitude had been less than that of the star, the sum of the corrections, subtracted from the apparent distance, will give the true distance nearly. See page 152.

V.

July 12, 1804, in latitude 71° 10′ N. and longitude 44° E. by account; the following observations were made in order to determine the ship's true longitude; the height of the eye being 30 feet.

Times per watch.	Dist. nearest limbs ☉ & ☽	Alt. ☽'s l. l.	Alt. ☉. l. limb.
0h. 24′ 10″	64° 8′ 30″	8° 9′	41° 58′
26 46	9 40	18	56
29 4	10 35	25	54
31 33	11 40	34	51
34 7	12 45	44	48
45 40	190	130	17

Mean -	0 29 8		64 10 38	8 26	41 53.4
Lon. in t.	2 56 0	☉'s sem.	+ 15 47	sem. + 15.2	+ 15.8
		Moon's	+ 15 14	dip — 5.2	— 5.2
Red time	9 33 8	Augm.	+ 2		
				ap. alt. 8 36 -	- 42 4
Apparent central dist. -			64 41 41	ap. alt. moon -	- 8 36
Moon's horizontal parallax			55′ 53″	Difference -	- 33 28

Apparent central dist. -	64 41 41	N.V.S. 572558	
Diff. app. altitudes -	33 28 0	N.V.S. 165793	
			l. d. 9.999122
Corr. ☽'s altitude -	— 49 12	Diff. - 406765	log. 5.609343
Corr. ☉'s altitude -	— 0 57	N. No. 405943	5.608465
Diff. true altitudes -	32 37 51	N.V.S. 157838	
True distance - -	64 8 14	N.V.S. 563781	
Distance at XXI. hours -	63 56 20	diff. 0° 11′ 54″	P. L. 1.1797
Distance at noon - - -	65 23 9	diff. 1 26 49	P. L. .3167
Proportional part - - -		0 24 41	P. L. .8630 Proportional

Proportional part - - - -	0 24 41	P. L.	.8630
Preceding time - - -	21		
Apparent time at Greenwich -	21 24 41		

Since the apparent time at the ship cannot be inferred with sufficient accuracy from the Sun's altitude, taken at the same time with the distance, because of the proximity of that object to the meridian; the following altitudes of the Sun were, therefore, observed in the afternoon, the ship's latitude reduced to that time being 70° 57′ N. and height of the eye as before.

Times per watch -	2h. 55′ 42″	Alt. ☉'s l. l.	35° 1′
	57 16	- - -	34 55
	58 54	- - -	34 48
	3 0 34	- - -	34 41
	3 2 19	- - -	34 33
	2 94 45		238
Mean, - -	2 58 57		34 47.6
Time p. w. of ob. of dist.	0 29 8	Semid. +	15.8
		Dip - —	5.2
Interval - -	2 29 49	Correc. —	1.2
App. time at Green. of ob. dist.	21 24 41	True alt.	34 57
		Polar dist.	68 0 co-sec. 0.03283
App. t. at Gr. of ob. alt.	23 54 30	Latitude	70 57 secant 0.48626
		Sum -	173 54
		Half -	86 57 co-sine 8.72597
		Difference	52 0 sine 9.89653
		Sum - - -	19.14159

Half horary angle in degrees - -	21 51⅛	sine 9.57079
Multiply by - - - - -	8	
Apparent time of observation of altitude	2 54 49	
- - at Greenwich - -	23 54 30	
Longitude in time - - - -	3 0 19 = 45° 4¾′ E. of	

that meridian, where the Sun's altitude was observed, for the purpose of finding the apparent time at the ship.

In place of finding the interval of time, between the observations of the distance and Sun's altitude, &c. as above, this part of the operation may be performed as follows:

Time per watch of ob. Sun's altitude -	2h 58′ 57″	P. M.
Apparent time of observation -	2 54 49	
Watch fast - - - - - -	4 8	

Watch

Watch fast - - - -	0h. 4′ 8″
Time per watch of observ. of dist. -	0 29 8
Apparent time at ship - -	0 25 0
Apparent time at Greenwich -	21 24 41
Longitude in time - - -	3 0 19 = 45° 4¾′ E.

When the apparent time cannot be ascertained from the Sun's altitude, observed at the same time with the distance, in consequence of the proximity of that object to the meridian, especially in high latitudes; and, if it has been impossible to obtain altitudes of the Sun for that purpose, either in the morning, or in the afternoon of that day; it then becomes necessary to deduce the apparent time from the Moon's altitude, taken at the same time with the distance. If the Sun's altitude be carefully observed, the apparent time may be deduced from two sets of observations of that object, although taken near the meridian.

VI.

☽ February 3, 1800, in latitude 57° 7′ N. and longitude 2° 6′ W. by account, the following observations were taken in order to ascertain the true longitude; the height of the eye being 18 feet, and index error of the sextant 2′ additive.

Times p. watch.	Dist. nearest limbs ☉ & ☽.	Alt. ☽'s l. l.	Alt. ☉'s l. l.
11h. 44′ 40″	107° 21′ 0″	8° 8′	16° 3′
46 50	22 10	8 26	4
48 55	22 45	8 38	5
50 26	23 40	8 48	6
52 10	24 30	9 5	7
171	245	185	25
Mean 11 48 34	107 22 49	8 37	16 5

Index error	+ 2 0	Semid.	+ 15	Sem.	+ 16
Sun's semid.	+ 16 16	Dip.	− 4	Dip.	− 4
Moon's sem.	+ 14 51				
Augmenta.	+ 2	Ap. alt. ☽	8 48	Ap. a. ☉	16 17 0
				Ap. a. ☽	8 48 0
Apparent central distance	107° 55′ 58″	cor. ☽ 47′ 55″ − c. ☉ 3′ 5″			+ 44 50

Apparent altitude Moon	8 48		
Apparent altitude Sun -	16 17	Sum true altitudes	25° 49′ 50
		Half sum - -	12 54 55
Sum - - -	133 0 58	Log. dif. t. XLII. XLIII.	9.999132
Half - - -	66 30 29	- co-sine -	9.600559
Difference - -	41 25 29	- co-sine -	9.874960
			19.474651
Arch - - -	56 53 44	- co-sine -	9.737325

Half

Half sum true altitudes -	12 54 55			
Arch - - -	56 53 54			
Sum - - -	69 48 39	- sine - -		9.972461
Difference - -	43 58 49	- sine - -		9.841616
				19.814077
Half true distance -	53 50 1	- sine - -		9.9070384
True distance - -	107 40 2			
Distance at XXI. hours	106 26 57	Diff. 1° 13′ 5″	P. L.	3915
Distance at noon -	107 48 9	Diff. 1 21 12	P. L.	3457
Proportional part - - -		2 41 59	P. L.	0458
Preceding time - - -		21		
Apparent time at Greenwich -		23 41 59		

☽'s app alt 8° 48′	☽'s dec. at noon 23° 14′	Right ascen. 59° 42′
Correction + 48	Eq. to 18′ from n. — 3	Equation - — $9\frac{1}{2}$
☽'s true alt. 9 36	Reduced declin. 23 11 N.	Red. R. A. 59 $32\frac{1}{2}$
In time - - -		3 58 10

Declination of Moon	23° 11′ N.	- secant	- 0.03657
Latitude - -	57 7 N.	- secant	- 0.26526
Difference - -	33 56 N. ver. s.	17031	
Moon's altitude - -	9 36 N. co-v. s.	83323	
Difference - - - -		66292 log.	4.82146
Moon's mer. dist. -	7h. 16′ 40″	rising -	5.12329
Moon's right asc. -	3 58 10		
Right asc. mer. -	20 41 30		
Sun's right asc. -	21 7 55		
Apparent time -	23 33 35		
App. t. at Green. -	23 41 59		
Long. in time	8 24 = 2° 6′ W.		

VII.

In lat. 38° 24′ N. and longitude by account 46° W. upon the 28th of September 1812, at 8h. 10′ A. M. per watch, let the apparent distance between the centers of the Sun and Moon be 79° 38′ 20″; the

the altitude of the Sun 23° 10′, and that of the Moon 62° 40′. Required the true longitude?

Time per watch - 8h 10′ A. M. Horizontal parallax of the Moon
Longitude in time 3 4 W. - 57′ 10″
And semidiameter - 15 35
Reduced time - 11 14 A. M.

App. distance - 79 38 20 N. V. S. 820148
Diff. app. alt. - 39 30 0 N. V. S. 228375
Log. diff. 9.993757
Corr. Moon's alt. + 25 46 Diff. 591773 Log. - 5.772155
Corr. Sun's alt. + 2 5
Nat. N°. 583327 - - 5.765912
Diff. true alt. - 39 57 51 N. V. S. 233554

True distance - 79 27 15 N. V. S. 816981
Diff. - 1 6 16 Prop. log. - 4340
Dist. at XXI hours 80 33 31
Diff. - 1 30 45 Prop. log. - 2974
Prop. part - 79 2 46
- - 2h. 11′ 25″ Prop. log. - 1366
Preceding time - - - 21 0 0

Apparent time at Greenwich 23 11 25
Latitude - 38° 24′ N. - secant - 0.10585
Declination - 2 3 S. - - secant - 0.00028

Sum - - 40 27 Nat. ver. sine 23903
True alt. ☉ - 23 8 Nat. co-ver. sine 60713

Difference - 36810 - log. 4.56597

Time from noon - 3h. 51′ 59″ - rising - - 4.67210

App. time at ship - 20 8 1
App. time at Greenw. 23 11 25

Longitude in time - 3 3 24 = 45° 51′ W.

Problem II.

To find the Longitude of a Place, by Means of Plate V.

Rule.

Reduce the apparent to the true distance, by the rule given for that purpose in page 163.

Draw a line over the Moon's relative motion in three hours, found in the side scales, No. 7. Take the difference between the true and preceding

preceding distances from the scale No. 9, and lay it off on the above line from the right hand; then a line drawn from the upper right hand corner through this point, and produced, will give the proportional part on the scale of hours No. 8; which being added to the preceding time, the sum will be the apparent time at Greenwich.

Put the vertical to 90°, and draw the parallel of declination, and put *m* at the intersection of these two lines; then put the vertical to the latitude of the place found in the quadrant to the right, and draw the parallel of altitude of the object, and at its intersection with the parallel of declination, put the character ✱. Now the interval between *m* ✱ being measured on the parallel of declination, scale No. 10, will give the time from noon or midnight, according as ✱ is to the left or right of *m*. Hence, the apparent time at the place of observation will be known; the difference between which, and the time at Greenwich, will be the required longitude in time.

Example.

In latitude 39° 32′ S. long. by account 115° W. July 8, 1786, about three quarters past XI, P. M. the distance between the Moon's remote limb and Fomalhaut was 69° 43¼′, the altitude of the Moon's upper limb 66° 38′, that of the star 43° 7′, and height of the eye 20 feet. Required the true longitude?

Estim. time	11h. 45′	Dist. ☽ & *	69° 43′ 15″	Alt. ☽ u. l.	66° 38′	Alt. *	43° 7′
Long. in time	7 40	Moon's sem.	— 15 20	Moon's sem.	— 15.8	Dip	— 4.3
				Dip	— 4.3		
Red. time	19 25	App. dist.	69 27 55			Ap. alt.	43 2.7
				App. alt.	66 18.4	Refr.	— 1.0
						True alt.	43 1.7

The vertical, in the moveable circle, being directed to the zenith and distance, and the parallels of altitude drawn, the several corrections from the scales will be as follow:

Apparent distance		69° 27′ 55″
Interval between ⊙ ☽	No. 1.	— 21 24
- - -	No. 2.	+ 24
Interval between ⊙ ✱	No. 2.	+ 1 0
Int. bet. par. in alt. and dist. scales No. 4. and 5.		0
True distance		69 7 55

Now, draw a line over 1° 23′ 38″, the interval between the preceding and the following distances, in the side scales No. 7; take 38′ 23″, the difference between the true distance, and that which immediately precedes it, from the scale No. 9, and lay it off from the right upon this line; and a line drawn from the upper corner to the right, through this point, will intersect the scale No. 8, in the point representing the point 1h. 22′ 37″, which being added to 18h. the

the time answering to the preceding distance, gives 19h. 22′ 37″, the apparent time at Greenwich.

Direct the vertical to the zenith, and draw the parallel of the star's declination; then direct the vertical to the latitude, and draw the parallel of the star's altitude; now, the interval between m ✻ being applied to the parallel of declination*, on the scale No. 10, will give 3h. 49′ 40″, the meridian distance of the star, which subtracted from its right ascension, 22h. 45′ 49″, the remainder 18h. 56′ 9″, is the right ascension of the meridian; from which the Sun's right ascension 7h. 14′ 15″ being subtracted, the remainder 11h. 41′ 54″, is the apparent time at the ship; the difference between which, and 19h. 22′ 37″, the time at Greenwich, is 7h. 40′ 43″, the longitude in time, which, reduced to degrees, is 115° 10¾′ W.

The scale No. 6. is intended to find the change of altitude answering to a given interval of time, independent of the rule in page 81, and is, therefore, very convenient for reducing the altitudes to the time of observation of the distance, when these observations are made by the same person. The change of altitude is found thus:

Take the nearest distance between the point of intersection of the latitude and azimuth, and the nearest horizontal line at the top of the scale, and apply it to the scale No. 9, as often as there are minutes in the given interval, and the change of altitude will be known.

CHAP. VI.

Of finding the Longitude at Sea or Land:

The necessary Data being,

The observed Distance between the Moon and the Sun, or a fixed Star, the Apparent Time, the Latitude of the Place of Observation, and its Longitude by Account.

Introduction.

The distance between the Moon and a star, and the altitude of the Moon, may be very accurately observed at the same time, that the horizon under the star is so indistinct, as to render it impossible to

* As the star's declination is 30° 43′, its parallel is, therefore, a little without the limits of the scale. This, however, can easily be estimated.

observe its altitude to any tolerable degree of accuracy. In this case, therefore, rather than lose an opportunity of determining the ship's longitude, when the distance and altitude of the Moon only can be observed, it will be proper to compute the true and apparent altitudes of the star. The remaining part of the operation is to be performed as usual. At the time of observation of the distance between the Sun and Moon, the horizon under one or both of these objects may be obscured by fog, which is often the case at the entrance of the Channel, and high latitudes; or the ship may be close to the land, so that it may not be safe to make any allowance for dip. It hence becomes necessary to compute the true and apparent altitudes of the objects.

This method will also be found particularly useful at land, especially when the observer is not provided with a proper artificial horizon.

Problem I.

Given the Apparent Time at a known Place, to find the True and Apparent Altitudes of a known Celestial Object.

Rule.

Find the meridian distance of the given object, which, in case of the Sun, is the interval between noon and the given apparent time. But if the object is the Moon or a star, add the Sun's right ascension to the apparent time; the sum will be the right ascension of the meridian, the difference between which, and the right ascension of the object, will be its meridian distance.

To the log. rising *, answering to the meridian distance of the object, add the log. co-sines of the object's declination, and of the latitude of the place of observation. Find the natural number answering to the above sum, which being added to the natural versed sine of the difference, or sum of the latitude and declination, according as they are of the same, or of a contrary name, will give the natural co-versed sine of the true altitude of the object's center.

If the object is the Sun, the quantity from Table x. answering to the true altitude, being added thereto, will give the apparent altitude. In the case of the Moon, take the reduction from Table xi. answering to its horizontal parallax and true altitude, which, subtracted therefrom, gives the apparent altitude. The apparent altitude of a star is found, by adding the quantity from Table xii. answering to its true altitude.

* If the meridian distance of the object exceed the limits of Table xlix. the log. rising may, therefore, be found in all cases, as follows:

Reduce the meridian distance to degrees, and the logarithm of the natural versed sine answering thereto, will be the log rising. Or twice the log. sine of half the meridian distance of the object in degrees, increased by the constant log. 0.30103, will be the logarithm rising.

Examples.

Examples.

I.

Required the true and apparent altitudes of the Sun's center 18th April, 1811, at 4h. 26′ 20″ P. M. in latitude 46° 20′ N. and longitude 60° W. ?

Apparent time	=	4h. 26′ 20″		Sun's declination at noon			10° 35.3
Lon. in time	=	4 0 0		Equation, Table XIII.		—	7.5
Reduced time	=	8 26 20		Reduced declination		-	10 42.8
Apparent time	-	4 26 20	-	- rising		-	4.78002
Declination	-	10 43.8 N.	-	- co-sine		-	9.99234
Latitude -	-	46 20.0 N.	-	- co-sine		-	9.83914
Difference	-	35 37.2	N V. S.	=	18692		
			N. No.		40879	-	4.61150
True altitude	-	23 50 48	N. co-ver. sine		59571		
Red. Table X. =		+ 2 1					
Apparent altit.		23 52 49					

II.

Required the true and apparent altitudes of Regulus. March 29, 1812, at 11h. 35′ 11″ P. M. apparent time in latitude 32′ 14′ S. and longitude 26° 40′ W. ?

Apparent time -	11h. 35′ 11″	Apparent time		11h. 35′ 11″		
Sun's right ascension	0 31 48	Long. in time		1 46 40		
Right asc. of meridian	12 6 59	Reduced time		13 21 51		
Right asc. of Regulus	9 58 21					
Mer. dist. of Regulus	2 8 38	- - rising	-		-	4.18588
Declination of Regulus	12 52 50	N. - co-sine	-		-	9.98893
Latitude - -	32 14 .0	S. - co-sine	-		-	9.92731
Sum - -	45 6 50	N. V. S. =		29430		
		N. No.	-	12651	-	4.19212
True altitude -	35 23 40	N. co-v. S. =		42081		
Equation, Table XII. =	+ 1 20					
Apparent altitude -	35 25 0					

III.

III.

Required the true and apparent altitudes of the Moon, December 13, 1804, at 10h. 40′ 27″ P. M. in latitude 21° 28 S. and longitude 57° 48′ E.?

Apparent time -	10h.	40′	27″		Apparent time	10h.	40′	27″
Sun's right ascension	17	23	39		Long. in time	3	51	12
Right asc. meridian	4	4	6		Reduced time -	6	49	15
Moon's right ascension	2	14	56					
Moon's merid. dist.	1	49	10		- - rising - -			4.04656
Moon's declination	18	56		N.	- - co-sine - -			9.97584
Latitude -	21	28		S.	- - co-sine - -			9.96878
Sum - -	40	24			N. V S. = 23846			
					N. No. = 9799 -			3.99118
True altitude -	41	34	16		N. co-v. S. = 33645			
Equation, Table XI.	—	44	46					
Apparent altitude -	40	49	30					

In this example the Moon's right ascension and declination are corrected by the equation of second difference. See the explanation of Table XXXVII.

Problem II.

Given the Apparent Distance between the Moon and the Sun, or a fixed Star, the Apparent Time, the Latitude and Longitude by account, to find the true Longitude.

Rule.

Compute the true* and apparent altitudes of each object's center by last problem; with which, and the apparent distance, find the true distance, and hence the apparent time at Greenwich as usual. Then will the difference between the apparent times of observation at the ship and at Greenwich, be the longitude in time; which is west or east, according as the time at the place of observation is earlier or later than at Greenwich.

* It is sufficient, in most cases, to compute the true altitude to the nearest minute; but the reduction of altitude must be taken out to the nearest second.

Examples.

Examples.

I.

☿ January 18, 1804, in latitude 16° 20′ N. and longitude 110° W. by account; at 1h. 46′ 18″ P. M. apparent time, the distance between the nearest limbs of the Sun and Moon was 76° 26′ 50″; the altitude of the Moon's upper limb 44° 6′; but the altitude of the Sun could not be observed, in consequence of that part of the horizon under the Sun being obscured by fog; the height of the eye was 14 feet. Required the true longitude?

Apparent time - -	1h. 46′	Sun's decl. at noon -	20° 43½′ S.
Longitude in time -	7 20	Equation Table XIII.	— 4½
Reduced time - -	9 6	Reduced declination	20 39 S.

Apparent time -	1h 46′ 18″	- - rising - -	4.02387
Declination Sun -	20 39 S.	- - co-sine - -	9.97116
Latitude ship - -	16 20 N.	- - co-sine - -	9.98211
Sum - -	36 58 N. V. S.	20101	
		9487	3.97714
True altitude -	44 46 30 n. co-v. s.	29588	
Red. Table X. -	+ 52		
App. altitude - -	44 46 22	Ob. alt. Moon's up. l.	44° 17′
		Semidiameter	— 16.4
		Dip - -	— 3.6

Observed distance -	76 26 50	App. alt. Moon -	43 57
Sun's semidiameter	+ 16 18	App. alt. Sun -	44 46 22
Moon's semidiameter	+ 16 10		
Augmentation -	+ 12	Difference -	0 49 22

Apparent distance -	76 59 30.	N. V. S. 774907	
Difference appar. alt	0 49 22	N. V. S. 000103	
			L. dif. 9.994963
Cor. ☽'s alt. -	— 41 43	774804	Log. 5.889192
Corr. ☉'s alt. -	— 52		
		765870	- 5.884155
Diff. true altitudes -	— 6 47	N. V. S. 000002	
True distance - -	76 27 35	N. V. S. 765872	
		Diff. - - 1° 37′ 57″	P. Log. 2643
Distance at VI. hours	74 49 38		
		Diff. - 1 38 9	P. Log. 2634
Distance at IX. hours	76 27 47		
Proportional part - - -		2 59 38 - -	P. Log. 0009

Proportional

Proportional part	- - -	2 59 38
Preceding time	- - -	6
Apparent time at Greenwich	-	8 59 38
Apparent time at ship	- - -	1 46 18
Longitude in time	- -	7 13 20 = 108° 20′ W.

II.

♄ March 24, 1804, in latitude 40° 36′ N. and longitude 143° E. by account, at 9h. 12′ 18″ apparent time, the distance between the Moon's west limb and Spica Virginis was 36° 41′ 20″, the altitude of the Moon's upper limb 47° 38′; but, by reason of an indistinct horizon, the altitude of the star could not be observed: the height of the eye was 16 feet. Required the true longitude of the ship?

Apparent time -	9h 12′ 18″	Apparent time -	9h 12′
Sun's right ascen.	0 13 25	Long. by acct. in t.	9 32
Right ascen. merid.	9 25 43	Reduced time -	11 40 A.M.
Rt. asc. Spica Virg.	13 14 53		

Mer. dist. Spica Vir.	3 49 10	- - rising	-	4.66241
Declin. Spica Virg.	10 8 S.	- - co-sine	-	9.99317
Latitude ship -	40 36 N.	- - co-sine	-	9.88040
Sum - -	50 44	N. V. S. -	36707	
			34354 - -	4.53598
True altitude -	16 49	N. co-v. s. -	71061	

Red. Table XII.	+ 3 6	Obs. alt. ☽'s up. limb -	47° 38′
		Moon's semidiameter	— 15 2
Apparent altitude	16 52 6	Dip - - -	— 3 8
Apparent distance	36h 41′ 20″	App. alt. ☽'s center -	47 14 0
Moon's semidiam.	— 15 3	App. alt. Spica Virg. -	16 52 6
Augmentation	— 11		
		Diff. appar. altitudes -	30 21 54

Apparent cent. dist.	36 26 6	N. V. S.	195469	
Diff. app. altitudes	30 21 54	N. V. S.	137178	Log diff. 9.995063
Corr ☽'s alt. -	+ 36 37	- Diff.	58291 - -	4.765601
Corr. ✱'s alt. -	+ 3 6			
			57632 - -	4.760664
Diff. true alt. -	31 1 37	N. V. S.	143076	
True distance -	36 56 15	N. V. S.	200708	

True

True distance	- 36 56 15			
Dist. at XXI. hours	38 16 20	- Diff. - 1° 20′ 5″	- P. L. -	3517
- at noon	- 36 44 4	- Diff. - 1 32 16	- P. L. -	2902
Proportional part	- - - 2 36 14		- P. L. -	615
Preceding time	- - - 21 0 0			
Apparent time at Greenwich	- 23 36 14			
Apparent time at ship	- - 9 12 18			
Longitude in time	- - 9 36 4 = 144° 1′ E.			

III.

♃ August 23, 1792, in latitude 48° 54′ S. and longitude, by account, 25° 40′ W. at 1h. 15′ 18″ apparent time, the distance between the nearest limbs of the Sun and Moon was 68° 21 20″. Required the true longitude?

Apparent time -	1h. 15′ 18″	Apparent time -	1h. 15′ 18″
Sun's right ascen.	10 12 25	Longitude in time -	1 42 40
Right ascen mer.	11 27 43	Reduced time - -	2 57 58
Moon's right ascen.	14 33 55	Horizontal parallax	57 48

Moon's mer. dist.	3 6 12	- rising -		4.49455
Moon's declination	11 10 S.	- co-sine -		9.99170
Latitude -	48 54 S.	- co-sine -		9.81781
Difference -	37 44	— N. V. S. - 20913		
		N. No. - 20140	- -	4.30406
True altitude -	36 7	— N. co-v. sine 41053		
Reduction -	— 45 48			
Apparent altitude	35 21 12			
Sun's merid dist.	1 15 18	- rising -		3.72829
Sun's declination	11 6.5, N.	- co sine -		9.99178
Latitude -	48 54.0, S.	- co-sine -		9.81781
Sum - -	60 0.5,	N. V. S. 50013		
		N. No. - 3450	-	3.53788
Sun's true altitude	27 44	N. co-v. sine 53463		
Reduction -	+ 1 40			
Sun's apparent alt.	27 45 40			

 Observed

Observed distance	68° 21′ 20″			Appar. altitude of ☽	35° 21′ 12″
Sun's semidiam.	+ 15 53			Appar. altitude of ☉	27 45 40
Moon's semidiam.	+ 15 45				
Augmentation	+ 8			Diff. of app. alt. -	7 34 32
Apparent distance	68 53 6	N. V. S.	639759		
Diff. app. altitude	7 34 32	N. V. S.	008728		
				Log. dif.	9.995947
Moon's true alt.	36 7	Diff. -	631031	Log. -	5.800051
Sun's true altitude	27 45				
		N. No.	625170	-	5.795998
Diff. true altitudes	8 22	N. V. S.	010643		
True distance -	68 38 33	N.V. S.	635813		
Distance at noon	67 6 52	Diff.	1° 31′ 41″	- P. L. -	2930
Dist. at III hours	68 39 55	Diff.	1 33 3	- P. L. -	2866
Apparent time at Greenwich	2 57 22			- P. L. -	0064
Apparent time at place of obs.	1 15 18				
Longitude in time -	1 42 4 = 25° 31′ W.				

CHAP. VII.

A New Method of finding the Longitude and Latitude of a Ship at Sea.*

Introduction.

In the preceding methods of finding the longitude at sea, the necessary elements, beside the observations, are, the latitude of the ship,

* This method was transmitted to Dr. Maskelyne, several years after the author discovered it; in consequence of which he received a letter from Dr. Maskelyne, dated Royal Observatory, Greenwich, May 22, 1787, from which the following is extracted.

" Last Sunday se'nnight, a paper of yours was left with me, entitled, *A New Method of finding the Longitude and Latitude of a Ship at Sea*, which I was desired to present to the Board of Longitude, if I thought it deserving of that honour. I shall be ready to present it to the Board at their next meeting, as I think it ingenious, and that you might reap the credit you deserve. Your method of correcting the errors arising from a wrong assumption of horizontal parallax of the Moon and semidiameter, are indeed ingenious, and your modes of calculation for finding the latitude, time, and longitude, are neat." For this method the author was honoured with the *thanks* of the Boards of Longitude of England and of France.

ship, and its longitude by account, together with the time at the ship nearly. It may, however, happen, that the latitude is not accurately known; hence the apparent time at the ship cannot be computed; and consequently the ship's longitude will remain unknown. To obviate this, the following method was invented, wherein neither the latitude, longitude, nor time, are required as necessary data, but are found directly from the same set of observations.

Problem.

The Apparent Distance between the Moon and the Sun, or a fixed Star, together with the Altitude of each being given, to find the Latitude and Longitude of the Place of Observation.

Rule.

Take the semidiameters of the Sun and Moon, and the Moon's horizontal parallax, from the Nautical Almanac for the noon of the given day; and let the Moon's semidiameter be increased by its augmentation from Table XXXI.

Reduce the observed altitude of each object's limb to the apparent central altitude, and find the correction of altitude answering to each; from whence, and the central distance of the objects, compute the correct distance as formerly; and find the corresponding time at Greenwich.

To this time, let the Moon's semidiameter and horizontal parallax be again taken from the Nautical Almanac, and find the sum of the corrections of the altitudes of both objects. Then to the ar-co. of the P. L. of the sum of the estimate corrections, add the P. Log. of the sum of the true corrections, and the P. Log. of the difference between the apparent and computed distances; the sum will be the P. Log. of the correction of distance, which correction, together with the change of the Moon's semidiameter, between noon and the Greenwich time of observation, being applied to the apparent central distance, will give the true distance; from which the apparent time at Greenwich is to be re-computed*.

From the N. V. sine of the zenith distance of the object nearest the meridian, subtract the N. V. sine of the difference between the true distance, and the zenith distance of the other object. To the log. of the remainder, add the log. co-secants of the two last quantities; the sum of these three logarithms being found in Table XLIX. gives *arch first*.

From the N. V. sine of the polar distance of the object nearest the meridian, subtract the N. V. sine of the difference between the true

* If the difference between the computed and true distances is a very small quantity, the difference between the approximate and the apparent time at Greenwich, expressed in seconds, will be nearly equal to twice the number of seconds in that quantity.

 distance,

distance, and the polar distance of the other object; to the log. of the remainder, add the log. co-secants of the two last distances; the sum, being found in Table XLIX. gives *arch second*. The difference between arches first and second is *arch third*.

Take the log. of arch third from Table XLIX. to which add the log. sines of the zenith and polar distances of the object farthest from the meridian; add the natural number answering to this sum, to the N. V. sine of the difference between the above zenith and polar distances; the sum will be the co-versed sine of the latitude of the place of observation.

Take the log. of arch third from Table XLVII. to which add the log. co-sine of the latitude, and the log. co-secant of the zenith distance, of the object farthest from the meridian; the sum being found in Table XLVII. gives the distance of that object from the meridian at the time of observation.

Now, if the object whose meridian distance is computed be the Sun, the apparent time at the ship will be known, the difference between which and the Greenwich time, will be the longitude of the ship from Greenwich. But if that object is a star, or the Moon, let the right ascension of the meridian be found, by applying the object's meridian distance to its right ascension, reduced to the Greenwich time of observation, by addition or subtraction, according as it is to the west or east of the meridian; from which subtract the Sun's right ascension, and the remainder will be the apparent time at the ship. Now, the difference between the apparent times of observation at the ship and Greenwich, will be the longitude of the ship in time; which is east, if the time at the ship is later than that at Greenwich, but west, if earlier.

Remark.

If the object from which the time at the ship is inferred be the Moon, or a star, and its altitude imperfectly observed, it will be proper to take altitudes of the Sun for that particular purpose, either previous, or subsequent to the observations for the longitude. In this case, the computed latitude must be reduced to the place of observation of the Sun's altitude, by applying thereto the change of latitude in the interval between the observations; and the longitude found by the comparison of the time thus computed, with the Greenwich time, will be that of the meridian of the place where the observations for the apparent time were made.

Examples.

I.

Being in south latitude ☉ June 24, 1792, in the afternoon, the distance between the nearest limbs of the Sun and Moon was 55° 48′ 34″, the altitude of the Moon's lower limb 43° 23′, that of the Sun's

17° 40′,

17° 40′, and height of the eye 12 feet. **Required the latitude and longitude of the place of observation?**

Obs. dist. n. l.	55° 48′ 34″	Alt. ☽'s l. l.	43° 23′ 0″	Alt. ☉'s l. l.	17° 40′ 0″
☉'s semidia	+ 15 47	Semidiam.	+ 15 7	Semidiam.	+ 15 47
☽'s semidia.	+ 14 58	Dip - -	— 3 18	Dip - -	— 3 18
Augment.	+ 9				
		App. alt.	43 34 49	Appar. alt.	17 52 29
Ap. cen. dist.	56 19 28	Correct.	+ 38 47	Correct.	— 2 47
Moon's app. alt.	43° 34′ 49″	Tr. alt.	44 13 36	True alt. -	17 49 42
Sun's app. alt.	17 52 29				

Difference -	25 42 20	N. V. S.	098965		
Appar. distance	56 19 28	N. V. S.	445510	Log. diff.	9.995391
Moon's true alt.	44 13 36	Diff. -	346545	Log. -	5.539760
Sun's true alt.	17 49 42	N. No.	342887	- -	5.535151
Difference -	26 23 54	N. V. S.	104284		
Computed dist.	56 26 19	N. V. S.	447171		
Dist. at III. h.	55 51 53	Diff.	0° 34′ 26″	- P. L. - -	7183
Dist. at VI. h.	57 15 18	Diff.	1 23 25	- P. L. -	3340

Proportional part - -	1 14 18	- P. L. -	3843
Preceding time - -	3 0 0		
Approximate time at Greenw.	4 14 18		

The Moon's hor. par. at the Greenwich time is 59′ 54″. Now the

Estimate corrections	38′ 47 / 2 47	True corr.	38′ 51″ / 2 47	App. distance / Comp. dist.	56° 19′ 28″ / 56 26 19
Sum -	41 34	Sum - -	41 38	Difference -	6 51

Sum of estimate corrections - - -	41′ 34″	Ar-co-P. L.	9.3635
Sum of true corrections - - - - -	41 38	- - P. L.	0.6358
Difference between app. and comp. dist.	6 51	- - P. L.	1.4196
Difference between app. and true dist.	6 52	- - P. L.	1.4189

Moon's semid. at noon	14′ 58″
— — at comp. time	14 59
Difference - - - -	1
Apparent distance - -	56 19 28
True distance - - -	56 26 21

The

The difference between the true and computed distances being only 2″, the error of the computed time will, therefore, be 4″; hence the apparent time at Greenwich is 4h. 14′ 22″. Moreover, the Moon's true altitude will be 44° 13′ 41″.

True distance -	56°	26′	21″	- - co-secant			-	0.07920
Sun's zen. distance	72	10	18	- - co-secant			-	0.02138
Difference -	15	43	57	N. V. S.	03746			
Moon's zen. dist.	45	46	19	N. V. S.	30248			
				Diff. -	26502	- Log.	-	4.42328
Arch first -	3h.	12	59	- - rising			- -	4.52386
True distance -	56	26	21	- - co-secant			-	0.07920
Sun's polar dist.	113	24	42	- - co-secant			-	0.03731
Difference -	56	58	21	N. V. S.	45496			
Moon's pol. dist.	99	13	0	N. V. S.	116017			
				Diff. -	70521	- Log.	-	4.84832
Arch second -	5	42	9	- - - rising			- -	4.96483
Arch third -	2	29	10	- - - rising			- -	4,31056
Sun's zen. dist.	72	10	18	- - - sine			- -	9.97860
Sun's pol. dist.	113	24	42	- - - sine			- -	9.96269
Difference -	41	14	24	N. V. S. =	24804			
					17859			4.25185
Latitude -	34	59	7	N. co-v. S. =	42663	co-sine		9.91344
Sun's zen. dist.	72	10	18	- - co-secant			-	0.02138
Arch third -	2	29	10	- - H. E. T.			-	0.21762
App. time at ship	2	58	59	- - H. E. T.			-	0.15244
App. t. at Greenw.	4	14	22					
Longit. in time	1	15	23	= 18° 50¾′ W.				

II.

In north latitude, January 10, 1791, about one o'clock P. M. the following observations were taken, the height of the eye being 16 feet. Required the latitude and longitude of the place of observation?

Dist.

Dist. ☉ and ☽'s n. limbs.		Alt. ☽'s low. limb.		Alt. ☉'s low. limb.
	70° 10′ 30″	17° 48′		90° 50′
	11 40	18 17		45
	13 0	18 49		39
	14 10	19 19		32
	9 20	133		166
Mean	70 12 20	18 33 15		9 41 30
Sun's semidiam.	+ 16 19	Semidiam. + 15 6	Semid.	+ 16 19
Moon's semid.	+ 15 2	Dip — 3 49	Dip	— 3 49
Augmentation	+ 4			
		App. alt. 18 44 32	App. alt.	9 54 0
Apparent dist.	70 43 45	Corrrection + 49 30	Correct.	— 5 9
Moon's ap. alt.	18 44 32			
Sun's ap. alt.	9 54 0	True alt. 19 34 2	True alt.	9 48 51

Difference	8 50 32	N. V. S. = 011884		
Apparent dist.	70 43 45	N. V. S. = 669966		
Moon's true alt.	19 34 2		Log. diff.	9.997941
Sun's true alt.	9 48 51	Diff. 658082	Log.	5.818280
		N. No. 654970		5.816221
Difference	9 45 11	N. V. S. 014453		
Correct. dist.	70 41 46	N. V. S. 669423		
Dist. at XXI. h.	69 35 20	Diff. 1° 6′ 36″	P. L.	4329
Dist. at noon	70 59 0	Diff. 1 23 40	P. L.	3327

Proportional part	2 22 55	P. L.	1002
Preceding time	21 0 0		
Time at Greenwich	23 22 55		

The time at Greenwich being so near noon, the correction arising from the change of the Moon's parallax and semidiameter, to be applied to the computed distance, is, therefore, almost insensible.

True distance	70° 41′ 46″	co-secant	0.02513
Moon's zen. dist.	70 25 58	co-secant	0.02583
Difference	0 15 48	N. V. S. = 00001	
Sun's zen. dist.	80 11 9	N. V. S = 82955	
		Diff. 82954 - Log.	4.91884
Arch first	5 44 36	Rising	4.96980

True

True distance -	70 41 46	- co-secant	-	0.02513
Moon's polar dist.	87 34 0	- co-secant	-	0.00039
Difference -	16 52 14	N. V. S. = 04304		
Sun's polar dist.	111 55 47	N. V. S. = 137347		
		Diff. - 133043 - Log.	-	5.12399
Arch second -	7 27 4	- rising	-	5.14951
Arch first -	5 44 36			
Arch third -	1 42 28	- rising	-	3.99252
Moon's zen. dist.	70 25 58	- sine	-	9.97417
Moon's polar dist.	87 34 0	- sine	-	9.99961
Difference -	17 8 2	N. V S. = 04438		
		N. No. = 09253	-	3.96630
Latitude -	59 40	N.Co-v. S. 1369 Co-sine		9.70332
Moon's zen. dist.	70 25 58	- - co secant	-	0.02583
Arch third -	1 42 28	- - H. E. T.	-	0,36417
Moon's mer. dist.	3 35 5	- - H. E. T.	-	0.09332
Moon's right asc.	0 0 20			
Right asc. merid.	20 25 15			
Sun's right asc.	19 27 48			
App time at ship	0 57 27			
Ap. time at Green.	23 22 55			
Longitude in time	1 34 32 = 23° 38′ E.			

The preceding method of computation would be facilitated, by taking the Moon's horizontal parallax and semidiameter from the Nautical Almanac, agreeable to the time answering to the apparent central distance. This time may be known with sufficient accuracy for the above purpose, by inspection. Moreover, if the latitude of the place of observation, and the azimuths of the observed objects, are known nearly, allowance may be made in the computation for the spheroidal figure of the earth. In this case, the computation gives the reduced latitude; and, therefore, the reduction from Table XXXVI. is to be added thereto. The following example is inserted, in order to illustrate the above.

III.

♂ October 16, 1804, the central distance between the Moon and Aldebaran, deduced from seven observations, and corrected by the index

index error, and the error of the line of collimation was 71° 14′ 30″, the apparent altitude of Aldebaran was 31° 58′, that of the Moon 37° 37′: and the azimuths of these objects 69° and 17° respectively, and the estimate latitude 54° N. Required the true latitude and longitude of the place of observation?

The time answering to the central distance is October 17, about 2h. A. M. and at this time the Moon's horizontal parallax is 59′ 42″.

App. alt. Aldeb.	31° 58′.0	App alt. ☽	37° 37′.0	Hor. par.	59′ 42″
Red. tab. XXVI.	+ 5.1	Red. t. XXVI.	+ 13.6	Re. t. XXXIII.	— 10
Red alt. ✱ - -	32 3.1	Red. alt. ☽	37 50.6	Red. par. -	59 32
Red. alt. ☽ -	37 50.6				
Difference -	5 47.5 = 5° 47′ 30″.				

App. distance -	71 14 30	N. V. S.	678423		
Diff. app. alt. -	5 47 30	N. V. S.	005105		
				Log. diff. -	9.995563
Corr. ☽'s alt. -	+ 45 48	-	673318	- Log. -	5.828220
Corr. ✱'s alt.	+ 1 31	-	666474	- -	5.823783
Diff true alt. -	6 34 49	N. V. S.	006587		
True dist. - -	70 55 1	N. V. S	673061		
Dist. at midnight	72 24 34	- Diff. -	1° 29′ 33″	- P. L. -	3032
at XV. hours	70 38 35	- Diff. -	1 45 59	- P. L. -	2300
Proportional part	- -		2 32 5	- P. L. -	732
Preceding time	- - -		12 0 0		
Apparent time at Greenwich	-		14 32 5		

True distance -	70° 55′	- -	Co-secant -	0.02455
Aldeb. zen. dist.	57 58.4	-	Co-secant - -	0.07071
Difference -	12 56.6	N. V. S. 02541		
Moon's zen. dist.	51 28.6	N. V. S. 37603		
		Diff. - 35062	- Log. -	4.54484
Arch first - -	8h 42′ 50″	-	rising -	4.64010

True distance	70° 55′		Co-secant	0.02455
Ald. polar dist.	73 55.6		Co-secant	0.01732
Difference	3 0.6	N.V.S. 00138		
Moon's polar dist.	87 31	N.V.S. 95667		
		Diff. 95529	Log.	4.98014
Arch second	6 11 55		Rising	5.02201
Arch first	3 42 50			
Arch third	2 29 5		Rising	4.31010
Ald. zen. dist.	57 58.4		Sine	9.92929
— polar dist.	73 55.6		Sine	9.98268
Difference	15 57.2	N.V.S. 03851		
		16675		4.22207
Latitude	52 37.8	N. co-v. s 20526	Co-sine	0.78316
Ald. zen. dist.	57 58.4		Co-secant	0.07071
Arch third	2 29 5		H. E. T.	0.21782
Ald. mer. dist.	3 51 55		H. E. T.	0.07169
Ald right asc.	4 24 43			
			Reduced latitude	52° 37′.8
Right ascen. mer.	0 32 48		Reduction Table XXXVI.	+ 14.4
Sun's right asc.	13 27 3		Latitude	52 52.2N.
Apparent time	11 5 45			
Ap. t at Green.	14 32 5			
Longit. in time	3 26 20 = 51° 35′ W.			

CHAP.

CHAP. VIII.

Of finding the Longitude at Sea or Land,

BY

An Observation of the Distance between the Moon and a Star, not used in the Nautical Almanac.

INTRODUCTION.

THE distances between the Moon and the Sun, and ten of the brightest fixed stars nearest the Moon's path, are given in the Nautical Almanac, when the Moon is in a proper position with respect to those objects. It may, however, happen, that some other star is in a more favourable position, for observation, than any of those given in the Ephemeris; the ship's longitude might therefore be determined by such an observation, when perhaps it would otherwise be impossible.

The difference of longitude between the Moon and the star with which it is to be compared, must not, however, be less than a certain quantity, otherwise the Moon's relative motion will be too slow, for the purpose of determining the longitude with that degree of precision with which it may be ascertained when the Moon is compared with one of the stars employed in the Ephemeris. Several other elements enter also into this method, which makes it necessary to treat of it in a particular manner.

PROBLEM

I.

To find that Quantity which the Difference between the Longitudes of the Moon and the given Star must exceed, that the Moon's relative Motion may not be too much diminished.

RULE.

Enter Table XVII. with the difference between the latitudes of the Moon and the proposed star, if both latitudes are of the same name; but with their sum, if of contrary names, and find the corresponding quantity.

quantity. Now, if this quantity is less than the difference between the longitudes of the Moon and given star, their distance may be observed, for the purpose of finding the longitude of the place of observation; but if it is greater than that difference, the Moon's relative motion will be too slow to derive any benefit for the above purpose, from such an observation.

Example.

Is it proper to compare the Moon with α Orionis, in order to determine the longitude, November 21, 1792, about 10h. P. M. reduced time?

Latitude of Betelguese =	16° 4′ S.	Long. of Betelguese =	2ˢ 25° 52′
Latitude of the Moon	1 3 N.	Long. of the Moon	11 10 4
Difference of latitude . .	17 7	Diff. of longitude -	3 15 48

Now in Table XVII. opposite to the diff. of latitude 17° 7′ is 33° 48′; which, being less than the diff. of longitude 105° 48′, therefore shows that this star may, in the present case, be employed for the purpose of finding the longitude.

Problem

II.

Given the true Distance between the Moon and a fixed Star, together with the Latitude of each, to find the Moon's Longitude.

Rule.

To the true distance, add the latitudes of the Moon and star, and find the difference between the half sum and distance.

Now to the log. secants of the latitudes of the Moon and star, add the log. co-sines of the half sum and difference, if the latitudes are of the same name, or the log. sines if of a contrary name; half the sum of these four logs. will be the log co-sine, or sine of half the difference of longitude, according as the latitudes of the Moon and star are of the same, or of a different name.

To the apparent longitude of the star, add the difference of longitude, if the Moon be east of the star, otherwise subtract it, and the sum or remainder will be the true longitude of the Moon.

Remark.

It will be sufficient in most cases to take out the latitudes of the Moon and star to the nearest minute.

Example.

Let the true distance between α Orionis and the Moon's center be 105°

105° 26′ 14″, November 21, 1792, at 10 hours reduced time. Required the Moon's longitude?

True distance	-	105° 26′ 14″				
Latitude of Moon		1 3 -	N.	- Secant	-	0.000073
Latitude of α Orionis		16 3 -	S.	- Secant	-	0.017267
Sum	- -	122 32 14				
Half	- -	61 16 7		- - Sine	-	9.942942
Difference	-	44 10 7		- - Sine	-	9.843091
						19.803373
		52 53 3		- - Sine	-	9.901686
Difference of long.		105 46 6				

Mean longitude of α Orionis, Nov. 21, 1792, -	2ˢ 25° 51′ 40″
Equation of equinoxes, Table LXVI. - -	— 3
Aberration - - - - - -	+ 19
Apparent longitude of α Orionis - - -	2 25 51 56
Difference of longitude, the ☽ being west of ✱	3 15 46 6
Moon's longitude - - - - - -	11 10 5 50

Problem

III.

Given the True Longitude of the Moon, to find the Apparent Time at Greenwich.

Rule.

From the Nautical Almanac, take four longitudes of the Moon, two of which immediately preceding, and two following the given longitude. Find the difference between each pair successively; find also the second difference, and let their mean be taken.

Now, to the constant log. 2.857332, add the ar-co. of the log. of the variation of the Moon's longitude in 12 hours, reduced to seconds, and the log. of the difference between the given and preceding longitudes in seconds; the sum, rejecting radius, will be the log. of the approximate time in minutes, to be reckoned from the preceding noon or midnight.

Take the equation of second difference from Table XXXVII. answering to the approximate time, and the mean second difference, with which enter Table XXXVIII. at the top, and find the equation corresponding thereto, and the Moon's motion in longitude in 12 hours, in the side column; which being applied to the approximate time, by addition or subtraction, according as the first difference of the Moon's motion is

is increasing, or decreasing, will give the apparent time at Greenwich.

Example.

November 21, 1792, the longitude of the Moon deduced from observation was 11ˢ 10° 5′ 50″: Required the apparent time at Greenwich?

☽'s lon.	20d. at midn.	10ˢ 27° 22′ 0″				
	21 at noon	11 4 18 52	Diff. 6° 56′ 52″			
	21 at midn	11 11 12 0	6 53 8	Dif. 3′ 44″	Mean
	22 at noon	11 18 1 27	6 49 27	Dif. 3 41	= 3′ 42.5

☽'s long. 21d. at noon	11ˢ 4° 18′ 52″		
Given longitude -	11 10 5 50		
Difference - -	5 46 58 = 20818″	Log. -	4.318439
Var. ☽'s long. in 12 hours	6 53 8 = 24788″	Ar-co-log	5.605758
Constant log. - - - - - - - .			2.857332
Approximate time -	10 4 41 = 604′.68		2.781529
The eq. from Tab. xxxvii. answer. to 15″ the eq. of 2d-diff. & 6° 53′ is	— 26		
App. time at Greenwich	10 4 15		

Problem IV.

Given the Latitude and estimate Longitude of the Place of Observation, the Distance between the Moon and a fixed Star, together with the Altitude of each, to find the true Longitude of that Place.

Rule.

With the given latitude, the corrected altitude, and declination of the star, compute the apparent time of observation by Prob. vii. page 133. Reduce this time to the meridian of Greenwich, and find the Moon's semidiameter and horizontal parallax agreeable thereto.

With the true and apparent altitudes, and the apparent central distance, compute the true distance by Prob. i. page 150; with which, and the latitudes of the Moon and star, find the difference of longitude by Prob. ii. page 204, and from thence find the apparent time at Greenwich, by the last problem. Now the difference between the apparent time at the place of observation, and that at Greenwich, will be the longitude of the place in time, as formerly.

Example.

Example.

November 21, 1792, in latitude 48° 50′ N. longitude by account 19° W. a little before 9h. P. M. the distance between the Moon's west limb and Betelguese was 106° 2′ 24″, the altitude of Betelguese 14° 44′, and that of the Moon's lower limb 26° 38′, height of the eye 10 feet. Required the true longitude?

Obs. alt. Betelguese =	14° 44′				
Dip - -	— 3				
Apparent altitude -	14 41				
Refraction - -	— 4				
True altitude -	14 37				
North polar distance	82 39	- -	Co-secant -	0.00358	
Latitude - -	48 50	- -	Secant -	0.18161	
Sum - -	146 6				
Half - -	73 3	- -	Co-sine -	9.46469	
Difference - -	58 26	- -	Sine -	9.93046	
				1958034	
	38 5	- -	Sine -	979017	
	8				
Mer. dist Betelguese	5 4 40				
R. A. Betelguese -	5 43 57				
R A of meridian	0 39 17				
Sun's R. ascension	15 51 2				
Approximate time	8 48 15				
Eq. to approx time Tab. XVII. -	— 1 32				
- to long. 19° W.	— 13			Obs. alt ☽'s l. limb.	26° 38′
				Semidiameter — dip	+ 13
Apparent time -	8 46 30				
Longitude in time	1 16			Apparent altitude -	26 51
				Correct. to red. par. -	+ 50 18
Reduced time -	10 2 30				
Observed distance	106 2 24			True altitude ☽ =	27 41 18
Augm. semidiam.	— 16 6			- - ✱ =	14 37 25
Apparent dist. -	105 46 18			Sum - - -	42 18 43
Apparent alt. ☽	26 51			Half - - -	21 9 21
Apparent alt. ✱	14 41				
Sum - -	147 18				
Half - -	73 49				

Log.

				Log. diff.	-	9.996844
Half	- -	73 39		Co-sine	-	9.449485
Difference	-	32 7		Co-sine	-	9.927867
						19.374196
Half sum true alt.		21 9 21				
Arch	- -	29 6 43	- -	Sine	- -	9.687098
Sum	- -	50 16 4	- -	Co-sine	-	9.805637
Difference	-	7 57 22	9 -	Co-sine	-	9.995799
						19.801436
		52 42 58	- -	Sine	-	9.900718
Computed dist.	-	105 25 56				
Seconds omitted	=	+ 18				
True distance	-	105 26 14				

The Moon's longitude, inferred from the true distance, and the latitudes of the Moon and star, by Prob. II. page 204, is 11^s 10° 5′ 50″, and the apparent time at Greenwich answering thereto, by Prob. III. page 205, is - - = 10h. 4′ 15″

And the app. time at the place of obs. is 8 46 30

Hence the longitude of that place is - 1 17 45 = 19° 26¼ W.

CHAP. IX.

Of finding the Longitude at Sea or Land,

BY

An Observation of the Distance between the Moon and a Planet.

Introduction.

This method of determining the longitude at sea will, for the most part, be attended with the desired success, provided the Moon is compared with Venus, Mars, Jupiter, or Saturn. The proximity of Mercury

Mercury to the Sun, and the smallness of the apparent magnitude of the Georgian planet, prevent them from being of any service for this purpose.

Venus and Jupiter may be compared with the Moon, even when the twilight is pretty strong, and in this case their altitudes may be very accurately observed. Hence, the longitude being computed from the mean of several sets of observations, may be safely depended on.

If the observer is provided with a circular instrument, he may measure the distance between the Sun and Moon when it exceeds 120°. As, however, the distance when above that quantity is not given in the Nautical Almanac, the following method of computation is to be used, in place of that formerly given. In this case, the difference of longitude between the Sun and Moon may be found thus:

To the logarithmic secant of the Moon's latitude, add the logarithmic co-sine of the true distance, the sum, rejecting radius, will be the logarithmic co-sine of the difference of longitude, of the same affection with the distance.

Problem.

Given the Latitude of a Place, and its Longitude by Account, the observed Distance between the Moon and a Planet, and their Altitudes, to find the true Longitude of the Place of Observation.*

Rule.

Let the error of the watch be inferred from the altitude of the Sun, observed the preceding evening, or from that of a fixed star, or from the altitude of the Moon or planet, taken at the same time with the distance. Hence, the apparent time of observation will be known; to which the longitude by account being applied, the reduced time will be obtained.

Find the apparent and true altitudes of the Moon and planet, with which, and their apparent central distance, compute the true distance. Now, with the true distance, and the latitudes of the Moon and planet, find their difference of longitude, by Prob. II. page 204, which being added to, or taken from, the geocentric longitude of the planet, found in the Nautical Almanac, page IV of the month, according as the Moon is east or west of the planet: hence, the

* The distance between the Moon's limb, and the center of the planet, is to be observed. In the rules, no allowance is made for the parallax of the planet, which in many cases is insensible. Those who wish to be rigorous in their calculations, may compute the geocentric place of the planet, its semidiameter, and horizontal parallax, from the best astronomical tables. M. de Lambre's tables of Jupiter and Saturn, and those of the other planets by M. de la Lande, will be proper for this purpose.

Moon's true longitude will be obtained. Find the time at Greenwich answering to this longitude by Prob. III. page 205, the difference between which, and the time at the place of observation, will be the longitude of that place in time.

Example.

April 2, 1792, in latitude 35° 10′ S. and longitude, by account, 42° E. at 7h. 24′ P. M. per watch, the distance between the Moon's west limb and Jupiter's center was 71° 12′ 8″, the altitude of the Moon's lower limb 41° 18′, that of Jupiter's center, 11° 13′, and height of the eye 13 feet. Required the true longitude?

Time p.w. 7h 24′	Obs. dist. ☽'s l. from ♃ 71° 12′ 8″	☽'s h. par. 55′ 35″
Long. in t. 2 48	Augm. semidiameter — 15 19	Reduction — 5
Red. time 4 36	App. central distance 70 56 49	Red. h. par. 55 30

Obs. alt. ☽'s l. limb	= 41° 3′	Observed altitude of ♃	11° 13′
Semidiameter -	+ 15	Dip - - -	— 3
Dip - -	— 3		
		Apparent altitude -	11 10
Apparent altitude	41 15	Refraction -	— 4 42
Correction -	+ 40 38		
		True altitude of ♃ =	11 5 18
True alt. ☽'s center	41 55 38	- - -	41 55 38
App. dist. + 11″	= 70 57		
Apparent alt. ☽	41 15	Sum - - -	53 0 56
Apparent alt. ♃	11 10	Half - - -	26 30 28
Sum -	123 22	- - Log. diff. - -	9.995559
Half - -	61 41	- - Co-sine - -	9.676094
Difference - -	9 16	- - Co-sine - -	9.994295
			19.665948
Half sum true alts.	26 30 28		
Arch - -	42 54 2	- - Sine -	9.832974
Sum - -	69 24 30	- - Co-sine - -	9.546179
Difference -	16 23 34	- - Co-sine - -	9.981977
			19.528156
	35 30 42	- - Sine	9.764078
Computed distance	71 1 24		
Seconds to be added	— 11		
True distance -	71 1 13		

True

True distance	-	71 1 13		
Latitude of ☽	-	3 48 S.	- Secant	- 0.000956
Latitude of ♃	-	1 32 N.	- Secant	- 0.000156
Sum	- -	76 21		
Half	- -	38 10 30	- - Sine	- 9.791034
Difference	-	32 50 30	- - Sine	- 9.734255
				19.526401
		35 25 45½	- - Sine	- 9.763201
		70 51 31		
Seconds omitted	-	+ 13		

Differ. of longitude	70 51 44	=	2ˢ 10° 51′ 44″
Geoc. long. of ♃ per Ephemeris			6 28 16 30
Moon's true longitude	- -		4 17 24 46
Moon's long. 2d April at noon	-		4 15 3 20
Difference of longitude	- -		2 21 26

☽'s long. 1st at midn.	=4ˢ 8° 54′ 30″	First diff.		
2d. at noon	=4 15 3 20	+6° 8′ 50″	Sec. diff.	
2d. at midn.	=4 21 16 5	+6 12 45	+3′ 55″	Mean
3d. at noon	=4 27 33 8	+6 17 3	+4 18	+ 4′ 6½″

Constant logarithm	- - - - - -		2.857332
Var. ☽'s long. in 12 hours	6° 12′ 45″ = 22365″	ar-co-log	5.650431
Diff. of long. of ☽ and ♃	2 21 26 = 8486	log. -	3.928703
Approximate time -	4h 33′ 11″ = 273′.19 -	log. -	2.436466
Eq. from Tab. XXXVIII.	+ 56		
App. time at Greenwich	4 34 7		

Altitude of Jupiter -	11° 3′		
South pol. distance of ♃	80 33	- Co-secant	- 0.00593
Latitude of ship	35 20	- Secant	- 0.08752
Sum - -	126 48		
Half - -	63 24	- Co-sine	- 9.65140
Difference - -	52 19	- Sine	- 9.89840
Sum - -	- - -	-	19.64289

Sum	- - - - - -			19.64289
Arch - -	41° 31½′	-	Sine - -	9.82144
Multiply by - -	8			
Meridian distance ♃	5 32 10			
Pass. ♃ over mer. ship.	12 55 3			
Appar. time at ship	7 22 53			
App. time at Greenw.	4 34 7			

Longitude in time 2 48 46 = 42° 11½′ E. the apparent time at the place of observation being later than that at Greenwich.

BOOK

BOOK IV.

CONTAINING

Various other Methods of finding the Longitude of Places.

CHAP. I.

Of finding the Longitude of a Place

BY

An Observation of the Transit of the Moon over the Meridian.

INTRODUCTION.

THIS method appears to have been first mentioned in Purchas' Account of Hall's Discovery of Greenland, and soon after it was clearly explained in Carpenter's Geography, printed at Oxford in 1635, "which method," says he, "for mine owne part, I preferre before all the rest both for certainty and facility."—Book I. page 247. It was again, in 1725, by M. Rodouay, by M. l'Abbé de Chappe D'Auteroche, in his edition of Dr. Halley's Solar and Lunar Tables. It is also mentioned by Dr. Maskelyne in his instructions relative to the transit of Venus in 1769; by M. J. Bernouilli, in his *Recueil pour les Astronomes*, vol. II. page 137; by the Abbe Toaldo, in a pamphlet publishedin 1784; and by Mr. Pigott in the Phil. Tran. for 1786 and 1790, who very strongly recommends it, "being convinced, that in a short time it must be universally adopted, having every advantage over Jupiter's first satellite, and but litttle inferior in precision to occultations."

This method of determining terrestrial longitudes (says M. Pigott, Phil. Trans. 1790, page 387,) "I have fully detailed in the Philosophical Trans-

Transactions, vol. LXXVI. and still think it cannot be too strongly recommended. The preceding additional set of results do further corroborate the reliance that may be put on it, though the observations were not made with that intention, and consequently several of them are deficient in many particulars. Their agreement, nevertheless, is conclusive, and infinitely more satisfactory than could be expected. Since the above-mentioned publication, I have been informed, that M le Marquis de Chabert, and others, many years ago, settled differences of meridians on similar principles, and, I dare say, with as much sagacity as the then imperfect state of the method would permit. At present it is certainly considerably improved, being susceptible of very great exactness and facility, which, perhaps, may be considered as the sole requisites for rendering it anywise useful."

The longitude of a place may be very accurately determined by this method at land; and a tolerable degree of precision may be obtained at sea, by observing equal altitudes of the Moon, as accurately as possible, and making an allowance in the time of transit, deduced from the equal altitudes, for the change of declination between the observations.

As the observation for the above purpose happens so often, it therefore seems to demand the particular attention of the practical astronomer, as well as of those who travel to distant countries. The Marquis de Chabert, in his travels, used this method with success. In 1753, he determined the longitude of Carthagena; and in order to facilitate the practice of it, he described, in the *Memoires de l'Academie* for the year 1766, a very simple method of placing and verifying the position of the transit instrument, which is nearly as follows.

The position of the meridian was first nearly ascertained by means of a mariner's compass, allowance being made for the variation. Then, the transit instrument being placed in this direction, at the distance of 70 or 80 toises from the instrument, a mark was erected; at which, the middle division of a rod, about 16 feet long, and divided into half inches, was placed so that the rod was perpendicular to the line joining the instrument and meridian mark. Now, this being prepared, the apparent time of noon was ascertained by equal altitudes of the Sun observed with a quadrant; the difference between which, and the time of noon observed the same day with the transit instrument, gave the error in the position of that instrument in time, answering to the latitude of the place and declination of the observed object: hence the horizontal deviation of the instrument may be computed*, with which, and the distance between the instrument and rod, that particular division on the rod which is in the meridian may be found; and hence, the transit instrument may be truly placed in the plane of the meridian. The various operations to facilitate the practice, with the method of verifying the position of the instrument at night, will be obvious to the practical astronomer.

* Horiz. dev. = secant alt. × co-sine decl. × ob. diff. of time × 15. to radius 1.

The

The necessary observation for determining the longitude by this method, is the apparent time of the Moon's passage over the meridian; or, in order to attain the utmost accuracy of which it is susceptible, the passage of a star, having the same declination nearly, is also to be observed; both at the place whose longitude is wanted, and at a place whose situation is known. It is not absolutely necessary, that the same star be observed at both places. In this case, it will be proper to use those stars only, whose places are accurately determined. The instruments necessary for this purpose at land are, a transit instrument, and an astronomical clock. At sea, the equal altitudes may be observed with a sextant for the sake of greater accuracy.

The meridian altitude of the Moon being observed at the same time with the transit, will afford another method of determining the longitude; and which, when the change of the Moon's declination is a maximum, or nearly so, will be found tolerably accurate. This method, however, can only be practised with success in an observatory.

PROBLEM I.

Given the Apparent Time of the Transit of the Moon's Limb over the Meridian, to find the Longitude of the Place of Observation.

RULE.

Reduce the apparent semidiameter of the Moon to time, by Tables XXXIX. and XL. which being applied to the apparent time of transit of the Moon's limb, by addition or subtraction, according as the western or eastern limb was observed; the sum or difference will be the apparent time of the transit of the Moon's center.

From the right ascension of the Moon in time, subtract that of the Sun, the remainder will be the approximate time of the Moon's passage over the meridian of Greenwich. Now, to the log. of the difference between the change of the Moon's right ascension in 12 hours, and that of the Sun's in the same time, reduced to seconds, add the ar-co-log. of the difference between the above quantity and 43200″, and the log. of the approximate time in seconds; the natural number answering to the sum of these three logs. will be the correction in seconds of the approximate time; which being added thereto, will give the apparent time of the Moon's passage over the meridian of Greenwich.

From the P. Log. of the difference between the apparent times of transit at Greenwich, and at the place of observation, subtract the P. Log. of the difference between the variations of the right ascensions of the Sun and Moon in 12 hours; the remainder will be the P. Log. of the longitude of the place of observation in minutes and seconds, which are to be esteemed degrees and minutes. If the time of transit at the place of observation is earlier than that at Greenwich, the longitude is east, otherwise it is west.

EXAMPLES.

Examples.

I.

July 2, 1792, the apparent time of the transit of the Moon's western limb was 10h. 43′ 58″. Sought the longitude of the place of observation?

☽'s R. A. at noon,	16h 58′ 20″	☽'s R. A. at noon,	16° 58′ 20″	Ap. time of tran. ☽'s west. limb,	10h. 43′ 58″
☉'s R. A. at noon,	6 48 30.4	☽'s R. A. at midnight,	17 30 0	☽'s eq. semidia. Table XXXIX.	+ 1 9.7
				Incr. sem. Tab. XL. +	3.36
Approx. time, 10	9 49.6	Vr. ☽'s R. A. in 12h.	31 40		
		Vr. ☉'s R. A. in 12h.	2 3.7	Ap. t. of tr. ☽'s cen.	10 45 11.3

Difference - - -	29 36.3 = 1776″.3	- log. -	3.249516
Constant quantity - - -	43200		
Difference - - - -	41423.7	ar-co-log.	5.382751
Approximate time of transit -	10h 9′ 49″.6 = 36589.6	- log. -	4.563358
Correction - - - -	+ 26 9.0 - 1569.0	- - -	3.195625
Appar. time of transit at Greenwich.	10 35 58.6		
App. time of tran. at place of observ.	10 45 11.3		
Difference - - - -	9 12.7	- P. L. -	1.2909
Diff. of varia. R. A. ☉ and ☽ in 12h.	29 36.3	- P. L. -	0.7839
Longitude - -	56° 0′	- P. L. -	0.5070

Which is west, because the time of transit by observation is later than the time at Greenwich.

II.

December 30, 1792, the Moon's eastern limb was observed to pass the meridian at 13h. 53′ 33″.8. Required the longitude at the place of observation?

☽'s R. A. at midn.	6h 42′ 20″	☽'s R. A. at midnight.	8h 42′ 20″	Appar. time of tr. ☽'s east limb.	13h 53′ 33″.8
- ☉'s R. A. at noon.	18 44 4.4	☽'s R. A. at noon.	9 6 12	Eq. sem. Tab. XXXIX. —	1 0.8
				Incr. of sem. Tab. XL. —	2.0
App. time	13 58 15.6	Vr. ☽'s R. A. in 12h	23 52		
	12	Var. ☉'s - -	2 12.6	Ap. t. of tra. ☽'s cen.	13 52 31.0

1 58 15.6 Difference - -	21 39.4 = 1299″.4	- log. -	3.113743
Constant quantity - - -	43200		
Difference - - - -	41900.6	ar-co-log. -	5.377780
Approximate time of transit -	1 58 15.6 A. M. 7095.6	- log. -	3.850989
Correction - - - - -	+ 3 40.0 = 220.0	-	2.342512
Apparent time of transit - -	2 1 55.6 A. M.		
Apparent time of observation -	1 52 31.0 A. M.		
Difference - -	9 24.6	- - P. L. - -	1.2817
Diff. var. R. A. ☉ and ☽ in 12 hours	21 39.4	- - P. L. - -	9197
Longitude - -	78° 13′ E.	- P. L. - -	3620

The

The above is a very ready method of finding the longitude of a place, especially as a single observation only is necessary. The longitude deduced by it is, however, affected by the error of the Moon's place in the Nautical Almanac. The position of the transit instrument must also be perfectly rectified, or allowance made in the observed time of transit for its deviation from the meridian, otherwise the conclusions cannot be depended on. In the following method, the error in longitude, arising from an error in the Moon's place, is avoided; and if the horizontal deviation of the instrument from the meridian does not exceed a few seconds of time, a very accurate solution may be obtained, provided the declinations of the Moon and star be nearly the same.

Problem II.

Given the Intervals of Time between the Transit of the Moon's Limb and a fixed Star, over Two different Meridians, to find the Difference of Longitude between the Places of Observation.

Rule.

The difference or sum of the given intervals, according as the Moon is on the same, or on opposite sides of the star, at the places of observation, reduced to sidereal time, will be the increase of the Moon's right ascension, answering to the difference of meridians of those places. To the log. of which, expressed in seconds, add the ar-co-log. of the increase of the Moon's right ascension in 12 hours, in minutes, taken from the Nautical Almanac, and the constant log. 5.20951; the sum will be the log. of the difference of longitude in seconds.

The westernmost place of observation answers to the greater or less interval, according as the Moon is to the east or west of the star. If the Moon be on opposite sides of the star, that place will be westernmost where the Moon was observed to precede the star.

Example.

Let the interval in sidereal time, between the transits of the Moon's west limb and δ Scorpionis, observed at Greenwich, be 1h. 24′ 3″.7; and the interval in sidereal time, as observed at another place, be 1h. 21′ 40″.3, the increase of the Moon's right ascension in 12 hours being 6° 12′. Required the longitude of the place of observation?

Obs. interval at Greenw.	1h 24′ 3″.7			
Obs. inter. at the other place	1 21 40 .3			
Difference - - - -	2 23 .4	= 143″.4	- log. -	2.15655
Increase ☽'s R. A. in 12h.	6° 12′	= 372′	- ar-co-log.	7.42946
Constant log. - - - - - - - -				5.20951
Longitude - -	17° 20′ 50″	= 62450″	-	4.79552

Remark.

From what has been said, it might be supposed, that the rules for computing the longitude by this method, and the examples to illustrate them, as given by late writers, would be accurate. However, that this is not the case, will be evident from the following remarks, which are here inserted, not with the most distant view to cavil, but only as a warning to seamen of the dangers they are liable to, by implicitly trusting to some rules, without previously examining them. In the fifth edition of a book upon Navigation, printed at London, in the year 1780, page 296, this method is proposed, and illustrated by an example, in which the longitude of the ship is found to be 10° 5′ W.; whereas the longitude, in round numbers, is 290° W. or rather 70° E. and in that case, the observation had been made on the 3d, in place of the 2d of December, as stated in the example. Again, in another treatise, printed in 1788, page 32, this method is announced as follows: *A new, concise, easy, and infallible Method to determine the Longitude at Sea, independent of the Dead Reckoning, by one Person only, and no other Instrument but Hadley's Quadrant well adjusted.* There are four examples given to illustrate this method; and their errors, were it possible for any of them to exceed 360°, are about 543°, 90°, 1719°, and 991° respectively.

Even in the more simple parts of Navigation, which, it is presumed, are universally known, very considerable errors are found to prevail; however, one of these only we shall take the liberty to mention here, which is as follows:—In the Encyclopædia Britannica, a rule is given to find the difference of longitude between two places, by computation, from their known latitudes and bearing; and in the example to that method, which is as follows—"A ship from a port in latitude 56° N. sails S. W. by W. till she arrives at the latitude of 40° N. Required the difference of longitude?" The difference of longitude is found to be 897′, whereas the true difference of longitude is 2172′; the error is, therefore, 1275′ = 21° 15′!

CHAP.

CHAP. II.

The Method of finding the Longitude at Sea or Land,

BY

An Observation of the Meridian Altitude of the Moon.

Introduction.

This method, which is well known to astronomers, is one of those mentioned by Dr. Maskelyne, in his instructions relative to the transit of Venus, page 41.

In order to ascertain the longitude as accurately as possible by this method, the time of observation of the meridian altitude of the Moon must be when the diurnal change of declination of that object is about its greatest. The altitude of the Moon ought to be observed with all imaginable care; and for this purpose a sextant, or circular instrument, becomes absolutely necessary. And although this method is not equal to some of the former in point of accuracy, yet in favourable circumstances, and when great care is taken, a very near approximation to the true longitude will be obtained.

Problem.

Given the Latitude of a Place, and its Longitude by Account, and the Meridian Altitude of either Limb of the Moon, to find the Longitude of that Place.

Rule.

Find the time of the Moon's passage over the meridian of the ship, by applying, to the time of transit given in the Nautical Almanac, the equation from Table xx. answering to the daily retardation, and longitude by account, by addition or subtraction, according as the longitude is west or east.

Reduce the observed altitude of the Moon's limb to the true altitude of its center, by Prob. xi. page 112; the difference between which and the complement of the latitude will be the declination of the Moon; of the same name with the latitude when the altitude exceeds the co-latitude; otherwise, of a contrary denomination.

Now,

Now, the difference between the declination of the Moon, and that at the preceding or following noon or midnight, according as the longitude is west or east, being multiplied by 12 hours, and divided by the change of declination in that interval, will give the apparent time at Greenwich, the difference between which and the apparent time of transit, reduced to the meridian of the place of observation, will be the required longitude.

Remarks.

In strictness, the apparent time at Greenwich ought to be corrected by the equation of second difference from Table XXXVII. reduced to time. Or, rather, calculate the Moon's declination to the nearest second, by Prob. II. page 44, from its latitude and longitude, inferred from the Ephemeris, for the apparent time of observation: and if this declination agrees with that deduced from observation, the apparent time is accurately determined; if not, the correction of the apparent time answering to the difference between the computed and observed declination, may be found with sufficient accuracy by even proportion.

As the Moon's passage over the meridian of Greenwich is given only to the nearest minute in the Nautical Almanac, and as it is necessary, for the sake of exactness, to have this element more accurately ascertained, it may, therefore, be found to the nearest second as follows.

From the right ascension of the Moon in time, at the noon or midnight preceding the Moon's transit, subtract the Sun's right ascension at the same time, and the remainder will be the approximate time of the Moon's passage over the meridian at Greenwich.

Now, as the difference between 12 hours, and the difference of the right ascensions of the Sun and Moon in that period, is to the approximate time of transit, so is the difference of the right ascensions of the Sun and Moon in 12 hours, to a fourth term; to which the equation of second difference, from Table XXXVII. and reduced to time, being applied, will give the correction; which, added to the approximate time of transit, the sum will be the apparent time of the Moon's passage over the meridian of Greenwich.

To reduce the time of transit at Greenwich, to that at the meridian of the ship, the following rule may be employed:

As the sum of 24 hours, and the daily retardation of the Moon,
Is to the longitude by account in time;
So is the daily retardation of the Moon,

To the reduction: which added to, or subtracted from, the time of the Moon's passage over the meridian of Greenwich, according as the longitude is west or east, will give the apparent time of transit at the ship.

If the apparent time at the ship be known, the computation of the Moon's transit becomes unnecessary.

It may be remarked, that in high latitudes, and when the change of the Moon's declination is considerable, the meridian altitude of the Moon is not always the greatest altitude.

Example.

November 13, 1804, in latitude 45° 35′ 25″ N. and longitude, by account, 20° W. the meridian altitude of the Moon's lower limb, observed with a sextant, was 48° 33′ 40″, and height of the eye 15 feet. Required the true longitude of the place of observation?

Moon's passage over meridian Greenwich - -	8h 42′ P.M.
Equation, Table xx. to longitude 20 W. - -	3
Moon's passage over meridian ship - - -	8 45 P.M.
Longitude in time - - - - - -	22 0 W.
Reduced time - - - - - - -	10 5 P.M.
Observed altitude Moon's lower limb - -	48° 33′ 40″
Semidiameter Moon - - -	+ 16 12
Augmentation - - - - -	+ 12
Dip - - - - - -	— 3 42
Apparent altitude Moon's center - -	48 46 22
Correction to apparent alt. and hor. par. 59′ 27″	+ 38 22
True altitude Moon's center - -	49 24 44
Complement latitude - - -	44 24 35
Moon's declination - - -	5 0 9 N.

Now, the Moon's declination at noon, per Nautical Almanac, is 2° 25′; the difference between which, and that deduced from observation, is 2° 35′ 9″; and the variation of declination in 12 hours is 3° 5′:

Hence, 3° 5′ : 2° 35′ 9″ :: 12h. : 10h. 4′, apparent time at ship.
App. time of ☽'s pass. over mer. ship. 8 45

Longitude in time - - 1 19 = 19° 45′ W.

We cannot omit mentioning in this place, that the true altitude of the Moon's center may be found without using the augmentation of semidiameter, as follows:

From the observed altitude of the Moon's limb subtract the dip, and from the remainder subtract the refraction answering thereto; then add the parallax of the Moon in altitude, corresponding to the altitude

altitude corrected by dip and refraction; to which the Moon's horizontal semidiameter being added, or subtracted, according as the lower or upper limb was observed, will give the true altitude of the Moon's center. Thus, in the preceding example, the observed altitude of the

Moon's lower limb was - - - - -	48° 33′ 40″
Dip - - - - - - -	— 3 42
Altitude Moon's limb corrected by dip - -	48 29 58
Refraction - - - - -	— 50
Altitude Moon's limb corrected by dip and refraction -	48 29 8
Parallax - - - - - - -	+ 39 24
True altitude Moon's lower limb - - -	49 8 32
Semidiameter - - - - -	+ 16 12
True altitude Moon's center - - -	49 24 44

CHAP. III.

Of finding the Longitude at Sea or Land:

The necessary Data being

The observed Altitude of either Limb of the Moon, the Apparent Time at the Place of Observation, together with its Latitude and Longitude by Account.

Introduction.

A like degree of accuracy is not to be expected in this, as in any of the former methods. In some of these, it was sufficient to take the altitude of the nearest minute, but in this, the altitude should be taken to the nearest second, and the apparent time of observation determined

determined to the tenth of a second. The Moon's right ascension and declination should be reduced to the apparent time of observation by the method of *Interpolation*; or rather computed by the rule given in page 44, from the latitude and longitude of the Moon reduced by interpolation to the time and place of observation; as these elements are given in the Nautical Almanac to the nearest second; whereas the right ascension and declination are given only to the nearest minute. It is indeed much to be wished, that they were given to, at least, the nearest tenth of a minute: and if the computed longitude of the place of observation differs considerably from that which was assumed, so as to make an alteration in the Moon's declination, the operation must be repeated.

Problem.

Given the observed Altitude of the Moon's Limb, to find the Longitude of the Place of Observation.

Rule.

Reduce the apparent time of observation to the meridian of Greenwich, by Prob. III. page 105. To this time take the Moon's declination from the Nautical Almanac; with which, the correct altitude of the Moon, and the latitude of the place, find the horary distance of the Moon from the meridian by Prob. IX. page 137. Now to the Moon's right ascension at the preceding noon, add its meridian distance, if in the western hemisphere,—otherwise subtract it; and from this sum, or difference, increased by 360° if necessary, subtract the Sun's right ascension at the preceding noon; reduce the remainder to minutes, and find its logarithm.

From the sum of 180°, and the variation of the Sun's right ascension in 12 hours, subtract the variation of the Moon's right ascension in 12 hours, and find the arithmetical complement of the logarithm of the remainder. Now, the sum of these two logarithms, and the constant quantity 2.857332, will be the logarithm of a certain quantity expressed in minutes.

From the P. Log. of the difference between this time, and the apparent time at the place of observation, subtract the P. Log. of the difference between the variation of the right ascensions of the Sun and Moon in 12 hours, and the remainder will be the P. Log. of the longitude in minutes and seconds, which are to be esteemed degrees and minutes.

Examples.

I.

October 17, 1791, in latitude 43° 18′ N. and longitude by account 24° 30′ W. at 11h. 37′ 53″ apparent time, the altitude of the Moon's lower

lower limb was 23° 31′, and the height of the eye 12 feet. Required the true longitude of the place of observation?

Observed alt. ☽'s l. l.	23° 31′ 0″	App. time of obs. 11h. 37′ 53″		
Augm. semidiameter	+ 14 .9	Longitude in time 1 38 0 W.		
Dip - -	— 3 .3			
		Est. time at Green. 13 15 53		
Apparent alt. ☽'s center	23 42.6			
Correction -	+ 47.6			
True altitude ☽'s center	24 30.2			
Moon's polar distance	71 41	- -	Co-secant -	0.02258
Latitude - . -	43 18	- -	Secant -	0.13800
Sum - -	139 29.2			
Half - - -	69 44.6	- -	Co-sine -	9.53936
Difference - . -	45 14.4	- -	Sine -	9.85130
				19.55124
	36 37.2	- -	Sine -	9.77562
Moon east of meridian	73 14.4			
Moon's right ascension	83 36.0			
Difference - -	10 21.6			
Sun's right ascension -	202 16.2			
Remainder - -	168 5.4	= 10085′.4 -	log. -	4.003693
180° + var. ☉'s R. A. in 12h. - - -	180 28.1	-	Constant log. -	2.857332
Var. ☽'s R. A. in 12h.	. 6 14.0		-	
Difference - -	174 14.1	= 10454′.1	ar-co-log.	5.980713
	11 34 36	= 694.6	- -	2.841738
Apparent time of observ.	11 37 53			
Difference - -	3 17	-	P. Log. -	1.7389
Diff. var. ☽ & ☉'s R. A. in t.	23 3.6	-	P. Log. -	0.8924
Longitude - -	25° 38′	-	P. Log. -	8465

Which is west, because the time at the ship is later than the computed time.

II.

December 26, 1792, in latitude 45° 28′ N. and longitude by account 88° 10′ E. at 14h. 20′ 10″ apparent time, the altitude of the Moon's

Moon's lower limb was 39° 2′.9, and the height of the eye 16 feet. Required the true longitude?

Apparent time	-	14h. 20′ 10″
Longitude in time		2 32 40
Reduced time	-	11 47 30

Observed alt. ☽'s l. limb.	39° 2′.9			
Semidiameter - -	+ 15.0			
Dip - - -	− 3.8			
Apparent alt. ☽'s center	39 14.1			
Correction - -	+ 41.3			
True altitude -	39 55.4			
Polar distance -	71 50.6	-	Co-secant -	0.02218
Latitude - - -	45 28.0	-	Secant -	0.15408
Sum - - -	157 14.0			
Half - - -	78 37.0	-	Co-sine -	9.29529
Difference - -	38 41.6	-	Sine -	9.79599
				19.26754
	25 29.2	-	Sine -	9.63377
Multiply by - -	2			
Moon west of meridian	50 58.4			
Moon's right ascension	74 25.0			
Sum - - -	125 23.4			
Sun's right ascension -	276 2.1			
Remainder - -	209 21.3 = 12561.3		-	4.099035
180° + var. ☉'s R. A. in 12 hours -	180 33.2	Constant log.	- -	2.857332
Var. ☽'s R. A. in 12h.	6 21.			
Difference - -	174 12.2 = 10452.2		ar-co-log.	5.980792
	14 25 17 = 865.28		-	2.937159
Apparent time at ship	14 20 10			
Difference - -	5 7	-	P. L. -	1.5463
Diff. var. ☉ and ☽'s R. As. in 12h. -	23 11.2	- -	P. L. -	8900
Longitude - -	39° 43′	- -	P. L. -	6563

Which is east, because the apparent time at the ship is earlier than the computed time.

CHAP. IV.

The Method of finding the Longitude of a Place at Sea or Land,

BY

An Eclipse of the Moon.

INTRODUCTION.

THIS method of finding the longitude of a place was mentioned by Hipparchus, about 150 years before the Christian æra. It is particularly described in Apian's Cosmography as follows:

"Longitudines regionum, civitatum, locorumque ex initio eclipseos lunaris indagare. Observa itaque principium alicujus eclipsis in oppido cujus longitudo tibi ignota fuerit, quod si in horis et minutis cum eclipsi ex tabula sequenti accepta concordaverit, proclamabis istum locum habere meridianum Leyszningensem, quia eclipses sequentes mathematica supputatione ad Leyszningen meridianum collegimus. Est enim civitas Misniæ, cujus longitudo est gra. 30 min. 20, secundum Ptolomæum autem long. 34 gra. 40 min. Si autem initium eclipsis differt, dic locum illum alium habere meridianum et diversam longitudinem, quam sic invenies. Aufer numerum horarum et minut. eclipseos, minorem de majori, et differentiam in gradus, et graduum minut. convertito hoc pacto. Pro qualibet hora accipe 15 gradus, pro 4 minu. horæ, 1 graduum, et pro quolibet minu. horæ, 15 minu. gra. Tandem numerum graduum et minutorum jam elicitum, adde ad gradus longit. meridiani Leyszningen, si orientalior, hoc est, si major horarum numerus in ea repertus fuerit. Aut subtrahe, si occidentalior, hoc est, si minor horarum numerus in ea quam in tabulis eclipsium reperitur, et habebis longitudinem hujus oppidi quæ antea ignota fuerat. Similiter etiam cum eclipsibus quæ ad alium meridianum sunt supputatæ, agito."—*Ex Cosmographia Petri Apiani. Ed. Antwerpiæ,* 1550 *.

This

* To find out the longitudes of countries, cities, and places from the beginning of a lunar eclipse. Observe, therefore, the beginning of some eclipse in the town whose longitude is unknown to you, which, if it shall agree in hours and minutes with the eclipse taken from the following table, you may be assured that that place has the meridian

This method is illustrated by Adrian Metius, in his *Doctrina Spherica*, in an example applied to find the difference of longitude between Regiomont and Marpurg, by the lunar eclipse of February 11, 1598. It is also mentioned by Ed. Harrison in the *Idea Longitudinis*, page 49, printed at London in the year 1696; *Abrége du Pilotage*, and since, it has been recommended by succeeding writers on this subject.

This is, perhaps, the most easy of the astronomical methods that have been proposed, to find the longitude at sea; as it is neither affected by refraction nor parallax, to which some of the other methods are liable. However, as lunar eclipses happen so very seldom, the longitude cannot be found by this method so often as is necessary, yet such observations should not, if possible, be neglected.

The beginning and end of the eclipse are the principal phases from which the longitude is to be found. If the observer is provided with a sextant, these phases may be observed with tolerable accuracy, with the telescope belonging to that instrument. The observations may also be multiplied, by measuring the versed sines of the enlightened part of the Moon's disc, for which purpose this instrument is well adapted at sea. At land, a different method is commonly used.

Problem I.

To find if a Lunar Eclipse will be visible at a given Place.

Rule.

To the times of the beginning and end of the eclipse, as given in the Nautical Almanac, let the longitude by account be added or subtracted, according as it is east or west, and the correspondent instants of these phases by the meridian of the place will be obtained.

Now, compute the times of sun-rising and setting by means of a table of semidiurnal arches, or otherwise; and if the reduced times of these phases happen while the Sun is above the horizon, the eclipse will not be visible; but one or both phases will be visible, if the Sun is under the horizon, and may, therefore, be observed, provided the Moon is not obscured by clouds or fog.

meridian of Leyszningen, because we have computed the following tables of eclipses by mathematical calculation to the meridian of Leyszningen. For it is a city of Misnia, whose longitude is 30° 20′, but, according to Ptolemy, 34° 40′. But if the beginning of the eclipse differs, say that that place has a different meridian, and a different longitude, which you will find out in this manner. Subtract the number of hours and minutes of the eclipse, the less from the greater, and convert the difference into degrees, and minutes of a degree, as follows. For every hour take 15 degrees, for 4 minutes of an hour one degree, and for every minute of an hour 15 minutes of a degree. Then add to the degrees of longitude, of the meridian of Leyszningen, the number of degrees and minutes now taken out, if more easterly, that is, if the number of hours in it be found greater: or subtract, if more westerly, that is, if a less number of hours be found in it, than in the tables of eclipses, and you shall have the longitude of this town, which had been formerly unknown to you. Proceed in like manner, also, with eclipses which are calculated for a different meridian."

Example.

Example.

Was the lunar eclipse of October 11, 1791, visible at Cape Comorin, in latitude 7° 56′ N. and longitude 78° 5′ E.?

App. time beginning at Green.	12h 0′	App. time of end	15h 11′
Long. Cape Comorin in time	5 12 E.	- -	5 12
Reduced time, October 11,	17 12	- - -	20 23
or October 12,	5 12 A. M.	- - -	8 23 A.M.

Now, to latitude 7° 56′ N. and declination of the Sun 7° 25′ S. the semidiurnal arch is about 5h. 56′. The Sun, therefore, rises at 6h. 4′. Hence, the beginning of the eclipse was visible; but its end was invisible, as at that time the Sun was considerably above the horizon, and, therefore, the Moon nearly as much below.

Problem II.

To observe a Lunar Eclipse, and from thence to determine the Longitude of the Place of Observation.

Rule.

Apply a suitable magnifying power to the telescope, and adjust it to distinct vision, by means of the lunar spots, or by any other convenient object. Put the watch in a proper place, or give it to an assistant to hold, who also may be provided with materials to mark down the times of observation. Direct the telescope to that part of the Moon which, it is expected, will first touch the shadow some minutes before the time of the beginning of the eclipse, if that phase be visible; and as the extremity of the shadow is not well defined, a certain degree of shade must be fixed on; which may be determined soon after the Moon has entered the penumbra. The end of the eclipse is also to be observed, the same degree of shade being used.

Reduce the times per watch of the beginning and end of the eclipse to apparent time; the mean of these will be the apparent time of the middle of the eclipse, the difference between which, and that given in the Nautical Almanac, will be the longitude in time of the place of observation; which is east or west, according as the time at the place of observation is later or earlier than the Greenwich time.

If only the beginning or end of the eclipse be observed, it must be compared with the correspondent phase in the Nautical Almanac, to find the longitude. If the eclipse is total, it will be proper to observe the immersions of both limbs of the Moon, and also their emersions. The mean of the first immersion and last emersion is to be taken, and also that of the last immersion and first emersion; and the mean of these will be the time of the middle of the eclipse, being more to be depended on than that deduced from a single pair of observations.

Now

Now this mean being compared with the time of the middle of the eclipse, as given in the Nautical Almanac, will give the longitude as before.

If the observer is provided with a sextant, it will conduce to greater accuracy to measure the versed sines of the uneclipsed part of the Moon, as often as is possible, at the ingress; and to take correspondent observations at the egress.

Examples.

I.

October 2, 1800, the beginning of the lunar eclipse was observed at 6h. 1½′ P. M. per watch, and the end was observed at 12h. 52′ P. M. the error of the watch for apparent time was 13¼′ slow. Required the longitude of the place of observation?

Beginning of the eclipse per watch	6h	1½′
End - - - -	7	52
		53½
Middle of eclipse per watch - -	6	56¾
Error of the watch - -	+	13¼
Apparent time of middle of eclipse	7	10
- - per Naut. Almanac -	9	56
Longitude in time - - -	2	46 = 41° 30′ W.

II.

April 18, 1791, being in lat. 11° 48′ N. and long. by account 80½ E. the undermentioned observations of the lunar eclipse were taken; and towards the middle of the eclipse, the following altitudes of Antares were observed, in order to ascertain the error of the watch, the height of the eye being 14 feet. Required the longitude of the place of observation?

Time p. watch	9°	53′	18″	Alt. =	11°	43′
		54	20		11	56
		55	12		12	8
		56	14		12	21
		19	4			8
Mean -	9	54	46	Mean	12	2
Dip and refraction - -						8
Altitude corrected - -					11	54

Altitude

Altitude corrected - -	11° 54′		
Polar distance - -	115 57	- Co-secant	- 0.04616
Latitude - - - -	11 48	- Secant	- 0.00928
Sum - - - - -	139 39		
Half - - -	69 49½	- Co-sine	- 9.53768
Difference - - -	57 55½	- Sine	- 9.02806
			19.52118
Half mer. dist. in deg. -	35 11⅛	- Sine	- 9.76059
Multiply by - - -	8		
Antares' meridian distance	4 41 29		
- - right ascension -	16 16 38		
Right ascension of meridian	11 35 9		
Sun's right ascension -	1 45 23		
Approximate time -	9 49 46		
Equation to approximate time	— 1 31		
Equation to longitude E.	+ 50		
Apparent time - -	9 49 5		
Time per watch - -	9 54 46		
Watch fast - - -	5 41		

Observations of the Eclipse.

Beginning	8h 44′ 43″	V. S. unecl. part.	End 11h 37′ 25″
	8 58 30	28′	11 2 30
	9 9 34	22½	11 12 40
	9 21 50	18	11 0 20
	9 35 40	13½	10 46 16
	9 49 50	10	10 32 30
	100 7		32 41
Mean -	9 16 41⅙		11 5 26⅚
			9 16 41⅙
			20 22 8

Time per watch of middle of eclipse -	10 11 4
Watch fast - - - - - -	5 41
Apparent time of middle of eclipse -	10 5 23
- - per Nautical Almanac -	4 42 20
Longitude in time - - - - -	5 23 3 = 80° 45¾′ E

If

If the eclipse be partial, observations of the versed sines of the enlightened part of the Moon, taken near the middle, will be of no service in determining the instant of that phase, as the change is then too slow. The following method of observing a lunar eclipse is commonly practised in observatories.

Set the assistant clock a going exactly with the transit clock; then, the telescope being prepared for observation, direct it to the Moon as before, and, as it is difficult to ascertain the precise instant of the beginning of the eclipse, a certain degree of shade is to be fixed on, after the Moon has entered the penumbra. Observe also the immersion of both edges of all the remarkable spots that lie in the path of the shadow. In like manner, the emersions of the same spots, and the end of the eclipse, are to be observed, the same degree of shade being still used.

Reduce the times per clock of observation to apparent time; compare the instants of the beginning and end of the eclipse by observation, with those given in the Nautical Almanac, and their difference will be the longitude of the place of observation in time.

If the eclipse has been observed at any other place, compare the instants of each corresponding pair of observations with each other; their difference will be that of the meridians in time. Take the mean of all the differences of longitude deduced from the ingress, and also the mean of those inferred from the egress; and the mean of both will be the correct difference of longitude. Hence, the longitude of either place, with respect to some given meridian, may be obtained, provided the longitude of the other place is known.

As it often happens, by reason of clouds, that the ingress or egress of the same spots of the Moon cannot be observed at two different places; and hence, the difference of longitude between these places cannot be ascertained by a comparison of similar observations; M. Cassini, therefore, proposed, in the Memoirs of the Academy of Sciences for 1692 *, to find the difference of longitude between two places, when the ingress or egress of those spots observed at one of the places had not been observed at the other. This is done by computing the instant, when a spot observed at one of the places, but not at the other, would have been observed at this other place, by means of the times of the observation of two adjacent spots, and a map of the Moon. M. Cassini applied this method to ascertain the difference of longitude between Lyons and Marseilles from incomplete observations of the lunar eclipse of 28th July, 1692, made at Lyons, and a small island near Marseilles.

* Page 190. Amsterdam, 1723.

DETER-

DETERMINATION

Of the Longitude of the late Observatory at Aberdeen, by Observations of the Lunar Eclipse of 10th September, 1783, made at Aberdeen, and at Chislehurst in Kent, 19″ in time East of the Royal Observatory at Greenwich.

Clock fast 10th Sept. at noon -	10′ 9″.5	Equation of time at noon - -	3′ 11″.7
and gained daily - -	5 .5	and daily increase - - -	20 .5
Hence clock fast at ix hours =	10 11 .3	Hence, eq. of time at ix hours =	3 19 .4
x - - - - -	10 11 .5	x - - -	3 20 .2
xi - - -	10 11 .8	xi - -	3 21 .1
xii - - -	10 12 .0	xii - -	3 21 .9
xiii - - -	10 12 .2	xiii - -	3 22 .8
xiv - - -	10 12 .5	xiv - -	3 23 .7

Names of Spots.	Time p. C.	Err. Clk.	Eq. Time	Appar. Time.	Ap. t. at Ch.	Dif Mer.
INGRESS.						
		—	+			
Aristarchus - -	9h 49′ 34″	10′ 11″.5	3′ 20″.0	9h 42′ 42″.5	9h 50′ 55″	8′ 12″.5
Kepler - - - -	9 51 1	10 11 .5	3 20 .1	9h 44 9 .6	9 52 20	8 10 .4
Copernicus - - -	10 1 0	10 11 .5	3 20 .2	9 54 8 .7	10 2 24	8 15 .3
Manilius covered -	10 14 27	10 11 .6	3 20 .4	10 7 35 .8	10 15 30	7 54 .2
Tycho covered -	10 15 49	10 11 .6	3 20 .4	10 8 57 .8	10 17 5	8 19 .2
Menelaus covered	10 17 43	10 11 .6	3 20 .5	10 10 51 .9	10 19 10	8 18 .1
Dionysius covered -	10 20 38	10 11 .6	3 20 .5	10 13 46 .9	10 21 38	7 51 .1
Plinius covered -	10 21 47	10 11 .6	3 20 .5	10 14 55 .9	10 22 40	7 44 .1
Mare Cris. east end	10 32 42	10 11 .6	3 20 .6	10 25 51 .0	10 34 34	8 43 .0
west end	10 37 44	10 11 .7	3 20 .7	10 32 53 .0	10 39 45	8 52 .0
Total darkness -	10 43 30	10 11 .7	3 20 .7	10 36 39 .0	10 46 34	9 55 .0
				Sum - - - -		254.9
				Mean - - -		8 23 17
EGRESS.						
Aristarchus - -	12h 31′ 54″	10′ 12″.1	3′ 22″.4	12h 24′ 4″.3	12h 33′ 52″	9′ 47″.7
Kepler - - - -	12 33 55	10 12 .1	3 22 .4	12 27 5 .3	12 37 26	10 20 .7
Copernicus - - -	12 42 24	10 12 .1	3 22 .5	12 35 34 .4	12 45 52	10 18 .6
Plato, east end - -	12 45 22	10 12 .2	3 22 .6	12 38 32 .4	12 47 22	8 49 .6
Tycho, east end -	12 46 54	10 12 .2	3 22 .6	12 40 4 .4	12 48 30	8 25 .6
west end -	12 47 55	10 12 .2	3 22 .6	12 41 5 .4	12 49 58	8 53 .6
Menelaus - - -	12 59 50	10 12 .2	3 22 .6	12 53 0 .6	13 1 40	8 39 .4
Dionysius - - -	13 1 35	10 12 .2	3 22 .8	12 54 45 .6	13 3 18	8 32 .4
Plinius - - - -	13 3 38	10 12 .2	3 22 .8	12 56 48 .6	13 5 40	8 51 .4
Mare Cris. east end	13 14 0	10 12 .3	3 22 .0	13 7 10 .7	13 16 35	9 24 .3
west end	13 19 10	10 12 .3	3 23 .1	13 12 20 .8	13 20 53	8 32 .2

Sum	95.5
Mean per egress - - - - - - - - - -	9 8.68
- - per ingress - - - - - - - - - -	8 23.17
	17 31.85
Difference meridians of Aberdeen and Chislehurst in time nearly -	8 45.92
Longitude of Chislehurst in time E. - - - - - - - -	— 19
Longitude of Aberdeen nearly - - - - - - - - -	8 26.92
Change of equation of time in 8′ 27″ - - - - - - -	— .12
Longitude Observatory at Aberdeen in time - - - - -	8 26.8 W.

CHAP.

CHAP. V.

The Method of finding the Longitude of a Place

BY

An Eclipse of the Sun.

INTRODUCTION.

THIS and the following methods are the most accurate of any that have hitherto been employed for the purpose of finding the longitude of a place. The difference of the meridians of two places may be found to the nearest second of time, by comparing correspondent observations of the same eclipse. These observations are, therefore, eagerly desired by the practical astronomer. The calculation is, indeed, longer than that in any of the former methods. This arises from parallax, the effect of which may be found by various methods; however, that by means of the *Nonagesimal* appears to be the easiest. This method was proposed by M. Cassini in the Histoire de l'Academie Royale des Sciences for 1700, page 131, Amsterdam, 1706; and has since been adopted by succeeding astronomers, as being one of the most accurate methods for this purpose.

As the ecliptic is a great circle, one half of it is always above the horizon, and the other half underneath it; the middle of the elevated part is, therefore, 90° distant from its intersection of the horizon, and is hence called the *Nonagesimal.* The elevation of the nonagesimal above the horizon is called its *Altitude*, and its distance from the equinoctial point Aries, is called its *Longitude*.

PROBLEM I.

Given the Apparent Time at a Place, its Latitude, and the Obliquity of the Ecliptic, to find the Altitude and Longitude of the Nonagesimal.

RULE.

From the given latitude, subtract the number answering thereto in Table XXXVI. the remainder will be the reduced altitude.

To the apparent time, add the Sun's right ascension, reduced to the

given time and place; the sum will be the right ascension of the meridian.

Let the difference between the right ascension of the meridian, and O. XII. or XXIV. hours, according to which it is nearest, be reduced to degrees, and called arch first.

Then to the log. co-tangent of the reduced latitude, add the log. sine of arch first; the sum will be the log. tangent of arch second. Now, if arch first is between 0° and 180°, the sum of arch second and the obliquity of the ecliptic is arch third, otherwise their difference.

To the log. tangent of arch first, add the log. co-secant of arch second, and the log. sine of arch third, the sum is the log tangent of arch fourth; which, when the right ascension of the meridian is between 0 and VI. hours, is the longitude of the nonagesimal, or its supplement if between VI. and XII. hours. But arch fourth being added to 180°, if the right ascension of the meridian is between XII. and XVIII. hours, or subtracted from 360°, if between XVIII. and XXIV. hours, will give the longitude of the nonagesimal.

To the log. tangent of arch third, add the log. co-secant of arch fourth, the sum will be the log. tangent of the altitude of the nonagesimal.

EXAMPLE.

Required the altitude and longitude of the nonagesimal, in latitude 57° 8′ 9 N. and longitude in time 8′ 40″ W. Nov. 26, 1787, at 11h. 18′ 8″. apparent time * ?

	Apparent time - -	11h 18′ 8″
	☉'s R. A. at red time -	16 10 57
Latitude = 57° 8′.9		
Reduction — 13.8	R. A. of meridian -	3 29 5 = 52° 16¼′
Red. lat. - 56 55.1	Co-tang. 9.81387	
Arch first 52 16.¼	Sine - 9.89813 tang.	0.11143
Arch 2d - 27 15.½	Tangent 9.71200 co-sec.	0.33913
Obliq. ecl. 23 28.0		
Arch 3d - 50 43.½	- Sine -	9.88881 tang. 0.08737
Long. non. 65 24.¼	- Tangent -	0.33937 co-s. 0.04131
	Altitude nonagesimal 53° 22′	tang. 0.12868

The altitude and longitude of the nonagesimal may also be readily found by the following rule.

* It is sufficient, in most cases, to find the altitude and longitude of the nonagesimal to the nearest minute.

Find

Find the reduced latitude of the place, and the right ascension of the meridian, as before. Then, if the right ascension of the meridian is less than VI. hours, increase it thereby, and the sum will be the arch first; but if it exceeds VI. hours. the difference between it and XVIII. hours is arch first.

Take the log. corresponding to arch first from column of rising, to which add the log. co-sine of the reduced latitude, and the log. sine of the obliquity of the ecliptic; the sum will be the log of a natural number; which being added to the natural co-versed sine of the sum of the reduced latitude and obliquity of the ecliptic, will give the nat. versed sine of the altitude of the nonagesimal.

Take the log. corresponding to arch first, or its supplement, if it exceeds VI. hours, from Table XLVII.; to which add the log. secant of the reduced latitude, and the log. sine of the altitude of the nonagesimal; the sum will be the log. secant of the longitude of the nonagesimal, if the right ascension of the meridian is less than VI. hours. If between VI. and XII. hours, its supplement is the longitude of the nonagesimal; but being added to 280°, if the right ascension of the meridian is between XII. and XVIII. hours, or subtracted from 360° if above XVIII. hours, will give the longitude of the nonagesimal*.

Remark.

If arch first exceeds the limits of the tables, take the log. from Table XLIX answering to its supplement, to which add the log. co sine of the latitude, and the log. sine of the obliquity of the ecliptic; the natural number answering to the sum of these three logs being subtracted from the natural co-versed sine of difference of the latitude and obliquity of the ecliptic, will give the natural versed sine of the altitude of the nonagesimal.

Example.

Required the altitude and longitude of the nonagesimal at Greenwich, June 3, 1788, at 19h 24′ 56″.5 apparent time?

Apparent time =	19h	24′	56″.5	Lat. of Greenwich	51°	28′	40″	N.
Sun's right asc.	4	51	30 .5	Reduction - -		14	33	
R. A. meridian –	0	16	27 .0	Reduced latitude	51	14	7	N.
	6							
Arch first - -	6	16	27					

* The altitude and longitude of the nonagesimal may be found independent of each other by the following formulæ.

Co-sine alt. nonagesimal = sine obl ecl. cos. lat sine R. A. M. from near equinoctial point + cos. obliquity ecl. sine latitude. } to radius 1.
Tang. long. non. from nearest eq. = sine ob. ecl. secant R. A. M. tang latitude. — cos. obliquity ecl. tan. R. A. Meridian. }

Arch first - -	6 16 27	- -	Rising	-	5.03007
Latitude - -	51 14 —	- -	Co-sine	-	9.79668
Obliq. ecliptic -	23 28 —	- -	Sine	- -	9.60012
Sum - -	74 42 —	n. co. ver. sine = 03544			
			26722	-	4.42687
Alt. nonagesimal	45 47 —	n. ver. sine -	30266 - Sine	-	9.85534
Suppl. arch first	5 43 33	- -	H. E. T.	- -	0.00112
Reduced latitude	51 14 —	- -	Secant	- -	0.20332
Long. nonages.	29 23 —	- -	Secant	- -	0.05978

The effect of parallax depends on the altitude of the nonagesimal, and on the distance of the object therefrom. When the object is in the nonagesimal, its latitude only is affected; but in other positions, both the latitude and longitude of the object are altered by parallax. These effects may be computed as follows.

Problem II.

Given the Latitude of a Place, the Altitude and Longitude of the Nonagesimal, together with the Longitude, Latitude, and Horizontal Parallax of the Moon, to find the Moon's Parallax in Longitude and in Latitude.

Rule.

From the horizontal parallax of the Moon, subtract the number from Table XXXIII. answering to the given latitude, the remainder will be the *reduced parallax*.

To the proportional log. of the reduced parallax, add the log. co-secant of the altitude of the nonagesimal, the log. co-sine of the true latitude of the Moon, and the log. co-secant of the difference between the longitudes of the Moon and nonagesimal, the sum will be the prop. log. of the approximate parallax in longitude; which added to the difference between the longitudes of the Moon and nonagesimal, the sum will be the apparent difference of longitude nearly. Now, to the sum of the three first logarithms, add the log. co-secant of the apparent difference of longitude of the Moon and nonagesimal, and the sum will be the prop. log. of the parallax in longitude: which is additive to the Moon's longitude when that object is east of the nonagesimal, otherwise it is subtractive.

To the prop. log. of the Moon's reduced parallax, add the log. secant of the altitude of the nonagesimal, and the log. secant of the Moon's latitude, the sum will be the prop. log. of the approximate parallax in latitude: and since the parallax increases the Moon's distance from the elevated pole of the ecliptic, it is, therefore, to be subtracted

subtracted from the Moon's true latitude, when the latitude of the Moon, and the elevated pole of the ecliptic are of the same name; but added when of a contrary denomination, in order to obtain the approximate apparent latitude of the Moon.

Then to the log. secant of the approximate apparent latitude of the Moon, add the log. secant of the altitude of the nonagesimal, and the prop. log. of the Moon's reduced horizontal parallax, the sum will be the prop. log. of part first of the parallax in latitude.

Again to the log. co-secant of the Moon's approximate latitude, add the log. co-secant of the altitude of the nonagesimal, the prop. log. of the reduced parallax, and the log. secant of the apparent difference of longitude between the Moon and nonagesimal, the sum will be the prop. log. of part second of the parallax in latitude. If the Moon is between the ecliptic and its elevated pole, and the apparent difference of longitude between the Moon and nonagesimal less than 90 degrees, part second subtracted from part first gives the parallax in latitude; but if the Moon be on the opposite side of the ecliptic with respect to the elevated pole, their sum will be the parallax in latitude. If the apparent distance of the Moon from the nonagesimal exceeds 90 degrees, then the sum or difference of the two arches, according as the Moon is upon the other side of the ecliptic, or between the ecliptic and the elevated pole, will be the parallax in latitude.

When the parallax in latitude is to be subtracted from the Moon's latitude, and equal to it, or nearly so, the approximate parallax in latitude may be taken for the true parallax in latitude.

Example.

Let the longitude and altitude of the nonagesimal be 29° 23′ and 45° 47′ respectively, the Moon's longitude 2ˢ 13° 19′ 41″, latitude 0° 20′ 9″ N. and equatorial horizontal parallax 60′ 33″, and latitude of the place of observation 51°.14′ N. Required the parallax of the Moon in longitude and latitude?

Equat hor. par. ☽ -	60′ 33″			Moon's longitude -	2ˢ 13° 20′
Reduction Table XXXIII. —	9			Long. nonagesimal	0 29 23
Reduced parallax - -	60 24	P. L.	0.4742	Diff. of long.	1 13 57
Altitude nonagesimal -	45 47	co-sec.	0.1447		
Moon's latitude - -	20	co-sine	0.0000		
			0.6189	- -	0.6189
Difference of longitude	43 57	co-sec.	0.1586		
Appr. par. longitude -	+ 30	P. L.	0.7775		
Approximate diff. long.	44 27	- -		- co-secant -	0.1547
Parallax in longitude - -			30′ 19″	- P. L. -	0.7736

Parallax

Parallax in longitude -	-		30′	19″
Moon's true longitude -	2	13	19	41
Moon's apparent longitude	2	13	50	0

Reduced parallax -	60′	24″	P. L.	0 4742
Alt. nonagesimal 45	47		secant	0.1565
Moon's latitude	20	9	secant	0.0000
Appr. par. latitude	42	7	P. L.	0.6307

Appr. app. latitude	21	58	secant	0.0000	- co-secant	- -	2.1945
Alt. nonagesimal 45	47		secant	0.1565	- co-secant	- -	0.1447
Moon's red par.	60	24	P. L.	0.4742	- - - - - -		0.4742
Part first par. in lat.	42	7	P. L.	0.6307	App. D L. 44° 27′	sec.	0.1464
Part second - -		12	- - -		P. L.	-	2.9598

Parallax in latitude	41	55	
Moon's true latitude	20	9	N.
Moon's app. latitude	21	46	S.

Problem III.

To find the Apparent time at Greenwich of the Ecliptic Conjunction of the Sun and Moon.

Rule.

Find the difference between the longitudes of the Sun and Moon, at the noon or midnight immediately preceding the conjunction, and also that which follows it. To the logarithm of the first of these differences, expressed in seconds, add the ar co-log. of the sum of the two differences in seconds, and the constant log. 2.857325, the sum will be the log. of the approximate time in minutes, reckoned from the preceding noon or midnight.

Take the equation from Table XXXVIII. answering to the sum of the two differences, and to the mean second difference; which being applied to the approximate time, by addition or subtraction, according as the Moon's relative motion in longitude is increasing or decreasing; hence the apparent time of conjunction will be obtained.

Example

Example.

Required the apparent time of the conjunction of the Sun and Moon, June 3, 1788.

	Moon long.	Sun's long.	Difference.	First diff.	Second diff.	Mean
June 3, noon	2s 1° 29′ 3″	—2 13 26 43	—11 57 40			
midn.	2 8 47 5	—2 13 55 26	— 5 8 21	+6 49 19		
4 noon	2 16 8 50	—2 14 24 8	+ 1 44 42	+6 53 3	+3 44	
midn.	2 23 33 29	—2 14 52 50	+ 8 40 39	+6 55 57	+2 54	+ 3 19

Preceding difference = 5° 8′ 21″ = 18501″	- log. -	4 267195
Sum of preced. & fol dif. 6 53 3 = 24783	- ar-co. log. -	5.605846
Constant log. - - - - - - - -		2.857332
Approximate time - 8h 57′ 29″.6 = 537′.494	– log. -	2,730373
Equation Tab. XXXVIII. + 32 .8		
Ap. t. conj. past midn. 8 58 2.4 or June 3, at 20h. 58′ 2″.4.		

Problem IV.

Given the Apparent Times of the Beginning and End of a Solar Eclipse, to find the Longitude of the Place of Observation.

Rule.

Reduce the apparent times of the beginning and end of the eclipse to the meridian of Greenwich, by applying thereto the estimated longitude, by Prob. III. page 105; and to these times let the Moon's horizontal parallax, longitude and semidiameter, the Sun's right ascension and semidiameter, be taken from the Nautical Almanac; hence the reduced parallax of the Moon, and the right ascension of the meridian, will be known. From the Sun's semidiameter subtract the quantity of irradiation, which is estimated at 3½ seconds; and diminish the latitude of the place by the quantity from Table XXXVI. and from the Moon's reduced parallax subtract the Sun's horizontal parallax from Table VII.

Compute the altitude and longitude of the nonagesimal by Prob I. and, thence, the parallaxes in longitude and latitude be the instants of observation by Prob. II. using the difference between the Moon's reduced parallax and the Sun's horizontal parallax.

From the Moon's relative motion in longitude during the observed interval, subtract the difference between the parallaxes in longitude, if the Moon is on the same side of the nonagesimal at the beginning and end of the eclipse; but if on opposite sides, subtract the sum of the parallaxes, and the remainder will be the apparent motion in longitude in the observed interval.

If the true motion of the Moon is towards the elevated pole of the ecliptic, then to the motion in latitude in the given interval, add the parallax in latitude at the beginning of the eclipse, and the difference between

between this sum and the parallax in latitude at the end, will be the apparent motion of the Moon in latitude; which will be towards the elevated pole of the ecliptic, if the parallax in latitude at the end of the eclipse be less than the above sum: but if greater, the apparent motion of the Moon in latitude will be towards the opposite pole. Again, if the true motion of the Moon in latitude is from the elevated pole of the ecliptic, then, to the true motion of the Moon in latitude in the given interval, add the parallax in latitude at the end of the eclipse, and the difference between this sum and the parallax in latitude at the beginning, will be the apparent motion of the Moon in latitude; which will be towards the elevated pole of the ecliptic, if the above sum is less than the parallax in latitude at the beginning; but if the above sum is greater than the parallax in latitude at the beginning, the apparent motion of the Moon in latitude will be from the elevated pole of the ecliptic.

From the prop. log. of the apparent difference of longitude, its index being increased by 10, subtract the prop. log. of the apparent difference of latitude; the remainder will be the log. tangent of the apparent inclination; to the logarithm co-sine of which add the prop. log. of the apparent difference of longitude, and the sum will be the prop. log. of the apparent motion of the Moon in its relative orbit.

With the altitude and longitude of the nonagesimal, find the augmentation of the Moon's semidiameter by Table xxx. Now, the Moon's semidiameter being increased by the augmentation, and diminished by the inflexion, which is about $3\frac{1}{2}$ seconds, will give the corrected semidiameter of the Moon.

Find the sum of the corrected semidiameters of the Sun and Moon at the beginning, and also at the end of the eclipse; and from the prop. log. of their sum, its index being increased by 10, subtract the prop. log. of the Moon's relative motion in its proper orbit, the remainder will be the log. co-sine of the central angle.

If the apparent motion of the Moon in latitude is decreasing, that is, when the Moon is approaching towards the ecliptic, the sum of the central angle and the apparent interval is *arch first*, and their difference *arch second*; but if the apparent latitude of the Moon is increasing, the difference between the central angle and the apparent inclination is *arch first*, and their sum *arch second*.

To the log. secant of arch first, add the prop. log. of the sum of the corrected semidiameters of the Sun and Moon at the beginning of the eclipse, the sum will be the prop. log. of arch third; and to the log. secant of arch second add the prop. log. of the sum of the corrected semidiameters of the Sun and Moon at the end, the sum is the prop. log. of arch fourth.

Now, if the Moon is east of the nonagesimal, to arch third add the parallax in longitude at the beginning of the eclipse, and from arch fourth

fourth subtract the parallax in longitude at the end; if the Moon is west of the nonagesimal, apply the contrary rules. Then, to the prop. logs. of each of these quantities, add the ar-co. of the prop log. of the Moon's relative motion in longitude in the observed interval, and the prop. log of the observed interval; the sums, rejecting 10 from each of the indices, will be the prop. logs. of the intervals of time between the ecliptic conjunction, and the beginning and end of the eclipse, respectively: hence the apparent time of the ecliptic conjunction at the place of observation will be known.

Compute the apparent time of the ecliptic conjunction of the Sun and Moon at Greenwich by the last problem; the difference between which, and the apparent time of conjunction at the place of observation, will be its longitude in time; which is east or west, according as the time of conjunction, at the place of observation, is later or earlier than that at Greenwich.

If the apparent time of conjunction at Greenwich, found directly from observation, be used instead of that as found by the last problem, the longitude of the place of observation will be more accurately determined.

Example.

June 3, 1788, in latitude 57° 9′ N. and estimate longitude 8′ 32″ in time, the beginning of the solar eclipse was observed at 19h. 33′ 19″ apparent time, and the end at 20h 49′ 29″. Required the true longitude of the place of observation?

App. t. beg. eclipse	19h 33′ 19″	App. time of end -	20h 49′ 29″
Est. long. in time -	+ 8 32	- - - -	+ 8 32
Reduced time -	19 41 51	- - - - -	20 58 1
Moon's hor. par. at beg.	60′ 33″	- - - at end - -	60′ 34½″
Reduction, Tab. XXXIII.	— 11	- - - - - - - - -	11
Reduced hor. parallax -	60 22	- - - - -	60 23½
Sun's horizontal parallax	8½	- - - - -	8½
Difference - - - - -	60 13	- - - - -	60 15
Sun's semidiameter -	15 48½	Latitude -	57 9
Irradiation - - -	— 3½	Red. Table XXXVI. -	— 13.7
Corrected semidiameter	15 45	Reduced latitude -	56 55.3
Sun's long. at beg.	2s 14° 14′	Sun's long. at end -	2s 14° 17′
Semidiameter - -	— 16	Semidiameter - -	+ 16
Long. ☉'s w. L at beg.	2 13 58	Long. ☉'s east l. at end	2 14 33

Ap. t. beg. 19h 33′ 19″ - - App. time of end - - 20h 49′ 29″
☉'s R.A. bg. 4 57 34 - - Sun's right asc. at end 4 51 47

R. as. mer. 0 24 53 - - Right asc. of mer. at end 1 41 16
6

Arch first 6 24 53 rising - 5.04467 Sup. 5h 35′ 7″ H.E.T. 0.00256
Red. lat. 56 55.3 - co-sine 9.73702 - - secant - - 0.26298
Obl. ecl. 23 28. - sine - 9.60012

Sum - 80 23.3 n.c.v.s. 01404
24089 4.38181

Alt. nonag. 41 50 n. v. s. 25493 - - sine - - 0.82410

Long. non. 35 34 - - - - secant - 0.08864
L. ☉'s w. l. 78 58

Difference 38 24 - co-secant - 0.2068
Alt. nonag. 41 50 - co-secant - 0.1759 - - secant - - 0.1278
Dif. ☉ & ☽'s h.p. 60 $13\frac{1}{2}$ - P. L. 0.4755 - - - 0.4755

Par. in long. 24′ 57″* P. L. - 0.8582 Par. in lat. 44′ 52″ P. L. 0.6033

R.A.M. at e. 1h 41 16
6

Arch first 7 41 $16\frac{1}{2}$ rising - 5.15460 Sup 4h 18 44′ H.E.T. 0.04385
Red. lat. - 56 55.3 - co-sine 9.73702 - - secant - 0.26298
Obl. ecl. - 23 28 - sine - 9.60012

Sum - 80 23.3 n.c.v.s. 01404
31027 4.49174

Alt. non. 47 $29\frac{1}{2}$ n.v.s. 32431 - - - - - sine - - 9.86758

Long. non. 47 59 - - - - - - - - secant - - 0.17441
Lon. ☉'s c. l. 74 33

Difference 26 $33\frac{1}{2}$ co-secant 0.3496
Alt. non. - 47 $29\frac{1}{2}$ co-secant 0.1324 - - secant - 0.1702
Diff. ☉ & ☽'s h.p. 60 15 P. L. 0.4753 - - P. L. - - 0.4753

Par. in long. - $19' 31\frac{1}{2}''$ P. L. - 0.9573 Par. in lat. 40′ 43″ P. L. 0.6455

* In computing the parallaxes in latitude and longitude, the Moon's latitude may be supposed = 0, by which means the computation will be rendered more simple, and the accuracy at the same time will not be diminished.

Par.

Par. in long. - 19′ 51″ Par. in lat. - - 40′ 43″
Par. in long. at beg. 24 57 - Change ☽'s lat. in ob. int. 4 20

Difference - - 5 5½ Sum - - - - - 45 3
Par. in lat. at beg. - 44 52

Rel. mot. in long. 43 46 Difference - - - 11 P. L. 2.9920

Ap. mot. in long. 38 40½ - P.L + radius - - - - - 10.6678

App. inclination 0° 16′ - Tangent - - - 7.6758

Hence, the apparent motion in relative orbit is nearly equal to the apparent motion in longitude.

☽'s sem. at beg. 16′ 30″ - at end - 16′ 30″
Augment. - + 9 - - - - + 11½
Inflexion - — 3½ - - - - — 3½

Correct. sem. 16 35½ - - - 16 38
Sun's semid. 15 45* - - 15 45

Sum - - 32 20½ + - - 32 23 = 64′ 43½″ P.L. + rad. 10.4442
Apparent motion in proper orbit - - 38 40½ P.L. - - 0.6678

Central angle - - - - 53° 18′ - co-sine - 9.7764
Apparent inclination - - - 16

Arch first - - - - - 53 34
Arch second - - - - 53 2

Ar. 1st 53° 34′ - second 3.2263 Arch sec. 53° 2′ second 0.2209
Sum se. be. 32 20½ - P.L. 0.7455 Sum sem. at end 32 23 P. L. 0.7449

19 12 - P.L. 0.9718 19 28½ P.L. 0.9658
Par. l. at b. 24 57 - Par. lon. at end 19 51½

Sum - 44 9 - P.L. 0.6103 Difference - - 23 P.L 2.6717
☽'s r. m. l. 43 46 ar-co. P.L. 9.3859 - - - - - - - - - - 9.3859
Obs. inter. 1h 16 10 P.L. 0.3735 - - - - - - - - - - 0.3735

Int. b. be. & ☌ 1 16 50 P.L. 0.3697 Int. be. bg. & ☌, 0h 0′ 40″ P.L. 2.4311
Apt. t. beg. 19 33 19 Ap. t of end, 20 49 29

Ap. t. conj. 20 50 9 Ap. t. conj. 20 50 9
Ap. time of conj. at Green. by Prob. III. p. 239 50 58 2

Longitude in time - - - 7 53 W.

* The Sun's semidiameter is diminished by the quantity of irradiation.

But the apparent time of conjunction at Greenwich, as deduced from observation, was 20° 58′ 45″; the difference between which, and 20h. 50′ 9″ is 8° 36′, the longitude in time, = 2° 9′ W.

The computation being performed upon the assumption that the figure of the earth is an exact sphere, the longitude of Aberdeen will be 8′ 32″.8 in time. See a paper by the author, upon the Determination of the Latitude and Longitude of the late Observatory at Aberdeen, in the Philosophical Transactions of the Royal Society of Edinburgh, vol. IV. page 150.

PROBLEM V.

Given the Apparent Time of the Beginning, or of the End of a Solar Eclipse, to find the Longitude of the Place of Observation.

RULE.

Reduce the apparent time of observation to the meridian of Greenwich; to which time, find the Moon's semidiameter, horizontal parallax, and latitude; and the Sun's semidiameter, longitude, and right ascension; find also the difference between the Moon's reduced parallax and the Sun's horizontal parallax.

Now, compute the altitude and longitude of the nonagesimal, and the parallaxes in latitude and longitude as usual. Hence, the Moon's apparent latitude will be known; to the P. Log. of the sum of which, and the sum of the semidiameters of the Sun and Moon, add the P. Log. of their difference; half the sum will be the P. Log. of arch first.

If the Moon is east of the nonagesimal, and the observation that of the beginning of the eclipse, the sum of the above arch and the parallax in longitude is to be taken; but if the end of the eclipse be observed, find their difference. If the Moon is west of the nonagesimal, use the contrary rules. Then to the Prop. Log. of this sum or difference, add the ar-co. of the Prop. Log. of the Moon's relative motion in longitude, and the constant log. 0.4771; the sum will be the Prop. Log. of an arch.

Now, if the Moon is east of the nonagesimal, the above arch must be added to the time of the beginning of the eclipse, or subtracted from that of the end, if the parallax in longitude is less than arch first; otherwise added. If the Moon is west of the nonagesimal, the above arch must be applied to the apparent time of the beginning, by addition or subtraction, according as it is greater or less than the parallax in longitude; but if the observation is that of the end of the eclipse, it must be subtracted therefrom, and the sum or difference will be the apparent time of conjunction at the place of observation; the difference between which, and that by the meridian of Greenwich, inferred

inferred from the Nautical Almanac, by Prob. III. or from observation as above, will be the longitude of the place in time.

Example.

At the Observatory at Aberdeen, the author observed the end of the solar eclipse of June 15, 1787, at 5h. 32′ 2″ apparent time; and at the Royal Observatory at Greenwich, Dr. Maskelyne observed the beginning at 4h. 10′ 58″ apparent time*. (The beginning of the eclipse could not be observed at Aberdeen, nor the end at Greenwich, by reason of clouds.) Required the longitude of Aberdeen.

Computation of the Apparent Time of Conjunction at Aberdeen.

Ap.t. end	5h 32′ 2″	Moon's semid. 16′ 45″	Sun's semid. 15′ 47″.5
Lon. in t.	8 32	Augmentation + 7	Irradiation — 3 .5
		Inflexion - — 3.5	
Red. time	5 40 34		Corr. semid. 15 44
		Corr. semidia. 16 48.5 - - -	16 48.5
☽'s h. par.	61 26		
Reduction	11	Sum - - -	32 32.5
		Moon's lat. June 15, at noon	+ 1° 13′ 6″ N.
Red. par. -	61 15	Prop. part. to 5h 40′ 34″ -	— 19 38
Sun's h. par.	— 8½	Equation of second differ.	+ 6
Difference	61 6½	Moon's lat. at reduced time -	0 53 34 N.
☽'s rt. asc.	5h 35′ 39″	Latitude -	57 9.0 N.
Ap.t. of ob.	5 32 2	Reduction -	13.7
Rt. as. mer.	11 7 41	Reduced latitude	56 55.3 N.
	18		
Arch first	6 52 19	- rising - 5.08860 Su. 5h 7′ 41″ H.E.T.	0.01141
Red. latit.	56 55.3	- co-sine 9.73702 - secant - -	0.26298
Obl. ecl.	23 28	- - sine - 9.60012	
Sum -	80 23.3	n. co. v. s. 01404	
		26652 4.42574	
Alt. non.	43 59½	n. v. s. 28056 - - - sine - -	0.84170
	40 3	- - - - secant - -	0.11609
Lon. non.	139 57		

* In Dr. Maskelyne's letter to the Author, dated Royal Observatory, Greenwich, December 29th, 1787, he says, "I observed the beginning of the eclipse of the Sun, "June 15th last, at 4h. 11′ 3″ apparent time, or rather a small impression of ☽ on ☉; "the true beginning may be reckoned 5″ sooner, or at 4h. 10′ 58″. The end was hid "by clouds."

Lon.

Lon. non. 139 57
L. ☉'s e. l. 84 40½

Difference 55 16½ - co-sec. 0.0852
Alt. non. 43 59½ - co-sec. 0.1583 - - secant - 0.1430
Dif. ☉ & ☽'s p. 61 6½ P.L. 0.4692 - - P. L. - 0.4692

Par. in long. 34 52½ P.L. 0.7127 P. in lt. 43′ 57½″ - P. L. - 0.6122
Moon's true lat. at end - - 53 34

Moon's app. latitude - - - 9 36½
Sum of ☉ & ☽'s sem. - - 32 32½

Sum - - - - - 42 9 - P. L. - 0.6305
Difference - - - - - 22 56 - P. L. - 0.8948

1.5253
Arch first - - - - - 31 5½ - P. L. - 0.7626
Par. in longitude - - - 34 52½

Sum - - - - - - - - 65 58 - P. L. - 0.4359
☽'s rel. hor. mot. - - - 35 34 ar-co. P.L. - 9.2958
Constant logarithm - - - - - 0.4771

Diff. bet. end & ☌ - - - - 1 51 18 - P. L. - 0.2088
App. time of end - - - 5 32 2

App. time of conj. - - - 3 40 44

Computation of the Apparent Time of Conjunction at Greenwich.

Moon's semid. 16′ 45″ Sun's semid. 15′ 47″.5 ☽'s hor. par. 61′ 27″
Augment. + 10 Irradiation — 3 .5 Reduction — 9.5
Inflexion - — 3.5
——— Correct. sem. 15 44 .0 Red. hor. par. 61 17.5
Cor. semid. - 16 51.5 - - - - 16 51 .5 Sun's hor. par. 8.5

Sum - - - 32 35 .5 Difference - 61 9

☉'s r. as. 5h 35′ 24″ Lat. Green. 51° 28′.7 ☽'s lat. at noon 1° 13′ 6″N.
Ap. t. of b. 4 11 31 Reduct. — 14 .6 P. p. to 4h 10′ 58″ — 14 30½
——— Eq. second dif. + 5½
R. as. mer. 9 46 55 Reduced lat. 51 14 .1
18 ☽'s lat. at beg. 0 58 41 N.

Arch 1st 8 13 5

Arch

Arch 1st 8h' 13' 5" rising - 5.18993 Sup. 3h 46' 55" H.E.T. 0.07775
Red. lat. 51 14.1 - co-sine 9.79666 - - secant - - 0.20334
Ob. ecl. 23 28 - sine - 9.60012

Sum - 74 42.1, n.c.v.s. 03543
38611 4.58671

Alt. no. 54 39½ n. v. s. 42154 - - - - sine - 9.91153

50 4½ - - - - - - - secant 0.19262

Lon. non. 129 55½
L. ☉'s w. l. 84 5½

Differ. 45 50 co-secant - 0.1443
Alt. no. 54 39½ co-secant - 0.0885 - - - secant - 0.2377
Dif. ☉ & ☽'s p. 61 9 P. L. - 0.4689 - - - P. L - - 04689

Par. in long. 35' 46" P. L. 0.7017 Par. in lat. 35' 22½" - P. L. 0.7066
Moon's latitude at beg. - - 58 41

Moon's app. latitude - - 23 18½
Sum of semidiameters - - 32 35½

Sum - - - - - - 55 54 - P. L. 0.5079
Difference - - - 9 17 - P. L. 1.2875

1.7954
Arch - - - - - 22 47 - P. L. 0.8977
Parallax in longitude - - 35 46

Difference - - - 12 59 - P. L. 1.1419
Moon's rel. hor. mot. - - 35 34½ a.c. P. L. 9.2959
Constant logarithm - - - - - 0.4771

Diff. bet. beg. and ☌ - - 21 54 - P. L. 0.9149
App. time of beg. - - 4 10 58

App. t. ☌ at Greenwich - - 3 49 4
- - at Aberdeen - 3 40 44

Longitude in time - - - 8 20 = 2° 5′ W.

The apparent time of conjunction at Greenwich, computed by Prob. III. is 3h. 49′ 28″, and hence the longitude in time is 8′ 44″. Now the mean of this and 7′ 53″, the longitude by last example, deduced from a like comparison with the Nautical Almanac, is 8′ 18½″.

Since the observations of the above eclipse were, the beginning at one

one place and the end at the other, the longitude deduced is, therefore, affected by the sum of the errors, arising from the error of the Moon's latitude If both observations had been the beginning, or the end, the resulting longitude would be affected by the difference of these errors, and consequently would be nearer the truth.

An observer at land, provided with proper instruments, besides observing the beginning and end of the eclipse, takes as many measures as possible of the versed sines of the lucid part of the Sun's disc, or of the distance between the cusps. Hence, by the method of interpolation, he can find the time of the middle of the eclipse, and the nearest approach of the centers of the Sun and Moon; from whence the error of the Moon's latitude, and the longitude of the place of observation, are determined.

CHAP. VI.

The Method of finding the Longitude of a Place

BY

An Occultation of a fixed Star by the Moon.

Introduction.

Of the method of applying observations of occultations of the fixed stars by the Moon, which is practised by astronomers, Dr. Halley observes, in the Philosophical Transactions, No. 354, as follows: " Of all the methods hitherto proposed for finding the longitude of places for geographical uses, none seems more adapted to the purpose than that by the occultations of the fixed stars by the Moon, observed in distant parts. For those *Immersions* of the stars, which happen on the dark semicircle of the Moon, and their *Emersions* from the same, are perfectly momentaneous, without that ambiguity to which the observations of the eclipses of the Moon, and those of Jupiter's satellites, are subject." By observations of the occultation of the planet Mars, 21st August, 1676, Dr. Halley determined the longitude

longitude of Oxford and Dantzic; and the longitudes of many other places have since been determined with great accuracy.

The observations necessary for finding the longitude of a place by this method are, the instant when the Moon's eastern limb covers a star called the *Immersion*, and that of the re appearance of the star from behind the Moon's west limb, called the *Emersion*.

This method of ascertaining the longitude is given in the Memoires de l'Académie Royale des Sciences, pour l'année 1705, by M. Cassini, page 255, printed at Amsterdam in the year 1707, and has since been employed by astronomers for the same purpose

The immersion of a star is easily observed, as the sight may be directed to the star till that observation happens. In an emersion, the observer should direct his sight to that part of the Moon's limb, from which the star is expected to emerge, some minutes before the supposed time of emersion, and continue looking till the star appears. This time may be found nearly, by adding to the observed time of immersion, the time required by the Moon to pass over a space equal to the estimated chord of the segment cut off by the star.

In the calculations, the altitude and longitude of the nonagesimal, and the parallaxes in latitude and longitude, are necessary, as in a solar eclipse. These are, therefore, calculated by Prob. I. and II. of last chapter.

Problem I.

To find the Apparent Time at Greenwich, of the Ecliptic Conjunction of the Moon and a fixed Star.

Rule.

Reduce the mean longitude of the star, to its apparent longitude by the rule given for that purpose in Vol. II.

From the Nautical Almanac, page V. of the month, take the longitudes of the Moon, immediately preceding and following that of the star; find also the mean second difference of the Moon's place.

Now to the ar-co. of the log of the change of the Moon's longitude in XII. hours, add the logarithm of the difference between the star's longitude and the preceding longitude of the Moon, and the constant log. 2.557332; the sum will be the log. of the approximate time in minutes * to be reckoned from the preceding noon or midnight.

* When the proportional part is less than 3 hours, the computation may be facilitated by using P. logarithms. In this case, the constant log. is 1.1761; and the degrees and minutes in the change of the Moon's longitude are to be esteemed minutes and seconds.

Take the equation of second difference from Table XXXVII. answering to the approximate time, and the mean second difference, with which enter Table XXXVIII. at the top, and the change of the Moon's longitude in twelve hours in the side column, and take out the corresponding equation; which being added to the approximate time, if the Moon's motion in twelve hours is increasing, or subtracted if decreasing, will give the apparent time at Greenwich of the ecliptic conjunction of the Moon and star.

Example.

Required the apparent time of the ecliptic conjunction of the Moon and η ♊, November 26, 1787?

Mean long η ♊ at giv. t.	3^s	$0°$	$28'$	$45''$						
Equat. of equinoxes +				18						
Aberration - - +				18						
Apparent longitude	3	0	29	21	-				Constant log. -	1.1761
Moon's long. at midn.	3	0	13	39	Diff.	0	15	42	P. L.	1.0594
- - - at noon	3	7	43	20	Diff.	7	29	41	ar-co. P. L.	8.6195
Proportional part - - -						0	25	8	P. L.	0.8550
Preceding time - - - -						12				
Approximate time - - - -						12	25	8		
The mean 2d diff. is $1'\ 57''$; hence, in Tab. XXXVII. the equat. of 2d diff. is $1''.9$, to which, and $7°30'$ in T. XXXVIII. the correction is - - - - -						—		3		
Apparent time of conjunction - -						12	25	5		

Problem II.

To find the Longitude of a Place by an Occultation of a fixed Star by the Moon, when both the Immersion and Emersion are observed.

Rule.

Reduce the apparent times of observation to the meridian of Greenwich, by applying thereto the estimated longitude. Find the Moon's horizontal parallax and semidiameter, and, hence, the reduced parallax and semidiameter as before.

Find the apparent longitude of the star; from thence, and the Moon's longitude, compute the apparent time at Greenwich of the ecliptic conjunction of the Moon and star, by the last problem. Diminish the latitude of the place of observation by the quantity from Table XXXVI.

With

With the reduced latitude, and the right ascension of the meridian, compute the altitude and longitude of the nonagesimal at the instants of observation; and find the difference between the longitudes of the nonagesimal and observed star, which will, therefore, be the apparent differences of longitude between the nonagesimal and the Moon's observed limb. Compute the parallaxes of the Moon in latitude and longitude by Prob. II. page 236, using the apparent latitude of the star in the computation.

Find the Moon's motion in latitude and longitude during the observed interval, and from the change of longitude subtract the difference of the parallaxes in longitude, if the Moon is either east or west of the nonagesimal at both observations; but if east of the nonagesimal at the time of immersion, and west at that of emersion, the sum of the parallaxes in longitude is to be subtracted from the change of the Moon's longitude in the above interval. Hence, the apparent difference of longitude will be obtained.

If the Moon is approaching the elevated pole of the ecliptic, find the difference between the parallax in latitude at the emersion, and the sum of the change of the Moon's latitude in the observed interval and the parallax at the immersion. But if the Moon is receding from the elevated pole, let the difference between the parallax in latitude at the immersion, and the sum of the change of the Moon s latitude and the parallax in latitude at the emersion, be found Hence, the apparent change of the Moon's latitude in the observed interval will be known.

From the P. Log. of the apparent difference of longitude, its index being increased by 10, subtract the P. Log. of the apparent difference of latitude; the remainder will be the log. tangent of the apparent inclination; to the log co-sine of which, the P. Log. of the apparent difference of longitude being added, the sum will be the P. Log. of the apparent motion of the Moon in its relative orbit.

From the P. Log. of the Moon's apparent motion in its proper orbit, subtract the P. Log. of the Moon's augmented diameter; the remainder will be the log. secant of the central angle *. Now, if the Moon be approaching the elevated pole of the ecliptic, the sum of the apparent inclination, and the central angle, will be *arch first*, and their difference *arch second*; the star being between the Moon and the elevated pole of the ecliptic; but, when the Moon is between the star and the elevated pole of the ecliptic, the difference of these angles is *arch first*; and their sum *arch second*. If the Moon is receding from the elevated pole of the ecliptic, the sum of the apparent inclination and central angle is *arch first*, and their difference *arch second*; the Moon being between the star and elevated pole of the ecliptic; otherwise, their difference is *arch first*, and their sum *arch second*.

Now, to the log. secant of arch first, add the P. Log. of the Moon's

* In strictness, this angle ought to be computed both at the immersion and emersion, using the Moon's apparent diameter at these times. This degree of precision is, however, almost unnecessary.

 semi-

semidiameter at the immersion; the sum will be the P. Log. of arch third; and to the log. secant of arch second, add the P. Log. of the Moon's semidiameter at the emersion, the sum is the P. Log. of arch fourth.

Then, if the Moon is east of the nonagesimal, to arch third add the parallax in longitude at the immersion, and from arch fourth subtract the parallax in longitude at the emersion; but if the Moon is west of the nonagesimal, use the contrary rules. Now, to the P. Logs. of these quantities, add the ar-co. of the P. Log of the Moon's motion in longitude in three hours; the sums will be the intervals of time between the ecliptic conjunction, and the immersion and emersion respectively. Hence, the apparent time of conjunction of the Moon and star will be known; the difference between which, and that inferred from computation, by Prob. 1. page 249, will be the longitude of the place in time; which is east or west, according as the time of conjunction at the place of observation is later or earlier than that at Greenwich.

Example.

☽ November 26, 1787, in latitude 57° 9′ N. longitude by account 8′ 32″ W in time, the immersion of η ♊ was observed at 11h. 18′ 8″ apparent time, and the emersion at 12h. 23′ 12″. Required the true longitude of the place of observation?

App time of immers.	11h	18′	8″	App. time of emers.	12h	23′	12″
Long. by estimation		8	32	- - - -		8	32
Reduced time -	11	26	40	- - -	12	31	44
☽'s hor. par. at imm.		61	11	- at emersion -		61	10
Reduction - -		—	11	- - - - -		—	11
Reduced parallax -		61	0	- - - - -		60	59
☽'s semidiam. at immers.		16	40	- at emersion -		16	40
Augmentation - -		+	13	- - - -		+	14
Inflexion - - - - -		—	3½	- - - -		—	3½
Apparent semidiameter		16	49½	- - -		16	50½

The apparent time at Greenwich, of the ecliptic conjunction of the ☽ and η is found by Prob. 1. to be 12h 25′ 5″.

App. time of immer.	11h	18′	8″	Latitude - -	57°	9′
☉'s R. A. at 11h. 26′ 44″	16	10	57	Reduction - - -	—	14
				Reduced latitude -	56	55
Right ascen. meridian	3	29	5			
	6					
Arch first - -	9	29	5			

Arch

Arch first - -	9h 29′ 5″		
Supplement - -	2h 30′ 55	- - Rising - -	4.32033
Reduced latitude -	56 55	- - Co-sine - -	9.73708
Obliquity ecliptic -	23 28	- - Sine - -	9.60012
Difference - -	33 27 nat. co-v. sine 44879		
	4545		3.65753
Alt. nonagesimal -	53 22 nat. ver. sine 40334	- Sine -	9.90443
Supplt. arch first -	2 30 55	- - H. E. T. - -	0.21330
Reduced latitude -	56 55	- - Secant - - -	0.26292
Long. nonagesimal -	65 24½	- - Secant - - -	0.33065
Longitude ♊ - -	90 29		
App. dif. longitude	25 5½	- - Co-secant -	0.3728
Alt. nonagesimal -	53 22	- - Co-secant -	0.09 6
Apparent latitude ✱	54 48″	- - Co-sine -	9.9[illegible]99
Reduced parallax ☽ -	61 0	- - P. L. - -	0.4699
Parallax in long. -	20 45	- - P. L. - -	0.9382
Apparent latitude ✱	54′ 48″ Secant 0.0001	- Co-secant -	1 7975
Alt. nonagesimal	53 22 Secant 0.2242	- Co-secant -	0.0956
Red. parallax of ☽	61 0 P. L. 0.4699	- - - - - -	0.4699
		D. lon. 25° 5½′ sec.	0.04[illegible]0
Part first par. in lat.	36 24 P. L. 0.6942		
Part second - -	+ 42′	- - - Prop. Log. -	2.4060
Parallax in lat. -	37 6½		
App. time emer.	12 23 12		
Sun's R. A. at 12h 31′ 48″	16 11 9		
R. A. of meridian	4 34 21		
	6		
Arch - - -	10 34 21		
Supplement -	1 25 39	- - Rising - -	3.83900
Reduced latitude	56 55	- - - Co-sine - -	9.73708
Obliquity ecliptic	23 28	- - - Sine - -	9.60012
Difference -	33 27 nat. co-ver. sine 44879		
	1500	- -	3.17620
Alt. nonagesimal	55 31 nat. co-ver. sine 43379		

Alt.

Alt. nonagesimal 55° 31′ - - Sine - - 9.91608
Supplt. arch 1st 1 25 39 - - H. E. T. - - 0.43761
Reduced latitude 56 55 - - Secant - - 0.26292

Long. nonages. 76 0½ - - Secant - - 0.61661
Long. η ♊ - 90 29

Difference long. 14 28½ - - Co-secant - 0.6021
Alt. nonages. - 55 31 - - Co-secant - 0.0839
Latitude η ♊ - 54 48 - - Co-sine - - 9.9999
☽'s reduced parall. 60 59 - - Prop. Log. - 0.4700

Parall. in longitude 12 34 - - Prop. Log. - 1.1559
App. latitude η ♊ 0° 54′ 48″ Secant 0.0001 - Co-secant - 1.7975
Alt. nonagesimal 55 31 Secant 0.2471 - Co-secant - 0.0839
☽'s reduced parallax 60 59 P. L. 0.4700 - - - - - - 0.4700

Part first par. in lat. 34 31 P. L. 0.7172 D.lon. 14° 28′ sec. 0.0140

Part second - - + 46½ - - - - - Prop. Log. - 0.3654

Parallax in latitude 35 17½

☽'s mot. in long. in 111. h. correct. } 1° 52′ 41″ P. L. 2034 Par. in long. at imm. 20′ 45″
Observed interval 1 5 4 P. L. 4419 - at emer. - - 12 34

True diff. longitude 40 44 P. L. 6453 Difference - 8 11
Difference of parall. 8 11

App. diff. of long. - 32 33
☽'s mot. in latit. in 111. h. correct. } 0° 10′ 22″ P. L. 1.2396 Par. in lat. at imm. 37′ 6¼″
Observed interval 1 5 4 P. L. 0.4419 - at emer. - 35 17½

True differ. of latitude 3 45 P. L. 1.6815 Difference - 1 49
Difference of parallax 1 49

Apparent diff. latitude 1 56 P. L. 1.9689
Apparent diff. longit. 32 33 P. L. 10.7427 - - - 0.7427

Appar. inclination 3° 24′ tang. 8.7738 - Co-sine - 0.9992

☽'s mot. in rel. orbit 32 37 - - - - P. L. - - - - 0.7419
☽'s diameter at imm. 33 39 - - - - P. L. - - - - 0.7283

Central angle - 14 16 - - - Secant - - - 1.0136
Apparent inclinat. 3 24

Arch first - 10 52

Arch

Arch first 10° 52′	- - - -	Secant - - -	0.0079
☽'s semid. at immer.	16 49½ - -	P. L. - - -	1.0293
Arch third - -	16 31 - -	P. L. - - -	1.0372
Par. in long. at imm.	20 45		
Sum - -	37 16 - -	P. L. - - -	0.6839
☽'s mot. in long. in 111 hours.	1 52 41 - -	Ar-co. P. L. -	9.7966
Int. bet imm. & ☌	59 32 - - -	P. L. - - -	0.4805
Ap. time of imm.	11 18 8		
App. tim. of con.	12 17 40*		
Ap. t. of ☌ at Gr.	12 25 5		
Longitude in time	7 25 W.		

Since the time of conjunction, deduced from observation, is compared with that found by last problem; the longitude of the place is, therefore, affected by the errors arising from those in the longitudes of the Moon and star. If the time of conjunction at Greenwich had been also inferred from observation, the longitude of the place would have been accurately determined. Thus the apparent time of immersion, as observed at Greenwich, was 11h. 22′ 51″.7; and that of the emersion, 12h. 31′ 45″; with these, the apparent time of conjunction at Greenwich will be found to be 12h. 26 6′. 5: also the preceding operations being performed more accurately, by using common logarithms in place of proportional logarithms, and taking out the several terms to the nearest tenth of a second, the apparent time of conjunction at Aberdeen will be 12h. 17′ 35′.2: hence, the difference of longitude, in time, is 8′ 31″.3.

In the supposition of the figure of the Earth being a perfect sphere, the longitude in time will be 8′ 32″.2. See the fourth vol. of the Transactions of the Royal Society at Edinburgh, page 155.

In may be necessary to remark that, since the apparent place of the Moon's center, at the time of observation at Greenwich, was between the star and the elevated pole of the ecliptic, and the Moon receding from that pole, the sum, therefore, of the central angle and the apparent inclination is arch first.

It often happens, that the immersion, or the emersion only, is observed. In this case, the longitude may be found by the following problem, though not with equal accuracy, as it will now be affected by the errors arising from the error in the latitudes of the Moon and star, as well as from those in their longitudes.

* The apparent time of conjunction may be also inferred from the emersion, by using arch second and the Moon's semidiameter at that time.

PROBLEM

Problem III.

To find the Longitude of a Place, by an Occultation of a fixed Star by the Moon, when the Immersion or Emersion only is observed.

Rule.

Compute the apparent time at Greenwich of the conjunction of the Moon and star, by Prob. 1. page 249. Find the reduced latitude and parallax at the reduced time of observation, with which compute the altitude and longitude of the nonagesimal, and the parallaxes in latitude and longitude as usual. Hence, the Moon's apparent latitude at that time will be known.

Find the difference between the apparent latitudes of the Moon and star, at the instant of observation; to the P. Log. of the sum of which, and the Moon's semidiameter, add the P. Log. of their difference; half the sum will be the Prop. Log of arch first.

Now, if the Moon is east of the nonagesimal, and the observation that of an immersion, the sum of the above arch and the parallax in longitude is to be taken; but if an emersion, take their difference. If the Moon is west of the nonagesimal, use the contrary rules. Then, to the P. Log. of this sum or difference add the constant log 0.4771, and the ar co of the P. Log. of the Moon's horary motion in longitude; the sum will be the P. Log. of arch second Now, in case of an immersion, and the Moon east of the nonagesimal, arch second is to be added to the apparent time; but if the Moon is west of the nonagesimal, the above arch is to be applied to the apparent time of observation, by addition or subtraction, according as arch first is greater or less than the parallax in longitude. If an emersion be observed, and the Moon at that time east of the nonagesimal, arch second is to be subtracted from the apparent time of emersion; but if the Moon is west of the nonagesimal, arch second is additive or subtractive, according as the parallax in longitude is less or greater than arch first. Hence, the apparent time of the conjunction of the Moon and star will be obtained; the difference between which, and the computed time at Greenwich, will be the longitude in time, of the place of observation, as before.

Example.

♄ October 18, 1788, in latitude 57° 9′ N. longitude by account 8° 32′ W. in time, at 11h 47′ 17″ apparent time, the emersion of ι ♉ was observed. Required the correct longitude?

Mean

Mean lon. ♉ 2ˢ 13° 50′ 4″		Mean latitude -	1° 13′ 23″ S.
Eq. equinoxes + 17		Aberration -	0
Aberration + 13		Apparent latitude	1 13 23 S.
App. longitude 2 13 50 34			
☽'s lon at mid. 2 13 48 56	diff. 0° 1′ 38″ - -	P. L. -	2.0422
- - at noon 2 20 41 30	diff. 6 52 34÷4=1° 43′ 8½	P.L.	0.2418
Apparent time of conjunction	12h 2′ 51″ - -	P. L.	1.8004

App. time emer. 11h 47′ 17″	☽'s hor. par. 58′ 29″	Semidia.	15′ 56″
Long. in time 8 32	Reduction - — 11	Augm.	+ 10
		Inflex.	— 3
Reduced time 11 55 49	Red. parall. 58 18	Red. sem.	16 3

App. time emer. 11 47 17
⊙'s right ascens. 13 37 28

1 24 45
6

Arch first - 7 24 45	- -	Rising -	5.13399
Reduced latitude 56 55	- -	Co-sine -	9.73708
Obliq. ecliptic 23 28	- -	Sine - -	9.60012
Sum - - 80 23	Nat. co-ver. sine 01405		
	29593	- -	4.47119
Alt. nonagesimal 46 22	Nat. versed sine 30998	- Sine -	9.85960
Supplt. arch first 4 35 15	- -	H. E. T. - - -	0.03040
Reduced lat. - 56 55	- -	Secant - -	0.26292
Longitude nona. 45 19	- -	Secant - -	0.1529
Longitude of star 73 50½			
App. diff. long. 28 31½	- -	Co-secant - -	0.3210
Alt. nonagesim. 46 22	- -	Co-secant - -	0.1404
Latitude of star 1 13 23	- -	Co-sine - -	9.9999
Reduced parallax 58 18	- -	P. L. - -	0.4896
Parallax in long. 20 9	- -	P. L. - -	0.9509
Latitude of star 1° 13′ 23″	Secant 0.0001	- Co-sec. -	1.6707
Alt. nonagesim. 46 22	— Secant 0.1611	- Co-sec. -	0.1404
Reduced paral. 58 18	P. L. 0.4896		0.4896
Part first par. in lat. 40 13	P. L. 0.6508	D. long. 28° 31½ } Secant	0.0562
Part second - - 47	- -	P. L. - -	2.3569
Parallax in lat. - 41 0½			

Parallax in lat. -	41′ 01″			
Moon's true lat. -	28 47½			
Moon's app. lat.	1 9 48			
Star's latitude -	1 13 23			
Difference -	3 35			
Moon's semidiam.	16 3			
Sum - - - -	19 38	- -	P. L. -	0.9623
Difference - - -	12 28	- -	P. L. -	1.1595
				2.1218
Arch first - - -	15 39	- -	P. L.	1.0609
Parallax in long.	20 9			
Difference - - -	4 30	- -	P. L. -	1.6021
Constant log. - -		- - - - -	-	0.4771
☽'s hor. mot. in lon.	34 17	- -	Ar-co-P. L. - -	9.2798
Interval - - -	7 52	- - -	P. L. - -	1.3590
Ap. time emer.	11 47 17			
App. time conj.	11 55 9			
App. time at Gr.	12 2 51			
Long. in time	7 42 W.			

If the immersion and emersion of the same star be observed at another place, the error of the Moon in longitude and latitude may be found. Hence, the true latitude of the Moon at the instant of observation will be known; and hence, the apparent time of conjunction of the Moon and star, at the given place of observation, may be more accurately determined. If the other place of observation is Greenwich, the apparent time of conjunction, at that place is obtained by calculation: if not, the error of the Moon in longitude, being reduced to time, and applied to the apparent time of conjunction at Greenwich, inferred from a comparison of the longitudes of the Moon and star, will give the correct time of conjunction; and hence, the required longitude is found as before.

This occultation was observed at Greenwich; the apparent time of immersion, according to Dr. Maskelyne, being 10h. 48′ 36″, and that of emersion, 11h. 45′ 11″. From hence, the apparent time of conjunction may be correctly inferred; and, of course, the longitude more accurately ascertained than as above.

CHAP.

CHAP. VII.

The Method of finding the Longitude of a Place

BY

The Eclipses of the Satellites of Jupiter.

INTRODUCTION.

IN November 1609, Simon Marius, mathematician to the elector of Brandenburgh, discovered three of the satellites of Jupiter, and in January 1610, he observed the fourth satellite. January 8th, 1610, Galileo observed three of the satellites, two upon one side of Jupiter, and one on the other, which he supposed to be telescopic stars: the following evening, however, he found the satellites had changed their places with respect to each other, and they appeared to him to move along with Jupiter; hence, he inferred they were moons belonging to that planet. Soon after he observed the fourth satellite. In the beginning of March following, he published an account of the satellites, which he called *Astres de Medicis* in his *Nuncius de Syderius*, The satellites were also observed in January 1610, by Thomas Harriot, an English mathematician. Galileo, however, appears to be the first who applied them to the discovery of the longitude; and for this purpose dedicated a considerable part of his life in making observations to form a *theory* of their motions. This last discovery is, however, claimed by P. Herigone, in the fifth volume of his *Cursus Mathematici*, printed at Paris in 1644.

This method of finding the longitude is mentioned in Harrison's *Idea Longitudinis*, printed at London in 1696, page 71, as follows: "As by *lunar eclipses*, so also by the eclipses of Jupiter's satellites, the difference of the observed moments of the occultation or emersion of a satellite from his shadow, noted carefully in two distant places, will be the difference of meridians betwixt these two places in time." Also by Dr. Maskelyne, in his *Instructions relative to the Transit of Venus* in 1769, page 41; by M. Bezout, in his *Cours des Mathématiques*, vol. vi, page 247, &c.; and it is very strongly recommended by Mr. Emerson, in his *Astronomy*, page 367. "Of all methods," says he, "the eclipses of the satellites of Jupiter are the most proper means for finding the longitude, for that way requires no trouble, no intricate

 calculations.

calculations. The whole business depends on this, to have a true theory of the satellites, at least of the first of them."

Of this method of finding the longitude, the Abbé Rochon, page 40, of his *Opuscules Mathématiques*, says, "Les observations des éclipses des satellites de Jupiter fournissent la méthode la plus commode et la plus universelle de determiner les longitudes sur terre. C'est par leur moyen, qu'on a fixé la position de la plupart des lieux dont on connait le mieux la longitude; d'ailleurs, ces sortes d'observations n'exigent aucune espece de connaissance, il suffit seulement d'avoir une lunette qui amplifié environ 40 fois les objets en diametre, et d'avoir l'heure pour le lieu où l'on observe.

"Comme les lunettes ordinaires qui amplifient 40 fois, doivent avoir au moins 9 à 10 pieds de longueur, et que plus une lunette grossit, moins elle a de champ, il n'a pas été possible jusqu' à présent de faire sur mer ces sortes d'observations, la longueur des lunettes et la petitesse du champ ayant été des obstacles insurmontables à cause de l'extrême embarras et de la prise qu' a le vent sur des longues lunettes, et plus encore l'agitation perpétuelle du vaisseau, qui fait perdre continuellement l'objet de vue, sans qu'on puisse le retrouver promptement*."

The greatest inconvenience attending the practice of this method of finding the longitude of a ship at sea, is, that, in consequence of the motion of the ship, the eclipses of the satellites cannot be observed with telescopes of a sufficient magnifying power. In order, however, to remove this desideratum, various contrivances have been proposed; such as the Marine Chair of Mr. Irvine, which Dr. Maskelyne says he made a full trial of, in his voyage to Barbadoes, in 1763, but found it totally impracticable to derive any advantage from the use of it. The same method, has, since, been proposed by M. Kratzenstein †, by M. Fyot ‡, &c.

The observation necessary to find the longitude of a place by this method is, the apparent time of an immersion or emersion of either of the satellites, but particularly that of the first, upon account of its periodic time being shorter than that of either of the other satellites;

* The observations of the eclipses of Jupiter's satellites furnish the most convenient and universal method of determining the longitude on land. By their means, the position of most of the places whose longitudes are best known has been ascertained; besides, as these kinds of observations do not require any sort of knowledge, it is sufficient only to have a glass which magnifies objects about 40 times in diameter, and to have the hour at the place where the observation is made.

As the common glasses, which magnify forty times, ought to be at least 9 or 10 feet long, and the more a glass magnifies, the less is its field of view, it has not been possible hitherto to make upon sea those kinds of observations, the length of the glasses, and the smallness of the field of view, having been insurmountable obstacles, on account of the extreme embarrassment and power which the wind has upon long glasses, and still more the perpetual agitation of the vessel, which makes us lose continually the object of sight, without being able to find it again.

† Acta Regiæ Universitatis Hafniensis, Anno 1778.

‡ Nouvelle Theorie Astronomique, pour servir à la Determination des Longitudes, printed in 1788.—An idea of this machine is given in the *Cosmolabe des Jaques Besson*, printed at Paris in 1567.

and

and because its elliptic equation, and other inequalities, except that whose period is 437 days, are insensible. The instruments proper for making the above observation are, a clock or watch beating seconds, an achromatic telescope of three or four feet focal length, and a quadrant, or some other instrument, proper for determining the error of the watch.

An *immersion of a satellite* is, the instant of its entrance into the shadow of Jupiter; and an *emersion* is, that of its re-appearance out of the shadow.

When the time of Jupiter's passage over the meridian, as expressed in the Nautical Almanac, is less than XII. hours, the eclipses of the satellites happen on the west side of the planet; and when that time exceeds XII. hours, these observations happen on the west side of Jupiter. In the first case, the emersions of the first and second satellites are visible; and in the latter, the immersions only may be observed. Both the immersion and emersion of the other two satellites are generally visible, except when Jupiter is too near the opposition or conjunction.

In order to observe an immersion or emersion of any of the satellites, the observer must apply to his telescope a magnifying power suitable to the state of the weather; and adjust it to a distinct vision, previous to the time of observation. The watch must be contiguous to him, so that he may be able to catch the second, as soon as the observation of the satellite is completed. An assistant clock will, however, be found more convenient. The observer is then to place himself at the telescope, about three or four minutes before the expected time of an eclipse of the first satellite; twice as much before that of the second or third satellites; and a quarter of an hour before that of the fourth, the longitude of the place of observation being pretty well known. If its situation is uncertain, let four times as many minutes as there are degrees of uncertainty, be added to the above times. If the observation to be made is that of an immersion, the observer must keep his eye directed to the satellite till it disappears; but if it is an emersion, he must keep his sight directed to that part of the shadow from which he expects the satellite to emerge; and in the instant of the last appearance of the satellite, if the observation is that of an immersion, or that of its first appearance, if an emersion, must be wrote down, and the observation is completed.

If an astronomical eye-glass is used, it must be remembered, that it inverts objects, making that which is the right side apparently the left, *et contra*.

The third satellite is considerably larger than any of the rest; the first is a little larger than the second, and nearly of the size of the fourth; and the second is a little smaller than the first and fourth, or the smallest of them all.

The eclipses of Jupiter's satellites are given in the Nautical Almanac, in page III. of the month. Those visible at Greenwich are marked with

with an asterisk. The configurations, or apparent relative positions of the satellites, are contained in the last or XIIth page of the month.

PROBLEM I.

To find if an Eclipse of one of Jupiter's Satellites will be visible at a given Place.

RULE.

To the apparent time of the eclipse at Greenwich, as given in the Nautical Almanac, apply the estimate longitude of a given place, by addition or subtraction, according as it is east or west; hence, the apparent time, nearly, by the meridian of the place, will be known.

Now if, at the reduced time, the altitude of Jupiter exceeds six or eight degrees, and adjacent stars of the third magnitude visible to the naked eye, the eclipse may be observed; but if the Sun is above the horizon, it will be invisible.

EXAMPLES.

I.

Will the immersion of the first satellite of Jupiter, of January 19, 1812, be visible at Easter Island?

Apparent time of immersion at Greenwich -	12h 27′ 56″
Longitude of Easter Island in time - -	7 19 7 W.
Estimate time of observation - - - -	5 8 49

This eclipse will not be visible, because at the reduced time the Sun is above the horizon.

II.

Will the emersion of the second satellite of Jupiter of 11th May, 1804, be visible at Achem, in latitude 5° 22′ N. and longitude 95° 34′ E.

Apparent time of emersion at Greenwich -	5h 25′ 9″
Longitude in Achem in time - - - - -	6 22 16
Apparent time of emersion at Achem - - - -	11 47 25

The Sun is considerably under the horizon at the above time; this eclipse will therefore be visible, provided the altitude of Jupiter exceeds 7 or 8 degrees. Now, to the latitude of the place 5° 22′ N. and Jupiter's declination 9° 36′ S. the semidiurnal arch is about 5h. 56′; the time of Jupiter's transit over the meridian of the place is 10h. 54′; hence the time of the rising of Jupiter is 4h. 58′, and that of its setting

setting 16h. 50′. The altitude of the planet is, therefore, sufficient for the above observation.

Problem II.

To find the Longitude of a Place, from an Observation of an Immersion, or Emersion of one of the Satellites of Jupiter.

Rule.

To the observed time of the eclipse, apply the error of the watch for apparent time, deduced from an observation of the altitude of the Sun, or a fixed star, taken as near the time of the eclipse as possible. Hence, the apparent time of observation will be known. The difference between this time, and that given in the Nautical Almanac, page III. of the month, will be the longitude of the place in time; being east or west, according as the time by observation is later or earlier than the Greenwich time.

Examples.

I.

December 9, 1803, an immersion of the first satellite of Jupiter was observed at 16h. 58′ 35″ per watch, which was 3′ 58″ slow for apparent time. Required the longitude of the place of observation?

Time per watch of observation - -	16h	58′	35″	
Watch slow - - - -		3	58	
Apparent time of observation - -	17	2	33	
App. time at Greenwich, per N. Alm.	17	52	53	
Longitude in time - - -		50	20	= 12° 35

Which is west, because the time at the place of observation is earlier than the time at Greenwich.

II.

May 13, 1804, in latitude 31° 18′ N. the emersion of the first satellite of Jupiter was observed at 11h. 25′ 33″ per watch. In order to find the error of the watch, the following altitudes of Regulus were observed. Required the longitude of the place of observation?

Time

Time per watch	11h 28′ 20″	Altitude Regulus	19° 46′
	29 30	- - - - - -	19 31
	30 46	- - - - - -	19 17
	32 2	- - - - - -	19 0
	33 3	- - - - - -	18 46
	3 41	- - - - - -	80
Mean - -	11 30 44	- - - - - -	19 16
		Dip to 12 feet -	— 3.3
		Refraction - -	— 2.7
		Corrected altitude	19 10

Latitude - - - -	31° 18′ - - -	Secant - - -	0.06831
Declination of Regulus	12 55 - - -	Secant - - -	0.01113
Mer. zenith distance	18 23 Nat. ver. sine - 05103		
Altitude Regulus -	19 10 Nat. co-ver. sine 67168		
	Difference - 62065 log. -		4.79285
Meridian dist. Regulus	5h 0′ 58″ - -	Rising - -	4.87229
Right ascen. Regulus -	9 57 56		
Right ascen. meridian	14 58 54		
Sun's R.A. at noon -	3 20 10		
Approximate time -	11 38 44		
Eq. Tab. XVIII. to ☉'s R. A. & time of em. at Greenwich -	— 2 14		
Apparent time - -	11 36 30		
Time per watch - -	11 30 44		
Watch slow - - -	5 46		
Time per watch of em.	11 25 33		
Apparent time - -	11 31 19		
App. time at Greenw.	13 40 15		
Longitude in time -	2 8 56 = 32° 14′ W.		

III.

January 3, 1804, in latitude 39° 35′ S. the immersion of the first satellite of Jupiter was observed at 15h. 42′ 18″ per watch; and, previous thereto, the following altitudes of Canopus were observed, the height of the eye being 8 feet. Required the correct longitude?

Time

Time per watch	14h 56′ 50″	Altitude	53° 44′
	14 58 4	- - - - - -	33
	14 59 36	- - - - - -	19
	15 1 15	- - - - - -	.4
	35 45		100
Mean - - -	14 58 56	- - - - -	53 25
	Dip	- - - - - - -	— 2.7
	Refraction	- - - -	— .7
	Altitude corrected	-	53 21.6

True altitude - -	53° 21′.6		
Polar distance - - - -	37 24.4	- - Co-secant - -	0.21648
Latitude - - - - -	39 35	- - Secant - -	0.11312
Sum - - - - - -	130 21.0		
Half - - - - - -	65 10.5	- - Co-sine - -	9.62309
Difference - - - - -	11 48.9	- - Sine - -	9.31123
			19.26392
Arch - - - - - -	25 22¼	- - Sine - - -	9.63196
Multiply by - - -	8		

Mer. distance Canopus	3 22 58
Right ascen. Canopus	6 19 37
Right ascension merid.	9 42 35
☉'s R. A. at Green. at time of immer.	18 52 10
Apparent time -	14 50 25
Time per watch -	14 58 56
Watch fast - - -	8 21
Observed time of im.	15 42 18
App. time of immer.	15 33 57
App time p. N. Alm.	12 18 1

Longitude in time - 3 15 56 = 48° 59′ E.

Because the time at the place is later than the Greenwich time.

There are now added to the Nautical Almanac, by way of appendix, since the year 1800 inclusive, tables containing the eclipses of the satellites of Jupiter, expressed in mean time, calculated from the tables of M. de Lambre, inserted in the tables annexed to the first

volume of the third edition of M. de la Lande's Astronomy. And since these tables are far superior to any of the former, agreeing with the observations in a manner truly surprising *, it, therefore, becomes necessary to use them. For this purpose, the apparent time of observation must be reduced to mean time, by applying thereto the equation of time, from the Nautical Almanac, according to its title.

Thus, in the preceding example,

The apparent time of immersion was -	15h 33′ 57″
Equation of time - - - - -	+ 4 16
Mean time of immersion - - -	15 38 13
Mean time of imm. per M. de Lambre's tab.	12 22 36
Longitude in time - - - -	3 15 37 = 48° 54¼′ E.

When it is required to settle the longitude of a place, as accurately as possible, by this method, several immersions and emersions of the same satellites must be observed. The mean longitude, as given by the immersions, is to be found, and also that deduced from the emersions. The mean of these will be the longitude of the place, unaffected by the errors arising from the telescope; and is, therefore, more to be depended on, than that deduced from a single observation. If the comparison be made with corresponding observations taken at another place, the difference of the meridians of those places will be accurately found.

* M. de la Lande's Tables, page 230.

DETER-

Determination

Of the Longitude of the late Observatory at Aberdeen, by Observations of the Eclipses of the Satellites of Jupiter.

These observations were made with one of Dollond's four feet achromatic telescopes; and different magnifying powers were used according to the state of the atmosphere. The time per clock of observation was reduced to apparent time, and compared with the Nautical Almanac. Upon this account it is thought proper to subjoin only the results, as given by the observations of the first and second satellites.

First Satellite.

Year.	Month and Day.	Long. by Imm. ′ ″	Long. by Emer. ′ ″
1786	♂ Jan. 3		8 21
	♃ — 19		8 9
	☽ Sept. 18	8 36	
	☿ — 20	8 59	
	♃ Dec. 7		7 47
	♃ — 14		7 23
	♄ — 30		8 9
1787	☽ Jan. 8		8 32
	☽ — 15		8 25
	☽ — 22		8 11
	☿ — 31		8 28
	♀ Feb. 23		8 25
	♃ Oct. 25	9 13	
	☿ Dec. 26		8 53
	♀ — 28		8 23
1788	♀ Jan. 4		7 51
	☉ — 27		7 43
	☉ Dec. 14	9 30	
	☉ — 21	9 5	
1789	♃ Jan. 22		8 37
1791	♂ March 1	9 20	
	♄ April 9		8 45
Sum - -		43	242
Mean -		9 7.1	8 15 1
			9 7.1
Sum - - - -			- 17 22.2
Mean longitude			- 8 41.1

Second Satellite.

Year.	Month and Day.	Longitude by Imm. ′ ″	Longitude by Emer. ′ ″
1786	☽ Sept. 18	8 12	
	♂ Nov. 7	9 43	
	♄ Dec. 2		8 11
1787	☿ Jan. 3		9 29
	☉ Feb. 4		9 2
	♃ March 8		9 9
	♀ Dec. 21		7 36
	♀ Dec. 28		7 28
1788	♀ Jan. 4		8 25
	☉ May 11		8 52
1789	♃ Jan. 22		7 21
	☽ Dec. 9	8 49	
Sum - -		164	13
Mean - -		8 54.6	8 23.6
			8 54.6
			78.2
Mean longitude by Sec. Sat.			8 39.1
Mean by First Sat.			8 41.1
Long. of the Observatory			8 40.1

 CHAP.

CHAP. VIII.

The Method of finding the Longitude of a Ship at Sea,

BY

A Chronometer, or Time-Keeper.

Introduction.

WE are indebted to *Gemma Frisius* for the discovery of this celebrated method of finding the longitude of places. It is also mentioned by *Adrian Metius, Blundevil,* and others. This method is very particularly described in *Carpenter's Geography,* printed at Oxford in 1635, book I. page 242; and *Peter Martyr* prefers it to all other methods then known. The recommendations of this method are, simplicity of principle, and the shortness of the necessary calculations; and were it possible to obtain a watch, at a moderate price, that would keep an uniform rate in every different position and climate, it is more than probable, that no other method, for this purpose, would ever be practised at sea.

In the year 1662, Lord Kincardine, and Dr. Hook made several trials of a pendulum clock* at sea: the clock was loaded with lead; and suspended near the center of gravity of the ship, the pendulum vibrated half seconds; and these trials afforded a probability of success.

About the year 1664, Mr. *Huygens* applied himself very diligently to the improvement of time-keepers. In his paper on this subject in the Phil. Tran. No. 47, he recommends two watches, so that in case the one should meet with any accident, the other may be still safe. He then shows how carefully the watches should be kept; the manner of setting them to mean solar time under a given meridian, and of finding their daily rate; and the difference of longitude between the ship and the place to which the watches were regulated. Lastly, he gives a specimen of their utility in Major Holmes' Voyage to the island of St. Thomas.

* That the application of the pendulum to a clock, is the invention of Mr. Huygens, is very probable; but the balance spring of a watch was invented by Dr. Hook in the year 1657.

Ten

Ten years after the publication of the reward offered by the British Parliament for the discovery of the longitude at sea, namely, in 1724, Mr. Henry Sully, a celebrated English artist, having settled in France, applied himself to the improvement of watches for this express purpose. But he died at Bourdeaux while he was engaged in making experiments on his time-keeper. It appears the government of France was interested in the success of Sully, and enabled him to prosecute his labours. "I remember," says he, "with the highest gratitude, that I have already received distinguished marks of the royal favour; the protection of the first prince of the blood; the good-will of the prince who has the administration of affairs; and of the ministers of the court of France. These have also been seconded by the generous offices of many eminent persons of this court *."

The next who prosecuted this branch in France were M. Julien le Roy and Son, and M. Berthoud. And the first of these ingenious persons had made very considerable advances towards the improvement of time-keepers. An instrument constructed by him was placed in the Royal Observatory at Paris, of which M. Cassini de Thury gives the following account: "From the month of December, 1763, until July, 1764, it hath not varied one second upon the revolution of the fixed stars. During four months, and in the time of the greatest heat, when Reamur's thermometer † was as high as 30°, its greatest variation was never found more than one second, and it is doubtful whether or not that difference ought to be attributed to observation. And since the year 1748, it hath not differed more than three seconds."

M. le Roy, jun. engaged early in the improvement of watches. In the History of the Academy of Sciences for the year 1748, an account is given of a new escapement presented by him to that Academy; and on the 18th of December 1754, he presented a paper to the secretary of the Academy, containing *a description of a new time-keeper proper to be used at sea.*

About the middle of the year 1763, M. le Roy presented to the Academy a time-keeper three feet in length ‡, and the year following he made another time-keeper half the length of the former, which, by order of the Academy, was tried for near a year by M. le Monnier. In the year 1766, he presented a third to the King of France. This was suspended in a box a foot square, and eight inches high. M. le Monnier determined the rate of this instrument, by observations of the fixed stars, during the space of three months; from whom it was transferred to M. Cassini for farther examination; who placed it beside the instrument mentioned above, and at the same time observing, that *the son will be judged by his father.*

* Dissertation sur la Nature des Tentatives, pour la Decouverte de la Longitude sur Mer.

† In this thermometer the freezing point is at 0°, and the boiling-point at 80°

‡ Exposé succint des Travaux de M. M. Harrison et le Roy.

This

This machine gave so much satisfaction to these astronomers, particularly to M. Cassini, that he desired him not to give another for trial, as it could not be more perfect.

Agreeable to the order of the Academy of Sciences, M. le Roy embarked with two marine watches, on board the French frigate Aurora, at Havre de Grace, on the 20th of May, 1767, accompanied with the Marquis de Courtenvaut, M. Messier, and the Abbé Pingré, and on the 28th of the same month sailed for Calais, where they arrived next day. From thence they proceeded for Amsterdam, and in their way touched at Dunkirk, and Rotterdam, having experienced several very severe gales of wind. The ship sailed from Amsterdam 23d July, and arrived at Havre the 28th August. During the voyage, the watches were found to go very well, particularly the second, an accident having happened to the first; and to have given the longitudes of the several places they stopped at with great exactness: in a period of 46 days, including the time of their passage from Amsterdam to Havre, the first watch had gained 38″, being nearly equal to 3 leagues under the equator; and the second had varied only $7\frac{1}{4}''$of time, or about $\frac{2}{3}$ of a league.

M. Berthoud constructed time-keepers, two of which underwent a trial, of upwards of a year, on board the French frigate *Isis*: the commander of the frigate, *M. d'Eveux de Fleurieu*, and M. Pingré, of the Academy of Sciences, had the charge of these instruments. During this trial, one of the time-keepers gave the longitude within one third of a degree, in a period of 45 days; and in other periods of the same length, the longitude was obtained within a fourth and a sixth of a degree; and the error, in any of these periods, never exceeded half a degree. The other time-keeper, during the first six months of the trial, was found to go equally as well as the first. In consequence of the abilities of this artist, he was appointed watch-maker to the King, and to the Marine, with a pension of 3000 livres.

In England, Mr John Harrison applied himself with indefatigable industry to the improvement of time-keepers; and his success therein justly merited the attention of the public. In the year 1726, he made a pendulum clock, which kept time so exactly as not to err one second in a month during a period of ten years. In 1735, Mr. Harrison received a certificate, signed by Dr. *Halley*, Dr. *Smith*, Dr. *Bradley*, Mr. *Machin*, and Mr. *Graham*, stating, that the principles of his machine for measuring time promised a very great and sufficient degree of exactness. In the year 1736, he made trial of his first machine in a voyage to Lisbon, and found it to answer his expectation. In his return to England, he corrected the dead reckoning about a degree and a half. The following year, the Board of Longitude was pleased to give him a gratuity, and desired him to prosecute his discoveries. In 1739, Mr. Harrison, by order of the Commissioners of Longitude, completed a second machine more perfect than the first: and the result of the experiments made upon it was, that the motion of the machine

machine was sufficiently regular and exact for finding the longitude of a ship, within the nearest limits proposed by Parliament, and probably much nearer.

Being thus encouraged, Mr. Harrison engaged in a third, and more correct time-keeper, which was in such a state of forwardness, that in January 1741, twelve of the principal members of the Royal Society made the following report:—

"We, whose names are underwritten, are of opinion, that these machines, even in their present degree of exactness, will be of great and excellent use, as well for determining the longitude at sea, as for correcting the charts of the coasts. And as every step towards further exactness, and security, in a matter of such importance to the public, is greatly to be valued, we do recommend Mr. *Harrison* to the favour of the commissioners appointed by the act of Parliament, as a person highly deserving of such further encouragement and assistance, as they shall judge proper and sufficient, to finish his third machine."—This certificate was signed by Mr. *Folks*, the president of the Royal Society, Lord *Macclesfield*, Dr. *Smith*, Dr. *Bradley*, Prof. *Colson*, Mr. *George Graham*, Dr. *Halley*, Mr. *William Jones*, Dr. *Jurin*, Lord *Charles Cavendish*, Mr. *de Moivre*, and Mr. *John Hadley*, and delivered to the Commissioners of Longitude, 16th January, 1742.

In 1749, Mr. Harrison was honoured, by the Royal Society, with the gold medal, which is given annually for the most important or ingenious discoveries.

In 1758, Mr. Harrison finished his third time-piece, and at the same time he was employed in a fourth; but being fully persuaded that his third was sufficiently exact to entitle him to the highest reward, he applied to the Commissioners of Longitude, for orders to make a trial of that instrument, to some port in the West Indies, as directed by the act. Mr. Harrison accordingly received orders, dated 12th March 1761, for his son to proceed to Portsmouth, with his third instrument, and join the Dorsetshire, for Jamaica; but that ship being ordered on another service, Mr. Harrison jun. having remained at Portsmouth six months, returned to London.

Mr. Harrison having completed the examination of his chronometer, on the 3d of October, 1761, again requested the Commissioners of Longitude, that his son might be sent with it to Jamaica, in the same ship with Governor Lyttleton; and that every requisite precaution might be taken for making the necessary observations, &c. with accuracy, both in going out, and in the return home, so that there might be a sufficient proof of the exactness of the instrument for the purpose required by act of Parliament.

October 14, 1761, Mr. Harrison received the following instructions from the Admiralty:—

That

"That the watch be sent in charge of Mr. Harrison's son to Portsmouth; and that he proceed with it from thence in the Deptford, to Jamaica.

That in addition to the lock now upon the case of the watch, (and of which Mr. Harrison's son is to keep the key,) there be three other locks, of different wards, affixed to the case; the key of one of which should be in the possession of Governor Lyttleton, (who is going in the Deptford to Jamaica;) the key of another, in the possession of the captain of the Deptford; and the key of the third, to be kept by the first lieutenant of that ship.

That Mr. Robertson, master of the Royal Academy at Portsmouth, should be appointed to find the true time at that place, and to see the watch set to that time, and send exact information of the same to the lords of the Admiralty.

That observations of equal altitudes should be taken by the said Mr. Robertson; that the times of taking them should be marked down agreeable to the times shown by the watch; then sealed up, and sent to the lords of the Admiralty. All these matters to be done by Mr. Robertson, before commissioner Hughes, the captain of the Deptford, and Mr. Harrison's son, and they to attest the sealed accounts, which Mr. Robertson is to send to the lords of the Admiralty.

That Mr. John Robison, a person recommended by Admiral Knowles, as well skilled in mathematics, and particularly astronomy, be sent in the Deptford to Jamaica; who is to find the true time immediately upon the ship's arrival there, and note the same down before Governor Lyttleton, the captain, and first lieutenant of the Deptford, and Mr. Harrison's son, who are to attest the same; and also to note down the time given by the watch, immediately upon the ship's arrival, before the same persons, and to be attested by them; both which times so attested, are to be sealed up and sent to the lords of the Admiralty.

And Rear-admiral Holmes is to be directed, when the observations, made as aforesaid, to order Mr. Harrison, and Mr. Robison, a passage home in the first ship that sails for England, repeating the observations for finding the true time, &c. in the same manner, before the ship comes away." In consequence of these directions, the watch was sent by Mr. William Harrison to Portsmouth, and from thence, in the Deptford, to Jamaica.

Additional locks and keys were provided, and placed on the case in which the watch was kept, and the keys delivered to Governor Lyttleton, and to the captain and lieutenant of the Deptford.

Mr. Robertson, master of the Royal Academy at Portsmouth, ascertained the error and rate of the time-keeper. Its error on November 6, at noon, was 2′ 57″ fast; on the 7th, it was put back 3′; its error, therefore, reduced to the 6th, at noon, was 3″ slow, and

and it was found to have lost 24″ in 9 days, or 8″ in 3 days, on mean solar time.

These things were done in the presence of Commissioner Hughes, Captain Diggs of the Deptford, and Mr. Harrison's son; and they attested the sealed accounts, which Mr. Robertson sent to the Admiralty.

Mr. Harrison sailed from Portsmouth in the Deptford, November 18, 1761, and arrived at Madeira, December 9, following. During the course of this part of the voyage, the time-keeper was found of great utility, having corrected the longitude by account about a degree and a half. In the passage from Madeira to Jamaica, the longitude by account was found to differ from that given by the time-keeper above 3°; and the longitudes of the several islands, by which the ship passed, agreed with the same as given by the time-keeper. The Deptford arrived at Jamaica, January 19, 1762; and by equal altitudes, observed at Port Royal, the 26th of the same month, the time of mean noon, by the time-keeper, was found to be 4h. 59′ 7″ .5; to which 3″, its error, November 6, at noon, and rate 3′ 36″ .5, $\left(=\frac{8'' \times 81\text{d}.5\text{h}.}{3\text{d}.}\right)$ being applied, gives 5h. 2′ 47″, the mean time, at Portsmouth, of noon at Port Royal, and which is, therefore, the difference of longitude of Portsmouth and Port Royal in time, according to the time-keeper; but the difference of longitude in time between the same two places, deduced from the transit of Mercury observed at Kingston, and reduced thereto, is 5h. 2′ 51″; the difference of which from the former is only 4″ of time, which in the parallel of Jamaica is less than one nautical mile.

On January 28, 1762, Mr. Harrison sailed from Jamaica in the Merlin; and some time after met with a violent storm, which obliged him to remove the time-keeper to another place, where it was more exposed that before. The Merlin arrived at Portsmouth, March 26, and by equal altitudes, observed April 2, the time of mean noon was 11h. 51′ 31″ .5; to which its former error, 3″, and the accumulated rate during the interval, 6′ 32″ $\left(=\frac{8'' \times 147}{3} = 8'' \times 49\right)$ being applied, gives 11h. 58′ 6″ .5, the time of noon. Hence, from November 6, 1761, to April 2, 1762, during which period the time-keeper had passed through a variety of climates, and undergone violent agitations at sea, its error was no more than 1′ 53″ .5, or 28¼ minutes of longitude in time, which, in the parallel of Portsmouth, does not amount to 18 nautical miles.

Several objections were made to this trial; however, in consideration of the accuracy of the time-keeper in the above voyage, Mr. Harrison received 5000*l.* and was ordered to make a second trial, in a voyage to Barbadoes. On August 17, 1762, the Board of Longitude proposed to Mr. Harrison, to send his time-keeper to the Royal Observatory at Greenwich, in order that some observations might be

made at that place, for the trial of it, by Mr. Bliss, Astronomer Royal, previous to its being sent to the West Indies; he consented thereto, as soon as it shall have undergone some alterations, which, he thinks, will bring it to greater perfection, the doing of which would probably take up four or five months: and Mr. Bliss was desired to make the necessary observations, for the trial of the time-keeper, as soon as it should be sent to him. This, however, when proposed again by the Board, August 4, 1763, to Mr. Harrison jun. his father not being present, he said, that he did object to it, as he does not choose to part with it out of his own hands, till he shall have reaped some advantage from it. It was then proposed to Mr. Harrison, that he should send the rate of going of his time-keeper, immediately before he sails on his intended voyage, sealed up, to the secretary of the Admiralty, and abide thereby upon its trial; to which he consented. The following is a copy of Mr. Harrison's declaration, dated at Portsmouth, March 26, 1764.

"In obedience to your instructions, dated August 9, 1763, I humbly certify that I do expect the rate of going of the time-keeper will be as followeth, viz.

When the thermometer is at	42°, it will gain 3″ in every 24 hours.
- - -	52°, it will gain 2″ in every 24 hours.
- -	62°, it will gain 1″ in every 24 hours.
- - -	72°, it will neither gain nor lose.
- - -	82°, it will lose 1″ in every 24 hours.

Since my last voyage, we have made some improvement in the time-keeper; in consequence of which, the provision to counterbalance the effects of heat and cold has been made a-new; and for the want of a little more time, we could not get it quite adjusted, for which reason the above allowances are necessary. This is its present state; and as the inequalities are so small, I will abide by the rate of of its gaining, on a mean, one second a day for the voyage."

Upon the 13th of February 1764, the time-keeper was compared with Mr. Short's regulator, in Surry-street, whose error had been accurately ascertained with an excellent transit instrument; and Mr. Harrison jun. proceeded immediately on board his Majesty's ship Tartar, Sir John Lindsay commander, lying at Long Reach. The vessel sailed for Portsmouth, and remained there some time. During the stay at Portsmouth, the time-keeper was compared with the astronomical clock, set up in a temporary observatory, to which Mr. James Bradley had been appointed, for the above purpose; and also, to observe the eclipses of the satellites of Jupiter, particularly the first, to be compared with corresponding observations of the same satellite, to be made by Messrs. Maskelyne and Green, at Barbadoes, to ascertain the difference of longitude between these places; and which, upon a comparison of their observations, they found to be 3h. 54′ 20″ in time.

March 28, 1764, Mr. Harrison sailed from Portsmouth, in the Tartar, having previously ascertained the error of the time-keeper, from obser-

observations of equal altitudes of the Sun, taken at Portsmouth, between the 29th of February and the 26th of March. April 18th, at 4 P.M. Mr. Harrison, from observations of the Sun, found the mean time at the ship, which, compared with that by the time-keeper, allowance being made for its rate and error, gave the difference of longitude between the ship and Portsmouth; from whence he inferred, that the ship was 43 miles to the eastward of Porto Sancto: Sir John Lindsay then steered the direct course for it, and next morning, at one o'clock, saw the island, which exactly agreed with the distance mentioned above. Mr. Harrison arrived at Barbadoes the 13th of May; and upon the four following days, the time-keeper was compared with the clock at the Observatory near Bridgetown, and its error ascertained by equal altitudes of the Sun, observed by Messrs. Maskelyne and Green. The difference of longitude between Portsmouth and Barbadoes, as given by the time-keeper, was 3h. 55′ 3″; and that by astronomical observations being 3h. 54′ 20″, the error of the time-keeper was, therefore, only 43″ of time = $10\frac{3}{4}$′ of a degree. Mr. Harrison sailed from Barbadoes June 4th, in the New Elizabeth, Mr. Robert Manly commander. In the afternoon of July 18th, he was landed at Surry-stairs; and the time-keeper being compared with Mr. Short's regulator, whose error was that day determined by the Sun's transit, it was found to have gained only 54″ in 156 days, the interval of time between its leaving London and its return thither, allowing the time-keeper to have gained one second a day, being the rate by which Mr. Harrison declared he would abide, previous to his departure from Portsmouth. If, however, allowance be made for the variation of the thermometer, as stated by him before his departure, it will be found to have lost only 15″. In consequence of which, the Board of Longitude unanimously agreed, that Mr. Harrison's time-keeper had given the ship's longitude, in these trials, within the limits prescribed by act of Parliament; and, therefore, upon discovering the principles upon which his time-keeper was constructed, he was promised a part of the reward.

The above instrument*, together with its explanation, were delivered to the Board of Longitude, which being examined by a committee appointed for that purpose, the following report was made:—*That Mr. Harrison has taken his time-keeper to pieces in presence of us, and explained the principles and construction thereof, and every thing relative thereto, to our entire satisfaction; and that he also did, to our satisfaction, answer every question proposed by us, or any of us, relative thereto; and that we have compared the drawings of the same with the parts, and do find that they perfectly correspond.* He was then ordered 5000*l.*; and was promised the remainder of the reward, as soon as it was found that a proper time-keeper, constructed on the same principles by another person, was found to answer equally well.

* The author had the pleasure of seeing Harrison's machines, at the Royal Observatory, in the year 1799.

In the mean time, however, the Board of Longitude, at their meeting, April 26, 1766, resolved, that Mr. Harrison's watch should be tried at the Royal Observatory, under the inspection of Dr. Maskelyne. This instrument was accordingly delivered by Philip Stephens, Esq secretary to the Admiralty, to Dr. Maskelyne, 5th May, 1766; and from a comparison of its rate in six periods of six weeks each, terminating March 4, 1767, the watch being in an horizontal position, with its face upwards. he says, "that Mr. Harrison's watch cannot be depended upon to keep the longitude within a degree in a West India voyage of six weeks; nor to keep the longitude within half a degree for more than a fortnight, and then it must be kept in a place where the thermometer is always some degrees above freezing; that, in case the cold amounts to freezing, the watch cannot be depended upon to keep the longitude within half a degree for more than a few days; and perhaps not so long, if the cold be very intense; nevertheless, that it is a useful and valuable invention, and, in conjunction with the observations of the distance of the Moon from the Sun and fixed stars, may be of considerable advantage to navigation."

The Board of Longitude ordered Mr. Kendal. one of the committee, to construct a time-keeper upon the same principles This instrument was committed to the care of Mr. Wales, in his voyage round the world with Captain Cook, in the years 1772, 1773, &c. and its success was such, that in 1774, the House of Commons, to whom an appeal had been made, ordered Mr. Harrison the remaining part of the reward offered by Parliament. Indeed, from the East India Company and others, including also the parliamentary reward, he has received upwards of 24,000*l*.

Of late, several improvements have been made in chronometers; particularly that of a new balance spring, and an improved thermometer piece, or a compensation for the effects of heat and cold in the balance, by Mr. John Arnold, for which he claimed one of the new rewards offered for improvements in discovering the longitude. In 1780, he published an account of the going of a pocket chronometer, kept at the Royal Observatory, in which he observes, that its greatest error in the period of thirteen months, during which time it was under trial at the Royal Observatory, was 2′ 32″.2, which, in the Channel, would amount to about 25 geographical miles; and that, during the last eight months of trial, the extreme difference did not amount to one minute of time, or about 10 miles in the Channel. That during the whole period of thirteen months, there were only four days in which the daily difference amounted to three seconds per day; so that this machine, in its present improved state, is well adapted to show the effect of currents, and to determine with great exactness the run of a ship, from day to day. He concludes by observing, that the chronometer is better adapted for determining the relative longitude of places, by a comparison with the longitude of those which have been determined by a series of astronomical observations on shore, than any other method

method which can be practised at sea*. Mr. Arnold received a reward of 1300*l.* from the Board of Longitude. Upon December 26, 1805, Mr. Arnold junior received the sum of 1678*l.* which, together with the monies received by his late father, make the full sum of 3000*l.* voted by the Board of Longitude, as a reward for the excellence of the principle and the performance of his time-pieces.

Mr. Thomas Mudge has also obtained a part of the parliamentary reward for time-keepers, although they underwent no other trial than at the Royal Observatory. His first time-keeper was tried there between the 14th December 1774, and 13th March 1775, when it was found stopped, owing to the breaking of the main spring. The watch was taken away to be repaired, and brought back to the Royal Observatory November 11, 1776. At a Board held March 1, 1777, Dr. Maskelyne made a report of the going of the watch, from November 11, 1776, until February 28, 1777, a period of 109 days, during which time it had gained only 1′ 19″. In consequence of the above favourable report, and the Board being informed that Mr. Mudge had an intention of making two more, in order to obtain the greatest perfection, provided he could have assistance to do so, and to indemnify him, in case he should not obtain the great reward promised by act of Parliament, agreed to give him 500*l.* for this purpose.

Mr. Mudge, in consequence, made two time-keepers, which were tried at the Royal Observatory, three several times, in 1779, 1783,

* In the month of July, 1788, a chronometer of Arnold's construction, belonging to his grace the duke of Gordon, was put under the care of the Author, in order to ascertain its rate on mean solar time; in doing which, the greatest care imaginable was taken. It was always compared at noon with the Observatory clock, whose error was generally determined by transits of the Sun, which were observed with one of Ramsden's Transit Instruments; and which, previous to every observation, was levelled and adjusted to the meridian mark on the Grampian Hills, about 13900 feet distant from the Observatory. The mark is seen against the sky, and has a circular perforation in its top, of such a size, that the middle wire of the transit instrument covers about a third, and, therefore, a segment of light is seen on each side.

The observations began on the 15th of July, and were continued regularly for several months, during which time it was tried in every different position. In what follows, its rate, while in a horizontal position only, is given.

Daily rate.	Therm.	No. of days.	Daily rate.	Therm.	No. of days.
— 3″.99	65°	7	— 4″.60	60	7
— 5 .60	60½	7	— 4 .64	60	7
— 4 .89	59¾	7	— 4 .80	57½	5

The rate of the chronometer was ascertained when the hours XII. III. VI. and IX. and the bottom were uppermost successively. Its greatest loss upon mean solar time was when the hour XII. was uppermost, and the greatest gain when the hour VI. was uppermost, the difference between these two extremes amounting to upwards of one minute.

The price of this instrument is 60 guineas. It is in the form of an octagon, the radius of the circumscribing circle is 3.44 inches, and its depth is 3.3 inches—the case is mahogany; it has a bow handle for lifting it, and the whole is put into a mahogany box, 10.1 inches square, 6.4 inches deep, having its inside lined with soft elastic substances, to prevent the effects of sudden shocks. These chronometers beat half seconds.

The above-mentioned Observatory was taken down a few years ago, in consequence of the ground upon which it stood being wanted by government for the use of the barracks at Aberdeen, then newly built; and the instruments were removed to a room in the top of Marischal College, erected to receive them.

and

and 1789. On June 4, 1790, the Board of Longitude determined, unanimously, that as Mr. Mudge's time-keepers had not gone, upon the late twelve months trial, with the exactness required by the act of Parliament, the Board were not authorised to order further trial of them. On June 11, 1791, Mr. Mudge, in a memorial presented to the Board of Longitude, solicited a reward from them, on the ground of his time-keepers being superior to any that have been hitherto invented, and being constructed upon such principles as will render them permanently useful, and upon condition of making a discovery of these principles, his son, the father not being present, asked 2000*l.* The Board did not choose, in this case, to exercise the discretionary power intrusted to them by act of Parliament; namely, that of giving a less reward than any of the great rewards to any person who had made a less degree of improvement than was required by the act.

Mr. Mudge now finding his views frustrated with respect to obtaining a reward from the Board of Longitude for his time-keepers, applied to Parliament. This produced a letter from the president of the Royal Society, dated March 27, 1793, in which it is observed, that, "should the House grant a sum to Mr. Mudge, which will make his reward greater than that Mr. Arnold has received, they will manifestly reward the inferior, and discourage the superior artist; for no evidence has been brought forward to invalidate the fact of Mr. Arnold's time-keepers going better than Mr. Mudge's." However, in June 1793, Mr. Mudge received 1500*l.* from Parliament.

In 1792, Mr Mudge junior published a pamphlet entitled *A Narrative of Facts, &c.* in which it is asserted, that his father's time-keepers did not meet with good treatment while under trial at the Royal Observatory, and that an improper method was employed to find their daily rate. These, Dr. Maskelyne has refuted in his *Answer, &c.* to that pamphlet, published the same year; and Mr. Hellins, in his letter to Baron Maseres, dated September 25, 1792, shows that every manner of attention was paid to his time-keepers while he was assistant at the Royal Observatory.

Many other persons in Britain, as Messrs. *Earnshaw, Emery, Brooksbank, Combes, &c.* have constructed time-keepers, each artist introducing some improvement or alteration upon the preceding; so that it is to be hoped time-keepers will be brought to a much greater degree of perfection than hitherto, and afforded at a cheaper rate.

The utility of a time-keeper at sea is fully exemplified in the quarto edition of Cook's last Voyage, vol. I. page 46, vol. III. pages 321, 479, to which the reader is referred.

A time-keeper is certainly a most valuable appendage to a set of nautical instruments; if for no other purpose than that of connecting observations taken at different times; but like every other movement, is liable to be put out of order; and, therefore, astronomical observations are evidently to be preferred. Upon this subject Dr. Maskelyne observes as follows: "But I should prefer correspondent observations of an eclipse of a bright fixed star by the Moon, made by two astronomers,

nomers, furnished with proper instruments, at places not very remote from each other; and a number of correspondent observations of the transit of the Moon over the meridian, compared with those of fixed stars, made by two astronomers at two remote places, to any time-keeper whatever, for determining the relative situation of the two places*."

In order to find the longitude at sea by means of a chronometer, its daily rate on mean solar, or sidereal, time, must be established by observations made at some particular place, and its error ascertained, for the meridian of that, or of any other known place.

An observatory is the most proper and convenient place for this purpose, as there the rate and error may be both determined, with the utmost accuracy, by equal altitudes, or rather transits, of the Sun or fixed stars. But if an observatory is not adjacent, the rate and error of the chronometer may be found by altitudes taken daily for several days, from the horizon of the sea, or by the method of reflection.

If, by these observations, the daily rate is found to be nearly the same, that is, if the chronometer gains or loses nearly the same portion of absolute time daily, it may be depended on for finding the longitude; but if its rate is unequal, it must be rejected, as the longitude inferred from it cannot be expected to be accurate.

It would be proper to have two time keepers, and that they should be wound up at different stated times of the day, so that if one should be found stopt, either through neglect in winding up, or otherwise, it may be set by the other, observing to apply the former interval of time between them, and the change in their rates of going in that interval. It is absolutely necessary to keep a chronometer out of the reach of the magnetic influence of the ship's compass, magnetic bars, &c.

PROBLEM.

To find the Longitude of a Ship at Sea by a Chronometer.

RULE.

Let several altitudes of the Sun, or of any fixed star, be observed; and correct the mean altitude as usual; with which, the ship's latitude, and object's declination, compute the apparent time of observation, to which apply the equation of time, reduced to the time and place of observation, according to its title in the Nautical Almanac, and hence the mean time of observation will be known.

To the mean of the times of observation, as shown by the chronometer, apply its error and accumulated rate. Hence the mean time,

* *An Answer to a Narrative of Facts, &c.* Preface, page ix.

under

under the meridian of the place where the error and rate were established will be known; to which apply the difference of longitude in time between that place and Greenwich, and the mean time of observation under the meridian of Greenwich will be obtained. Now, the difference between the time at the place of observation and that at Greenwich will be the longitude of the place in time; and which is east or west, according as the time by observation is later or earlier than the Greenwich time.

Examples.

I.

May 19, 1804, in latitude 42° 15′ N. the following altitudes of the Sun's lower limb were observed in the afternoon, and the time of observation were given by a chronometer, whose error, as settled at the Royal Observatory at Greenwich, March 16, at noon, was 1′ 18″.4 fast for mean time, and its daily gain was 7″.83, height of the eye 26 feet. Required the longitude of the place of observation?

Time per chronometer	6h 58′ 40″	Altitude Sun's l. limb,	-	43° 24′
	6 59 36	- - - - -	-	43 33
	7 0 51	- - - - -	-	43 44
	7 2 12	- - - - -	-	43 57
	7 3 21	- - - - -	-	44 7
Sum - - -	4 40			225
Mean - -	7 0 56	Mean - - -	-	43 45
Error - -	— 1 18	Semidiameter -	-	+ 15.8
Accumulated rate -	— 8 23	Dip - - -	—	4.9
		Correction - -	—	.9
Mean time at Greenw.	6 51 15			
		Corrected altitude	-	43 55

To latitude 42° 15′ N. and reduced decli. 19° 51′ N. the number from

Table XXVII. is -	4.1572	Table XXVIII. - -	3288
Altitude 43° 55′ Sine -	9.8411		
	3.9983	Natural number - -	9961
Apparent time - -	3 12 34	Table XXIX. - -	6673
Equation of time -	— 3 51		
Mean time of observat.	3 8 43		
Mean time at Greenw.	6 51 15		
Longitude in time -	3 42 32 = 55° 38′ W.		

Remark.

Remark.

If the equation of time be applied, with a contrary sign, to the mean time of observation at Greenwich by the chronometer, the sum or difference will be the apparent time at Greenwich, the difference between which, and the apparent time at the ship, or place of observation, will be the longitude of that place in time as before.

II.

October 17, 1804, in latitude 57° 9′ N. the following altitudes of the Sun's lower limb were observed, the height of the eye being 13 feet; and suppose the time to be shown by a watch, whose error and rate were settled at the Observatory at Aberdeen; the error, October 1, was 3′ 11′ fast, and it lost 4″.6 daily. Required the ship's longitude?

Time per watch -	7h.	12′	8″	A. M. Alt. ⊙'s l. limb -		7° 14′
	7	13	0	- - - - - -		7 20
	7	14	7	- - - - - -		7 27
		9	15			61
Mean - -	7	13	5	- - - - - -		7 20.3
Error - -	—	3	11	Semidiameter -	+	16.1
Accumulated rate -	+	1	13	Dip and correction	—	10.4
Mean time at Aberdeen	7	11	7	Corrected altitude		7 26
Longitude of Aberdeen	+	8	32			
Equation of time -	+	14	31			
Apparent time Green.	7	34	10	A. M.		

To lat. 57° 9′ N. and reduced decl. 9° 13′ S. the number from

Table XXVII. is - -	4.2713	Table XXVIII. - - - -		2513
Altitude 7° 26′ Sine -	9.1118			
	3.3831	Natural number	=	2416
Time from noon -	4 1 52	Table XXIX. - - - -		4929
Apparent time -	7 58 8			
App. time at Greenw.	7 34 10			
Longitude in time -	23 58	= 5° 59½′ E.		

III.

February 3, 1804, being in latitude 15° 48′ N. the mean of several altitudes of Spica Virginis, east of the meridian, was 53° 24′, and that of the corresponding times 15h. 18′ 22″ per watch, which had

been set to mean solar time at Rio Janeiro, December 5, 1803, and was then gaining 53″.8 daily, on mean time. The height of the eye was 16 feet. Required the longitude of the ship?

Daily rate - - - - - - -			53″.8
No. of days between Dec. 5, 1803, and Feb. 3, 1804,			60
Gain in 60 days - - - - - - -		53	48″
Now 15h. 18′ — 54′ = 14h. 24′, in which time it gains			32
Accumulated rate - - - - - -		54	20
Time per watch of observation - - -	15	18	22
Mean time of observation at Rio Janeiro - -	14	24	2
Longitude of Rio Janeiro in time - -	2	50	55 W.
Mean time at Greenwich - - - -	17	14	57

Mean of ob. alt. = 53° 24′.0		Sun's R. A. at noon -	21h 4′ 3″
Dip and refraction —	4.5	Equation, Table XVIII.	+ 2 53
Altitude corrected 53	19.5	Reduced right ascension	21 6 56

To lat. 15° 48′ N. and reduced decl. 10° 8′ S. the number from

Table XXVII. =	4.0236	Table XXVIII. =	0506
Altitude 53° 19½′ Sine -	9.9042		
	3.9278	Natural number - - -	8468
		Table XXIX. - - - -	8974

Mer. dist. Spica Virg.	1h	44′	43
Right asc. Spica Virg.	13	14	53
Right ascens. merid.	11	30	10
Sun's right ascension	21	6	56
Apparent time at ship	14	23	14
Equation of time -	+	14	12
Mean time at ship -	14	37	26
Mean time at Greenw.	17	14	57
Longitude in time -	2	37	31 = 39° 29¼′ W.

IV.

August 16, 1804, in latitude 38° 19′ S. the mean of several altitudes of Antares, west of the meridian, was 14° 28′.4, the height of the eye being 12 feet, and the mean of the times per watch 11h. 41′ 38″ P. M. which had been compared with mean time at the Cape of Good Hope, June

June 22d, and was found to be 1h. 10′ 28″ slow, and gained 3″.54 daily. Required the ship's longitude?

Daily gain - - - - - - - - -			3″. 54
Number of days between June 22, and August 16 -			55
Gain in 55 days - - - - - -		3	14. 7
The Green. time of obs. is about 11h. 35′, and corr. gain			1. 7
Accumulated rate - - - - - - -		3	16
Error, 22d June - - - - - - -	1	10	28
Watch slow at time of observation - - -	1	7	12
Time per watch of observation - - - -	11	41	38
Mean time of observation at Cape of Good Hope	12	48	50
Longitude of Cape of Good Hope - -	1	13	33 E.
Mean time at Greenwich - - - - -	11	35	17

Observed alt. Antares =	14° 28.9′	Sun's R.A. at noon	9 42 40
Dip and refraction -	— 6.9	Equation, Table XVIII.	+ 1 44
Corrected altitude - -	14 22	Reduced right ascen.	9 44 24
Polar distance - -	64 1	- - Co-secant - -	0.04628
Latitude -	38 19	- - Secant - - -	0.10535
Sum - - ..	116 42		
Half - - -	58 21	- - Co-sine -	9.71993
Difference - -	43 59	- - Sine - - -	9.84164
			19.71320
Arch - - -	45 57¼	- - Sine - - -	9.85660
	8		
Mer. dist Antares -	6 7 38		
Right ascen. Antares	16 17 27		
Right ascen. meridian	22 25 5		
Sun's right ascension	9 44 24		
Apparent time at ship	12 40 41		
Equation of time -	+ 3 52		
Mean time at ship -	12 44 33		
Mean time at Green.	11 35 17		
Longitude in time -	1 9 16 = 17° 19′ E.		

Remark.

In practice, it will be found very convenient to have a table constructed, showing the error of the chronometer at the noon of every day for several weeks, or during the estimated time of the run to a place where its error and rate can be again settled. To this table a column should be added, containing its hourly rate continued up to 24 hours.

Thus, suppose the daily rate of a chronometer, deduced from a series of observations, was — 4″.72, and its error for mean time, May 9, 1820, at noon, to be 3′ 58″.6 slow; then,

Error of Chronometer at Mean Noon.							*Hourly Rate.*			
♂	May	9	Error	=	3′ 58″.6		1	hour	=	0″.2
☿	- -	10	-	-	4 3 .3		2	-	-	0 .4
♃	- -	11	-	-	4 8 .0		3	-	-	0 .6
♀	- -	12	-	-	4 12 .8		4	-	-	0 .8
♄	- -	13	-	-	4 17 .5		5	-	-	1 .0
☉	- -	14	-	-	4 22 .2		6	-	-	1 .2
☽	- -	15	-	-	4 26 .9		7	-	-	1 .4
♂	- -	16	-	-	4 31 .6		8	-	-	1 .6
☿	- -	17	-	-	4 36 .4		9	-	-	1 .8
♃	- -	18	-	-	4 41 .1		10	-	-	2 .0
♀	- -	19	-	-	4 45 .8		11	-	-	2 .2
♄	- -	20	-	-	4 50 .5		12	-	-	2 .4

Determination

Of the Longitude of the late Observatory at Aberdeen, by a Chronometer of Arnold's Construction.

The chronometer was set to mean solar time at Greenwich, June 16, 1788, and lost 7″.5 in eleven days. It was sent to Aberdeen by sea, and being compared with the Observatory clock, July 15, it was found to be 7′ 26″.6 fast; and was losing 6″.4 daily. It is hence probable, that the motion of the ship had altered its rate. Now, supposing this alteration to have commenced when the ship left London, which was July 8, its error at that time for the meridian of Greenwich would, therefore, be 15″ .0, from this time till July 15, it lost 44″.8 (= 6″.4 × 7) its rate being supposed to be uniform. Hence, its error for the meridian of Greenwich, July 15, at noon, was — 59″.8; but its error for the meridian of the Observatory at Aberdeen, at the same time, was + 7′ 26″.6. Hence, the longitude of the Observatory, in time, is 8′ 26″.4 W.

CHAP.

CHAP. IX.

The Method of finding the Longitude at Sea,

BY

The Variation Chart.

THE principle employed in the former methods of determining the longitude of a place, is that of finding the difference of time between the place whose longitude is wanted, and the first meridian. In this, the variation of the compass is used for the same purpose. In order, therefore, to discover the longitude by this method, charts must be constructed, in which the variation of the compass at all different places of the earth, is represented by curve lines, accurately drawn from observation. Now, the variation being observed at the ship, and compared with the correspondent variation on the chart, the longitude answering thereto will be obtained. The method of finding the longitude of any place, whose latitude, and the variation of the compass at that place, are given, is clearly explained and illustrated by examples in a small tract entitled the *Haven-finding Art*, annexed to Mr. Edward Wright's *Certain Errors in Navigation detected and corrected*, the first edition of which was printed at London in the year 1599, and the third edition in 1657.

Messrs. William Nautonnier, a Frenchman, Emmanuel Figueiredo, a Portuguese, Linton, Whiston, &c. have proposed the variation of the compass for the discovery of the longitude at sea. In 1676, Mr. Henry Bond published his *Longitude Found*, in 4to. But the want of a sufficient number of observations rendered this method of little or no service at that time.

Mr. Harrison, in his *Idea Longitudinis*, printed at London in 1696, recommends the variation of the compass as a good method of ascertaining the longitude. In the above tract, page 33, he says, "When I was in *East-India*, I understood what *variation* there was in most adjacent parts so well, that I have offered in discourse in company, to go in a ship that set sail from any part of the coast of *India*, bound any way, two, three, or four hundred leagues; I would keep no account of her way for a week, or ten days time, and any fair day, when I could have reasonable observations, I would have told them the place where the

the ship was, as well as they that kept the most exact reckoning, provided they had not seen the land since I saw it; and this I must have done by the *latitude* and *variation* observed*.

In 1700, Dr. Halley published the first variation chart, from a great number of observations, which for many years he had been collecting for that purpose. As the variation is, however, subject to an annual change, this chart, therefore, in a few years, became almost useless. In the years 1744 and 1756, it was re-published at London by Messrs. Mountaine and Dodson, from near one hundred thousand observations. It was also published at Paris in 1765, by M. Bellin, and again at London in 1788, and in 1794. Variation charts, adapted to different years, have been published by Mr. Samuel Dunn.

Although many of the observations used by Dr. Halley, and his followers, in the construction of variation charts, were accurate, yet it is not to be expected that this was the case with them all; and, therefore, it is not probable, that any curve drawn according to these observations, represented the actual variation at all places through which it passed, or which is the same, it will not be a line of equal variation. It should also be remembered, that these curves are subject to a change of place and figure.

This method of finding the longitude is impracticable at those places where the variation lines coincide with a parallel of latitude, or where they are nearly in that direction. This happens in the tract between the southern parts of Europe and North America, and in some parts of the Pacific and Indian Oceans. If the inclination between these lines be less than two or three points, this method of finding the longitude is liable to a very great degree of uncertainty. The less, therefore, the inclination of the magnetic curve to the meridian, *cæteris paribus*, the more accurately will the longitude be ascertained; and, hence, a tolerable exact solution to this problem may be obtained, provided a good chart, adapted to the time of observation, is used. This, however, will be difficult to be procured, until theory and observation are blended together in the construction of variation charts; and these charts should be republished at certain intervals.

Another and very particular objection against the practice of this method, was the want of a proper instrument to observe the magnetic azimuth with accuracy, in order to determine the variation of the compass†. This is, however, now removed, by the invention of a new azimuth compass by Mr. M'Culloch, which is so constructed, that the

* In a manuscript note, at the end of a copy belonging to the Author, and signed by the initials of Mr. Harrison's name, is the following paragraph —" The following words were forgot. viz. Supposing the variation and longitude to be two distinct arts, having no dependence on each other, save so much of the longitude as is found by the variation of the compass, the question is put, whether magnetic variation, well understood, be not as much for the public good of mankind, as the longitude, if it were known?

† See also Cook's last Voyage, second edit. vol. i. page 50.

centers

centers of motion, of gravity, and of magnetism, are brought almost all to the same point. This instrument has also many other advantages over the common azimuth compasses. An account of its great utility and superiority over them may be seen in his Report, printed at London in 1789.

PROBLEM I.

Given the Latitude of a Place, and the Variation of the Compass, to find the Longitude of that Place.

RULE.

Draw a line to represent the parallel of latitude of the given place, and its intersection with the line of the given variation will be the situation of the given place; and hence its longitude is known.

When the given variation falls between two of the lines of equal variation on the chart, allowance is to be made as usual, and, if the given time differs considerably from that to which the chart is adapted, allowance is to be made for the change of variation in the interval.

EXAMPLES.

I.

Let the ship's latitude be 18° 10′ S. and the variation of the compass 10° 30 W. Required the longitude?

By laying a rule over 18° 10′ S. the given latitude, in a direction parallel to the equator, it will be found to intersect the line representing 10° 30′ W. variation, in a certain point, the longitude of which is about 9° 20′ W.

II.

June 1, 1793, being in latitude 40° 0′ S. the variation of the compass deduced from the mean of several azimuths of the Sun was 10° E. and by comparing two variation charts, the latest of which being adapted to the year 1765, it is found that the annual decrease of variation is about 10 minutes. Required the ship's longitude?

The decrease of variation in the given time is about 4° 40′ (= 10′ × 28y.); hence the observed variation, reduced to the time to which the chart is adapted, is about 14° 40′ E. Now, by drawing the parallel of 40° S. its intersection with the line expressing the above variation is found to be in longitude 56° 30′ W. which is, therefore, that required.

Although the variation chart, in many cases, is not to be depended on for the purpose of determining the longitude at sea, yet its use in finding

finding the variation of the compass answering to a given latitude and longitude by inspection only, is evident. Upon this account, it demands the particular attention of the practical navigator.

Besides this method of finding the longitude of a ship, by the variation or declination of the needle, Mr. Whiston proposed to find the longitude by the dip, or inclination of the needle, the latitude of the place being given, or the latitude, if the longitude be known; and dates the time of his discovery to be about the end of the year 1718. If both these methods be combined, the latitude and longitude may be found; the declination, and inclination of the needle, being given. For this purpose, a chart, expressing the lines of equal variation and dip, is necessary. Then, the point of intersection of the line of the given variation, with that of the given dip, will be the ship's place; and, hence, its latitude and longitude are both known.

Or, if a set of tables were constructed, having the variation and dip to every degree of latitude and longitude, with their annual variations, the ship's place would be easily found.

In the year 1768, a chart, exhibiting the lines of equal dip, was published at Stockholm, by M. Wilcke. This chart was re-published by M. le Monnier, in his treatise entitled *Loix du Magnetisme*, printed at Paris in 1776.

Beside the methods which are here given, and illustrated, to find the longitude, others have been proposed; of which, however, the following only shall be briefly mentioned.

In the year 1714, Messrs. Whiston and Ditton published a pamphlet containing a method of finding the longitude by the explosion of bombs. Some time after, Sir Isaac Newton delivered a paper to a committee of the House of Commons, in which he observed, that this method "is rather for *keeping an account* of the *longitude* at sea, than for *finding* it, if it should be lost; and that it is easier by it to enable seamen to know their distance and bearing from the shore 40, 60, or 80 miles off, than to cross the seas." Whatever use, therefore, this method may be of near the land, yet it can be of no service in crossing the ocean.

M. Cassini proposed to ascertain the longitude of a place from observations on the spots of Jupiter. Many other methods might be given for finding the longitude, which, for various reasons, it is thought unnecessary to mention.

BOOK

BOOK V.

CONTAINING

The Demonstration of the preceding Rules and Formulæ.

Demonstration

Of Dr. Maskelyne's Rules for computing the Longitude and Latitude of a fixed Star, or Planet, from its known right Ascension and Declination; and for computing its right Ascension and Declination, its Longitude and Latitude being given, Pages 44, 45.

LET the primitive circle, fig. 2, represent the solstitial colure, EQ the equator, ♋ ♑ the ecliptic, and S the position of a star or planet; through which let circles of latitude and declination be drawn. Then is ♈ A the right ascension, AS the declination; ♈ B the longitude, and BS the latitude of the star: and the angle A ♈ B the obliquity of the ecliptic. Now, in the right-angled spherical triangle ♈AS, are given A♈ and AS, to find the angle A♈S.

Sine A♈ : R : : tangent AS : tangent A♈S.

$$\text{Hence, tangent A}\Upsilon\text{S} \quad \frac{\text{tangent AS,}}{\text{sine A}\Upsilon} \text{ to radius unity;}$$

And A♈S ∓ A♈B = B♈S.

Again, in the right-angled spherical triangles A♈S, B♈S, we have

R : co-sine A♈S : : tan. ♈S : tan. A♈

R : co-sine B♈S : : tan. ♈S : tan. B♈.

Hence, co-sine A♈S : co-sine B♈S : : tan. A♈ : tan. B♈

And tan. B♈ = ar. co. co-sine A♈S co-sine B♈S tan. A♈, or

= tan. A♈S ar. co. sine A♈S co-sine B♈S-tan. A♈

because the co-sine of an arch is equal to the rectangle under the co-tangent and sine of that arch, to radius unity.

Now, R : sine B♈ : : tan. B♈S : tan. BS.

Hence, tan. BS = sine B♈. tan. B♈S, or

= tan. B♈. co-sine B♈, tan. B♈S. *Q. E. D.*

If the positions of the ecliptic and equator be supposed to be mutually changed, the demonstration of the rule for computing the right ascension and declination will be the same as the above.

Theory *of* Hadley's Quadrant, *of the* Sextant, *and* Circular Instrument.

Lemma.

The angle contained between a ray of light falling on any reflecting surface, and a line perpendicular to that surface at the point of incidence, is equal to the angle contained between the same perpendicular and the reflected ray*.

Theorem I.

In the Fore Observation, the Altitude of an Object, or the angular Distance between two Objects, is equal to double the Arch on the Limb passed over by the Index.

Let ABC, fig. 27, represent a quadrant, D the index glass, F the fore-horizon glass, and O, G, two objects whose angular distance is the arch OG, or the angle GEO, the position of the eye being E. Now, let the index be so placed, that the reflected image of the object O, may coincide with the object G, seen directly. Then is the angle GEO equal to twice the angle CDH; that is, equal to twice the arch passed over by the index.

Demonstration.

Let the ray OD incident on the speculum at D, be reflected to the fore-horizon glass, and from thence to the eye at E; also, let DL be perpendicular to the index glass at the point of incidence D: and FM perpendicular to the fore-horizon glass at the point of incidence F. Then, by the above lemma, the angle LDF = LDO, and MFE = MFD. Let OD and GF produced meet in E, and FED will be the angular distance of the objects.

Now, since LD is parallel to EG, and perpendicular to the speculum, the angle EFD = FDL = LDO = DEF; and EFD is bisected by the perpendicular FM, therefore, the angle DEF = 2MFE. But because the right-angled triangles MFK, MFI, have the angle M common, the remaining angles MFI, MKF, are therefore equal: hence, the angle DEF = 2MKF; but MKF = KDC, therefore, DEF = 2KDC. Now, DEF is the angular distance of the objects, and KDC is measured by

* This is demonstrated by writers on optics.

the

the arch CH. The angular distance between any two objects is, therefore, double the arch on the limb passed over by the index. *Q. E. D.*

This also holds good with respect to the sextant, and circular instrument.

Theorem II.

In the back Observation, the Supplement of the Angle contained between any two Objects is equal to double the Excess of the Inclination of the Mirrors above a right Angle; and, therefore, equal to twice the Arch passed over by the Index.

Let D, fig. 28, be the speculum, N the back-horizon glass, whose planes produced meet in M; hence, the angle MNI is equal to the excess of the inclination of these planes above a right angle; also, let O, G, be any two distant objects, then PEO, the supplement of the angular distance GEO, is equal to twice the angle MNI = twice CDH = twice the arch CH.

Demonstration.

Let the index be so placed, that the reflected image of the object O may coincide with the direct object G, the eye being at P. Also, let DL be perpendicular to the speculum, and LN perpendicular to the back-horizon glass. Then is the angle LDN = LDO, and LNP = LND; hence, MND = RNP = MNI, therefore DNI = 2 MNI: and because the triangles MNI, MKN, are similar, and DC parallel to NK, therefore the angle MNI = (MKN = KDC =) the arch CK; hence DEN = twice the arch CH. Therefore, in the back observation, the supplement of the angular distance between two distant objects is equal to double the arch on the limb passed over by the index. *Q. E. D.*

Demonstration

Of the Rule for computing the Length of the Scale, or of any Portion of the Scale, on the Limb of Hadley's Quadrant, Page 62.

Let ABC, fig. 27, represent an octant, and therefore the angle BAC = 45°. Join BC, and draw AS perpendicular thereto. Now, in the triangle ASC, are given AC, and the angle CAS, to find CS.

R : sine CAS : : AC : CS.

Hence CS = AC sine CAS, and BC = 2AC sine CAS.

If AC = 1, then BC = 2 sine CAS. *Q. E. D.*

Demonstration

Of the Formula for computing the greatest Augmentation of the Moon's Semidiameter, Page 82.

Let ML, fig. 29, represent the Moon's semidiameter, E the Earth's center, and P a point on its surface: hence the angles MEL, MPL, will be the Moon's semidiameter, as seen from those points respectively; and their difference PLE is, therefore, the augmentation. Now, since these angles are small, the measure of each may be substituted for its sine. Let, therefore, the Moon's horizontal semidiameter MEL $= s$, the augmentation PLE $= x$, hence MPL $= s + x$. Also, let ME $= d$, and PE $=$ ME $-$ MP $= 1$, hence MP $= d - 1$.

Then $d - 1 : d :: s : s + x$

Convert ^{o} $d - 1 : 1 :: s : x$

Hence, $x = \dfrac{s}{a - 1}$.

If the Moon's horizontal parallax be expressed by h, then d becomes $= \dfrac{57^\circ\ 17'\ 44''.8}{h}$, and $d - 1 = \dfrac{57^\circ\ 17'\ 44''.8 - h}{h}$. But the ratio of the horizontal parallax to the semidiameter of the Moon is as $1 : 27249$ nearly: hence, $d - 1 = \dfrac{57^\circ\ 17'\ 44''.8 - h}{h} .27249$

$= \dfrac{15^\circ\ 46'\ 45'' - s}{s}$; which being substituted for $d - 1$ in the former equation, gives $x = \dfrac{s^2}{15^\circ\ 46'\ 45'' - s}$.

Let LP be produced to F, and EF being drawn parallel to ML, will therefore be the greatest augmentation to the radius EM. Let the Moon be in any other point A of its diurnal path; join AE, AF, make AG $=$ AE, and EG, which is the measure of the angle EAH to the radius AE $=$ LE, will be the augmentation corresponding to that altitude.

Now in the right-angled rectilineal triangle EFG, given the angle EFG $=$ complement of AEM $=$ ☽'s altitude, and EF the greatest augmentation, to find EG.

R : sine EFG :: EF : EG $=$ EF, sine EFG, to radius 1. *Q. E. D.*

It hence obviously follows, that if to the true altitude of the Moon's limb, the horizontal semidiameter of the Moon, as given in the ephemeris, be applied, the true altitude of the Moon's center will be obtained: and, therefore, in this case, the augmentation is unnecessary. Hence, also, the change of parallax at any given altitude, answering to a change of altitude equal to the Moon's semidiameter, will be the augmentation answering to that altitude.

Demon-

Demonstration

Of the Rule for computing the Dip of the Horizon, Page 84.

In fig. 21st, BE represents the height of the observer, and ECH = FEI the corresponding dip. Now in the right-angled rectilineal-triangle CHE, are given CH the semidiameter of the earth, and BE the height of the eye, to find the angle HCE.

Let CH = s, and BE = h, hence, CE = $s + h$.

Now, $\overline{HE}|^2 = \overline{2s+h} \times h$ per 36. III Euclid.

Hence, HE = $\sqrt{\overline{2s+h} \times h} = \sqrt{2sh}$, because, at any probable elevation, $2s + h$ is not sensibly different from $2sh$.

And $s : \sqrt{2sh} :: r :$ tan. HCE $= \frac{r\sqrt{2sh}}{s}$.

Since, in the present case, the arch may be substituted for its tangent, the radius, therefore, becomes 57° 17′ 44″.8.

Hence, the dip $= \frac{r\sqrt{2sh}}{s} = \sqrt{\frac{2r^2h}{s}} = \sqrt{\frac{2r^2}{s}} \times h$. *Q. E. D.*

Demonstration

Of the Rule for computing the Refraction in Altitude, Page 88.

(Fig. A, Plate IV.)

Let Z be the zenith, ZDH a vertical circle, and H a point in the horizon; also, let B be the true, and D the apparent place of a star; hence, BD will be the refraction of the altitude D: let the arch HA be the horizontal refraction, join CD, and draw the lines AK, BF, and DG parallel to CH and DE, AI parallel to CZ; then CI is the co-sine, and IH the versed sine of the horizontal refraction; CG is the sine, and GD the co-sine of the apparent altitude; and let the radius CH, or CD = 1.

Now, CH : CI :: BF : DG, per dioptrics.

Hence, IH : CI :: BE : DG, per division.

Therefore, BE $= \frac{IH \times DG}{CI}$.

And, since the triangle BDE may be considered as rectilineal and right-angled, it is, therefore, similar to the triangle CGD.

Hence, BE $= \frac{BD \times CG}{R}$.

Therefore, $\frac{BD \times CG}{R} = \frac{IH \times DG}{CI}$.

Whence,

Whence, BD $= \frac{IH \times DG \times R}{CI \times CG}$.

That is, the refr. in alt. $= \frac{\text{Ver. sine hor. refr.} \times \text{co-sine app. alt.} \times \text{rad.}}{\text{Co-sine hor. refraction} \times \text{sine app. alt.}}$

$= \frac{\text{Ver. sine hor. refr.} \times \text{rad.}}{\text{Co-sine hor. refr.}} \times \text{co-tang. app. alt.}$

But, since $\frac{\text{Ver. sine hor. refraction} \times \text{rad.}}{\text{Co-sine hor. refraction,}}$, is a constant quantity, being equal to the refraction at the altitude of 45 degrees; the refraction in altitude is, therefore, as the co-tangent of the apparent altitude, or as the tangent of the zenith distance. *Q. E. D.*

Remark.

It has been already observed, that Dr. Bradley, from a number of observations, inferred the refraction at 45° to be 57′; and also, that the refraction at any other altitude to be equal to 57″, multiplied by the tangent of the sum of that altitude, and thrice the refraction taken from the common tables. Many astronomers, however, differ with respect to the quantity of the mean refraction at 45°. At a place five and two-third degrees north of Greenwich, the author of this work, from many observations, found the mean refraction at 45° to be greater than 57″, and constructed a table for his own private use accordingly.—The observatory was about 100 feet above the level of the sea.

Demonstration

Of the Rule for computing the Parallax in Altitude, Page 69.

In the rectilineal triangles DAC, HAC, (fig 23,) the angle ADC is the horizontal parallax, AHC the parallax answering to the apparent altitude DAH.

Now DC : AC : : R : sine ADC.

And DC (=HC) : AC : : sine ZAH : sine AHC.

Ex. eq. R : sine ADC : : sine ZAH : sine AHC = sine ADC sine ZAH, to radius 1.

Since, in angles not exceeding one, or one and a fourth degrees, the arches are nearly proportional to their sines,

Therefore AHC = ADC. sine ZAH.

That is, par. in alt. = hor. par. × co sine of apparent altitude.

Or, by the nature of proportional logarithms,

P. log. Par. in alt. = P. L. hor. par. + log. secant app. alt. *Q. E. D.*

Demon-

Demonstration

Of the Rule for converting Degrees into Time, Page 104.

Let a be the given degrees, and x the correspondent time; then, since the Sun is in the plane of any given meridian once in 24 hours,

Therefore, $360° : a° :: 24h. : x$ h.

Hence, $x = \frac{24h.\ a}{360°} = \frac{1440'a}{360} = 4a.$ *Q. E. D.*

Demonstration

Of the Rule for reducing Time into Degrees, Page 105.

Let a be the given time, and x the degrees answering thereto.

Now, $24h. : ah. :: 360° : x°.$

Hence, $x = \frac{360}{24} \times a = 15\,a = 10\,a + \frac{10a}{2}.$ *Q. E. D.*

Demonstration

Of the Rule for reducing the Time under a given Meridian to the Time at Greenwich, and conversely, Page 105, 106.

The rotation of the Earth being from west to east, it is therefore evident, that the Sun, or any celestial object, will be sooner on the meridian of a place to the eastward, and later on that of a place to the westward, than on the meridian of Greenwich, by a quantity proportional to the longitude of the place from Greenwich. And conversely. *Q. E. D.*

Demonstration

Of the Rule to find the Correction of Noon arising from the Ship's Run between the Observations of the equal Altitude, Pages 120, 123.

Let $a =$ interval of time between the observations, and $b =$ hourly rate of sailing. Then, according to the principles of plane sailing,

$$\left.\begin{array}{l} \text{The diff. latitude} = \dfrac{ab.\ \text{co-sine course}}{1\ \text{hour.}} \\ \text{And departure} = \dfrac{ab.\ \text{sine course}}{1\ \text{hour.}} \end{array}\right\} \text{to radius unity.}$$

Now, the change in the horary angle answering to a given change of

of latitude, is = D. Lat. $\frac{\text{tan. azimuth}}{\text{co-sine lat.}}$ * = $\frac{ab. \text{ co-s. course. tan. azimuth}}{\text{co-sine latitude}}$

to radius 1, and therefore,

The equation of latitude in time $\frac{2ab. \text{ co-sine course. tang. azimuth}}{\text{co-sine of latitude.}}$

Or, if proportional logarithms be used, and the hours reckoned as minutes, then P. Log. of eq. of lat. = P. L. a + P. L. b + L. secant course + L. tan. azim. + L. co-s. lat. + L. 2. — P. L. 1 minute.

And the equation of long. = $\frac{2ab. \text{ sine course}}{\text{1 hour, co-s. lat.}}$.

Hence, P. L. eq. of long. = P. L. a + P. L. b + L. co-secant course + L. co-sine lat. + L. 2 — P. L. 1 minute.

Demonstration

Of the Formula for computing the Altitude of a circumpolar Star, when its Motion in altitude is a Maximum, Page 124.

Let P, fig 30, represent the pole, Z the zenith of the place of observation, DSd the parallel of the declination of the star, ZSA a vertical circle touching the parallel of declination in the point S, which is, therefore, the place of the star when its motion in altitude is quickest.

Now, in the right-angled spherical triangle ZSP, ZP and SP are given to find ZS.

Co-sine SP. : R : : co-sine ZP : co-sine ZS.

Hence, co-sine ZS = co-sine ZP. secant SP, to radius unity:

That is, sine altitude = sine latitude. co-secant declination.

Q. E. D.

Demonstration

Of the Rule for computing the Apparent Time from the Altitude of a Celestial Object.

Method First, *Page* 125.

In any Spherical Triangle ZPS, fig. 31,

$$\text{Sine } \tfrac{1}{2} \text{P} = \sqrt{\frac{\text{sine } \frac{1}{2}(\text{ZS}+\text{PZ}-\text{PS}) \text{ sine } \frac{1}{2}(\text{ZS}+\text{PS}-\text{PZ})}{\text{sine PZ. sine PS}}}. \text{ per spher.}$$

Wherein ZP may represent the co-latitude of the place of observation, ZS the zenith distance, and PS the polar distance of the observed object: and the angle ZPS the horary distance of the object from the meridian.

* Simpson's Fluxions, page 282, line 2 from bottom.

Now

Now, let the altitude $= a$, latitude $= m$, and polar distance $= p$.
Then $\frac{1}{2}(ZS + PZ - PS) = \frac{1}{2}(90 - a + 90 - m - p) = \frac{1}{2}(180 - a - m - p)$
$= 90 - \frac{1}{2}(a + m + p)$.
And $\frac{1}{2}(ZS + PS - PZ) = \frac{1}{2}(90 - a - \overline{90 - m} + p) = \frac{1}{2}(-a + m + p)$
$= \frac{1}{2}(a + m + p) - a$.

Hence, by substitution, we have

$$\text{Sine } \tfrac{1}{2} P = \sqrt{\frac{\text{co-s. } \frac{1}{2}(a+m+p) \cdot \text{sine } \overline{\frac{1}{2}(a+m+p) - a}}{\text{co-s. } m \cdot \text{sine } p}}$$

$$= \sqrt{\text{co-s. } \tfrac{1}{2}(a+m+p) \cdot \text{sine } \overline{\tfrac{1}{2}(a+m+p) - a} \cdot \text{secant } m \cdot \text{co-sec. } p}.$$

Method Second, *Page* 128.

In the spherical triangle ZPS. fig. 31.

Sine PS. sine PZ : R^2 :: v. sine ZS—v. sine $\overline{PS - PZ}$: v. sine P. per spherics.

But the difference of the versed sines is equal to the difference of the co-sines; therefore,

Sine PS. sine PZ : R^2 :: co-s. $\overline{PS - PZ}$—co-s. ZS : v. sine P.

$$\text{Hence, ver. sine } P = \frac{\text{co-s. } \overline{PS - PZ} - \text{co-s. } ZS.\, R^2}{\text{sine } PS \text{ sine } PZ}$$

$$= \frac{\text{co-sec.} PS. \text{co-sec. } PZ. \overline{\text{co-s.} \overline{PS - PZ} - \text{co-s. } ZS}}{R^2}$$

Whence, log. versed sine P = log. co-sec. PS + log. co-sec. PZ
+ log. $\overline{\text{co-s. } \overline{PS - PZ} - \text{co-s. } ZS}$ — 2 log. rad.

But PS — PZ = the meridian zenith distance = latitude $\pm$ declination, according as they are of different, or of the same name. And the column of rising is a table of log. versed sines, adapted to time. Therefore,

Log. secant lat. + log. sec. dec. + log. $\overline{\text{co-s. mer. zen. dist.} - \text{sine ob. alt.}}$ —2 log. rad. = log. rising of apparent meridian distance of the object.

Investigation

Of that Position in which an Object should be, that the Time deduced from its Altitude may be as little as possible affected by unavoidable Errors in the Observations and Latitude, Page 139.

When the latitude and declination are of the same name, the change of altitude, in a given time, is most rapid; first, when the star is in the prime vertical, the declination being less than the latitude; secondly, when the object is in that part of its diurnal path which is in contact

with an azimuth circle, in this case, the declination will exceed the latitude of the place of observation; and, thirdly, when the object is in the zenith, the latitude and declination being equal.

When the latitude and declination are of contrary names, the change of altitude is quickest when the object is in the horizon; which, therefore, is the most proper position for observation, abstracting from refraction. But because of the irregularity to which the horizontal refraction is subject, it is, therefore, improper to use altitudes observed near the horizon, for the purpose of determining the apparent time.

Let P, Z, S, fig. 32, represent the pole, zenith, and star, respectively; let MPS be a given change in the hour angle; and AM being a portion of a parallel of altitude, SA will be the correspondent change of altitude.

Let AS $= a$, and MPS $= h$.
Now R : sine PS : : h : SM $= h$. sine PS. to radius unity.
And R : sine AMS : : h. sine PS : $a = h$. sine PS. sine AMS.
$= h$. sine PS. sine ZSP. to rad. 1.

Now PS being constant, the change of altitude AS answering to a given change in the hour angle MPS, will be a maximum, when the sine of the angle ZSP is the greatest possible; that is, when ZSP is a right angle. Now ZSP is a right angle when the vertical ZS touches the parallel of declination of the star. And if ZSP be constant, the change of altitude will be quickest when PS $= 90^\circ$; that is, when the object is in the equator.

Again, in the triangle ZSP.
Sine ZP : sine PS : : sine ZSP : sine PZS.
Therefore, sine ZP. sine PZS $=$ sine PS sine ZSP.
Hence, sine ZP. sine PZS. being substituted for its equal in the former equation, we have $a = h$ sine ZP. sine PZS *.

Now ZP being constant, the change of altitude will be a maximum, when PZS is a right angle; that is, when ZS is the prime vertical †. And if PZS be constant, the change of altitude answering to a given change in the horary angle, will increase with the sine of PZ, and is, therefore, greatest when the latitude is least; that is, when the place is situated on the equator.

Since, sine ZSP $= \dfrac{\text{sine PZ. sine ZPS}}{\text{sine ZS}}$; which, substituted in the general equation, gives $a = h \dfrac{\text{sine PS. sine PZ. sine ZSP}}{\text{sine ZS}}$. Hence, the

* If proportional logarithms be used, the equation becomes
Prop. log. $a =$ Prop. log. $h +$ log. co-secant ZP $+$ log. co-secant PZS.
This is the demonstration of the rule in page 80, for computing the change of altitude answering to a given interval.

† If the latitude and declination are of contrary names, the parallel of declination will not intersect the prime vertical above the horizon; therefore, the change of altitude is quickest when the object is in the horizon.

latitude

latitude and declination being constant, the change of altitude will increase with the altitude and hour angle.

Demonstration

Of the Rules for computing the Correction of Apparent Time, answering to given Errors in the Altitude, Declination, and Latitude.—Prob. X. XI. XII. *Pages* 140, 141.

The error in altitude being given, the correspondent error of the hour angle may be easily deduced from the preceding formulæ.

For, since $a = h.$ sine PZ. sine PZS;

$$\text{Therefore, } h. = \frac{a}{\text{sine PZ. sine PZS. } 15} \text{ in time:}$$

And P. log. $h =$ P. log. a + log. sine PZ + log. sine PZS + log. 15.

Whence, in any spherical triangle, the change in any of its angles, occasioned by a change in the opposite side, is equal to this change, multiplied by the co-secant of one of the other sides, and the co-secant of the intervening angle, and conversely.

The Error in Declination being given, to find the resulting Error in the Hour Angle.

Let SB, SM, fig. 32, be portions of the parallels of altitude and declination of the given object, and BM the given change of declination: hence, MPS will be the correspondent change of the hour angle.

Now, in the right-angled triangle BMS, which may be considered as rectilineal, are given MB, and the angle BSM = ZSP, to find MS.
R : co-tan. ZSP : : MB : MS = MB. co-tan. ZSP, to radius unity.
And, sine PM : R : : MB. co-tan. ZSP : MPS = MB. co-tan. ZSP. co-secant PM*.

Or, by using P. logarithms, the change of the hour angle expressed in time is = Prop. log. BM + log. tangent ZSP + log. sine PM + log. 15.

Hence, in any spherical triangle, the change in any of its angles, arising from a given change in one of the adjacent sides, is equal to the above change multiplied by the co-tangent of one of the other angles, and the co-secant of the side contained between these angles; and conversely.

If l be the change of latitude, and h the corresponding change of the hour angle, then, by the preceding, we have

* Or, since co-tan. ZSP = co-sec. P. co-tan. PZ. sine PS — co-tan. P. co-s. PS, which being substituted in the former equation, we have

$$\text{MPS} = \text{MB. co-sec. } \overline{\text{P. co-tan. PZ} - \text{co-tan. P. co-tan. PS}}.$$

 $h =$ *log.*

$h =$ log. co-tangent PZS. log. co-secant PZ $\frac{1}{15}$.
And P. log. $h =$ P log. l + log. tan. PZS + log. sine PZ + log. 15.

Or, since co-tangent Z = co-secant P. co-tangent PS. sine PZ — co-tangent P. co-sine PZ. Hence, by substitution, we have

$h = \overline{l.\ \text{co-sec. P. co-tang. PS} - \text{co-tang. P. co-tang. PZ}}$. *Q. E. D.*

Demonstration

Of the Rule for computing the Apparent Time from an Observation of two Stars in the same Vertical, Page 142.

Let P, fig. 33, represent the elevated pole, Z the zenith, and S, A, two stars in the same azimuth circle ZSAN, to which let PB be perpendicular. Now, in the triangle APS are given, AP, SP, the polar distances of the stars, and the angle APS the difference of right ascension, to find the angle ASP.

Now, $\text{tang.S} = \frac{\text{sine APS}}{\text{sine PS. co-tan. AP} - \text{co-sine PS. co-sine P}}$, per spherics.

And in the right-angled spherical triangle PBS,

$\text{R} : \text{co-s. PS} :: \text{tang. S} : \text{co-tang. BPS} = \text{co-s. PS. tang. S} =$

$= \frac{\text{sine APS}}{\text{tang. PS. co-tang. AP} - \text{co-s. P}}$.

Hence, $\text{tangent BPS} = \frac{\text{tangent PS. co-tangent AP} - \text{co-sine P}}{\text{sine APS}} =$

$= \text{tang. PS. co-tan. AP. co-sec. P} - \text{co-tan. P}$.

Again, in the triangles PSB, PZB,

co-t. PS : co-tan. PZ :: co-s. BPS : co-s. BPZ = tan. PS. co-t. PZ. co-s. BPS.

∴ And BPS ∓ BPZ = ZPS. *Q. E. D.*

Demonstration

Of the Rule for computing the Hour Angle from the Interval of Time between the Rising of two known Stars, Page 143.

Let P, fig. 34, represent the pole, S the first observed star when in the horizon, and A the situation of the other at the same instant. The angle APS is the difference of right ascension. Let a be the position of the second star at the time of rising. Then will the angle APa be the interval of time between the instants when the stars were observed in the horizon; and the angle aPS will be the difference between the above interval, and the difference of right ascension.

In the spherical triangle aPS, are given Pa, PS, and the angle aPS, to find PSa. Now, per spherics,

$\text{Tangent PS}a = \frac{\text{sine } a\text{PS}}{\text{sine PS. co-tan. P}a - \text{co-s. PS. co-s. }a\text{PS}}$;

And

And, in the right-angled spherical triangle SPO,

R : co-s. PS :: tan. PSa : co-tan. SPO = co-s. PS. tan. PSa =

$$= \frac{\text{sine } a\text{PS}}{\text{tan. PS co-t. P}a\text{—co-s. } a\text{PS.}}$$

And tang. SPO = tan. PS. co-tan. Pa. co-sec. aPS — co-tangent aPS.

Again, R : co-s. SPO :: tan. PS : tan. PO

Hence, tan. PO = co-s. SPO × tan. PS

= co-s. mer. dist. star. × co-tan. decl.

= latitude required. *Q. E. D.*

Demonstration

Of the Rule for computing the Apparent Time from three Altitudes of the same Object, with the Intervals of Time between the Observations, Page 146.

Let A, B, C, fig. 35, be the positions of the object at the times of observation; and AD, BF, CG, the sines of the altitudes. Describe the semicircle KMR, and draw AN, BM, CL, perpendicular to KR, which, will, therefore, be the sines of the times of observation from noon to the radius VK. Join LM, LN, MN, and let MS, LU, be drawn perpendicular to LN and BM respectively. Now, the angle KVL = VLU = time from noon, when the greatest altitude was observed; the angle LVN = interval of time between the extreme observations, and consequently VLN = complement of half that interval. And because the triangles MTS, LTU, are similar, therefore, the angle TMS = TLU.

In the right-angled triangle LSM, are given, the angle MLS = half the arch MN, and LM = twice the sine of half the arch LM, to find MS and LS.

R : sine MLS :: LM : MS :: $\frac{1}{2}$ LM : $\frac{1}{2}$ MS.

Hence $\frac{1}{2}$ MS = sine $\frac{1}{2}$ arch MN. sine $\frac{1}{2}$ arch LM

R : co-s. MLS : $\frac{1}{2}$ LM :: $\frac{1}{2}$ LS = co-s. $\frac{1}{2}$ arch MN. sine $\frac{1}{2}$ arch LM.

And because the lines CX, CA, and LN, are cut proportionally in the points I, B, and T,

Therefore, CX : CI :: LN : LT :: $\frac{1}{2}$ LN : $\frac{1}{2}$ LT.

$$\text{Hence, } \tfrac{1}{2}\text{ LT} = \frac{\text{CI. sine } \frac{1}{2} \text{ arch LMN}}{\text{CX}},$$

And $\frac{1}{2}$ LS — $\frac{1}{2}$ LT = $\frac{1}{2}$ TS.

Now, in the right-angled triangle TSM,

$\frac{1}{2}$ TS : $\frac{1}{2}$ MS :: R : co-tangent TMS = TLU;

$$\text{Therefore, co-tangent TLU} = \frac{\frac{1}{2}\text{ MS}}{\frac{1}{2}\text{ TS}} = \frac{\text{sine } \frac{1}{2} \text{ arch MN. sine } \frac{1}{2} \text{ arch LN}}{\frac{1}{2}\text{ TS}}$$

And TLV — TLU = VLU = KLV,

Or, MTS — $\frac{1}{2}$ LVN = KVL. *Q. E. D.*

Demon-

Demonstration

Of the Methods of reducing the Apparent to the True Distances.

Method First, *Page* 150.

Given the apparent distance *ms*, (fig. 24,) the apparent zenith distances *Zm*, *Zs*, and the true zenith distances ZM, ZS, to find the true distance MS.

$$\left.\begin{array}{l} \text{Sine } Zs.\text{sine } Zm : R^2 :: \text{v. sine } ms - \text{v. sine } \overline{Zs-Zm} : \text{v. sine P} \\ \text{Sine } Zs.\text{sine ZM} : R^2 :: \text{v. sine MS} - \text{v. sine } \overline{ZS-ZM} : \text{v. sine P} \end{array}\right\} \text{per sph.}$$

$$\text{Sine } Zs.\text{sine } Zm : \text{sine ZS. sine ZM} :: \text{v. sine } ms - \text{v. sine } \overline{Zs-Zm} : \text{v. sine MS} - \text{v. sine } \overline{ZS-ZM}$$

$$\text{V. sine MS} - \text{v. sine } \overline{ZS-ZM} = \overline{\text{v. sine } ms - \text{v. sine } \overline{Zs-Zm}} \times \frac{\text{sine ZS. sine ZM}}{\text{sine } Zs.\text{ sine } Zm}.$$

That is, $\text{v. sine MS} = \overline{\text{v. sine } ms - \text{v. sine } \overline{Zs-Zm}} \times \frac{\text{sine ZS. sine ZM}}{\text{sine } Zs.\text{ sine } Zm} + \text{versed sine } \overline{ZS-ZM}.$

But table of log. diff. $= \log. \frac{\text{sine ZS.sine ZM}}{\text{sine } Zs.\text{ sine } Zm}$, which call *d*, and let *a* be the number answering to $\log. \overline{\text{v. sine } ms - \overline{Zs-Zm}} + d$.

Then, versed sine $\text{MS} = \text{v. sine } \overline{\text{ZS}-\text{ZM}} + a$. *Q. E. D.*

Method Second, *Page* 153.

$$\left.\begin{array}{l} \overline{\text{Co-sine}}|^2\,\tfrac{1}{2}Z = \dfrac{\text{sine } \frac{1}{2}(\text{ZM}+\text{ZS}+\text{MS}).\text{sine } \frac{1}{2}(\text{ZM}+\text{ZS}-\text{MS})}{\text{sine ZM. sine ZS}} \\ = \dfrac{\text{sine } \frac{1}{2}(Zm+Zs+ms).\text{sine } \frac{1}{2}(Zm+Zs-ms)}{\text{sine } Zm.\text{ sine } Zs} \end{array}\right\} \text{per spherics.}$$

Now, let A, *a*, B, and *b*, be the complements of ZM, *Zm*, ZS, and *Zs*, respectively; also, let C and *c* represent MS and *ms*.

Then, $\frac{1}{2}(\text{ZM}+\text{ZS}+\text{MS}) = \frac{1}{2}(90-\text{A}+90-\text{B}+\text{C}) = 90-\frac{1}{2}(\text{A}+\text{B}-\text{C})$

$\frac{1}{2}(\text{ZM}+\text{ZS}-\text{MS}) = \frac{1}{2}(90-\text{A}+90-\text{B}-\text{C}) = 90-\frac{1}{2}(\text{A}+\text{B}+\text{C})$

$\frac{1}{2}(Zm+Zs+ms) = \frac{1}{2}(90-a+90-b+c) = 90-\frac{1}{2}(a+b-c)$

$\frac{1}{2}(Zm+Zs-ms) = \frac{1}{2}(90-a+90-b-c) = 90-\frac{1}{2}(a+b+c)$

Hence, by substitution,

$$\frac{\text{Co-sine } \frac{1}{2}(\text{A}+\text{B}-\text{C}).\text{ co-sine } \frac{1}{2}(\text{A}+\text{B}+\text{C})}{\text{sine ZM. sine ZS.}} = \frac{\text{Co-sine } \frac{1}{2}(a+b-c)\text{ co-sine } \frac{1}{2}(a+b+c).}{\text{sine } Zm.\text{ sine } Zs.}$$

Hence, $\text{co-sine } \frac{\text{A}+\text{B}-\text{C}}{2}.\text{ co-sine } \frac{\text{A}+\text{B}+\text{C}}{2}$

=co-

$$= \text{co-sine}, \frac{a+b-c}{2}. \times \text{co-sine}, \frac{a+b+c}{2}. \times \frac{\text{sine ZM}, \text{sine ZS},}{\text{sine } Zm. \text{ sine} Zs.}$$

Now, log. of, $\frac{\text{sine ZM.sine ZS}}{\text{sine } Zm. \text{sine } Zs}$, is contained in Tab. XLII. which call d;

Also, let $\frac{a+b-c}{2} = \text{D}$, $\frac{a+b+c}{2} = \text{E}$, and $\text{A}+\text{B}=\text{F}$;

Then, co-sine, $\frac{\text{F}-\text{C}}{2}$. co-sine $\frac{\text{F}+\text{C}}{2} =$ co-sine D. co-sine E. d.

But co-sine $\frac{\text{F}-\text{C}}{2}$. co-sine $\frac{\text{F}+\text{C}}{2} = \frac{1}{2} \times \overline{\text{co-s.F}+\text{co-s.C}}$, to radius 1.

Hence, $\frac{1}{2} \cdot \overline{\text{co-sine F}+\text{co-sine C}} =$ co-sine D. co-sine E. d.
Let co-sine D. co-sine E. $d = \text{sine}|^2 \frac{1}{2} n$.
Then, $\frac{1}{2} \times \overline{\text{co-sine F} + \text{co-sine C}} = \text{sine}|^2 \frac{1}{2} n$.
And co-sine F + co-sine C $= 2\ \text{sine}|^2 \frac{1}{2} n = 1 -$ co-sine n.
But co-sine C $= 1 - 2\ \text{sine}|^2 \frac{1}{2}$ C:
Hence, $1 - 2\ \text{sine}|^2 \frac{1}{2} \text{C} = 1 -$ co-sine $n -$ co-sine F;
Therefore, $\text{sine}|^2 \frac{1}{2} \text{C} = \frac{1}{2} \times \overline{\text{co-sine } n + \text{co-sine F}}$,

$$= \text{co-sine } \frac{n+\text{F}}{2}. \text{ co-sine } \frac{n-\text{F}.}{2}$$

Q. E. D.

METHOD FOURTH, *Page* 157.

Let the versed sine of the supplement of the sum of the apparent altitudes = A', that of the true altitudes = a: the versed sine of the difference of the apparent altitudes = B; and of the true altitudes = b; the versed sine of the apparent distance = D; and of the true distance = d.

Now, A — B : A — D :: $2r$: v. sine supp. angle at zen. } per trig.
And $a - b$: $a - d$ :: $2r$: v. sine supp. angle at zenith. }
Hence, A — B : A — D :: $a - b$: $a - d$.

$$\text{And } a - d = \frac{\overline{\text{A}-\text{D}} \times \overline{a-b}}{\text{A}-\text{B}}$$

Whence, $d = a - \frac{\overline{\text{A}-\text{D}} \times \overline{a-b}}{\text{A}-\text{B}} =$ true distance. *Q. E. D.*

METHOD SIXTH, *Page* 159.

Let sm, fig. 25, be the apparent distance, Sn the distance cleared of refraction, and SM the true distance. From Z draw ZD perpendicular to the distance, which let be bisected in E.

Then,

Then, $\tan.\frac{sm}{2} : \tan.\frac{Zs+Zm}{2} :: \tan.\frac{Zs-Zm}{2} : \tan.\frac{DE}{2}$, per spherics.

Hence, tan. arch first ($= \tan.\frac{DE}{2}$) = co-tan. half sum alt. tan. half diff. altitudes, co-tan. half distance, to rad. I.

And, half distance ± arch first = sD and Dm respectively = arches second and third.

Now in the right-angled triangles, sDZ, Sas.

Tangent Zs : tan. Ds :: R : co-sine s.

And R : co-s. s :: Ss : Sa.

Hence, Ex. eq. pertur. tan. Zs : tan. Ds :: Ss : Sa = $\frac{Ss. \tan. Ds.}{\tan. Zs.}$

But, Ss = 57″. tang. Zs, nearly;

Hence, Sa = 57″. tang. Ds;

And hence, also, nb = 57″. tang. Dm.

Sa and nb may, therefore, be taken from a table of refraction.

Again, in the triangles mDZ, M c n;

R : co-tan. Zm :: tan. Dm : co-s m = co-t. Zm. tan. Dm,

R : co-tan. Zm. tan. Dm :: Mn : nc = Ma. co-t. Zm. tan. Dm.

But Mn = hor. par., sine Zn to radius I. And as Zm may be assumed = Z n, therefore nc = hor. par. tan. Dn. co-sine Zn.

Hence, P. log. par. in distance = P. log. hor. par. + log. co-tan. Dn + log. co-secant altitude.

The last correction of parallax is found by means of Table XXII. or LXXII. It is the difference between Sc and SM, and may be computed by various methods The reason for entering the table twice will be obvious, from a due consideration of the triangles S c M, M c n.

Q. E. D.

Method Seventh, *Page* 161.

In the triangle Zsm, fig. 25, are given the three sides, to find the angle at each object. Thus, per trigonometry,

$$\text{co-s. } Zms = \frac{\text{co-s. } Zs - \text{co-s. } Zm \times \text{co-s. } ms}{\text{sine } Zm \times \text{sine } ms} =$$

$$= \text{co-s. } Zs \times \text{co-sec. } Zm \times \text{co-sec. } ms - \text{co-tan. } Zm \times \text{co-tan. } ms.$$

$$\text{And co-s.} Zsm = \text{co-s.} Zm \times \text{co-sec.} Zs \times \text{co-sec.} ms - \text{co-tan.} Zs \times \text{co-t.} ms.$$

Hence, first correction = corr. ☽'s alt. × co-sine Zms.

And second correction = corr. ✳'s alt. × co-sine Zsm.

The third correction is found as in the former method.

From an inspection of the figure it is evident, that the first correction is subtractive, and the second additive, when the angles at the Moon and at the star are acute; and the contrary, when these angles are obtuse. For the sake of rendering the operation more simple, the rule is adapted to proportional logarithms.

Demon.

Demonstration

Of the Rule for computing the true Altitude of the Sun, Moon, or a fixed Star, Page 188.

In the spherical triangle ZPS, fig. 31, are given PZ the co-latitude, PS the polar distance, and ZPS the horary angle, to find ZS the co-altitude. Then per spherics,

Sine PS. sine PZ^2 : R :: v-sine ZS — v-sine $\overline{\text{PS} - \text{PZ}}$: v-sine P.

Hence, v-sine ZS=v-sine $\overline{\text{PS} - \text{PZ}}$ + v-sine P. sine PS. sine PZ. to radius 1.

Now, $\overline{\text{PS} - \text{PZ}}$ = mer. zenith distance, and ZS = co-altitude.

Hence, co. v-sine alt. = v-sine, mer. zen. dist. + v-sine P. co-sine declination. co-sine latitude.

Or, because the difference of the versed sines is equal to that of the co-sines,

Therefore, sine alt. = co-sine mer. zen. dist. — v-sine P. co-sine declination. co-sine latitude.

And column of rising is a table of log. versed sines: hence, the rule is obvious. *Q. E. D.*

Demonstration

Of the Method of computing the Latitude, Longitude, and Time, from the same Set of Observations, Page 194.

Let P, fig. 36, represent the elevated pole, Z the zenith, A, B, *a*, *b*, the true and apparent places of the objects. Hence, ZA, ZB, Z*a*, Z*b*, the true and apparent zenith distances: AB, *ab*, the true and apparent distances, and PA, PB, their respective polar distances.

The demonstration of the method of reducing the apparent to the true distance, is given in page 302.

In the spherical triangle BAZ, given ZA, ZB, the true zenith distances, and AB the true distance of the objects. Required the angle ZAB.

Sine ZA. sine AB : R^2 :: v-sine BZ — v-sine $\overline{\text{AZ} - \text{AB}}$: v-sine ZAB.

Or, by using a table of natural sines,

R : co-sec. ZA. co-sec. AB :: co-sine $\overline{\text{AZ} - \text{AB}}$ — co-sine BZ : v-sine ZAB.

Log. co-secant ZA + log. co-sec. AB + log. co-sine $\overline{\text{AZ} - \text{AB} - \text{co-s. BZ}}$. — 2 log. R = log. v-sine ZAB.

Now, the co-sine of the difference of any two arches is equal to the sine of the sum of the less, and complement of the greater; and column of rising is a table of log. versed sines. Wherefore, log. secant alt. object farthest from mer. + log. co-secant distance + log. sine alt. obj. farthest from mer. + dist. — sine other objects alt. — 2 log. R =(log. versed sine angle ZAB =) log. rising of arch first.

In the spherical triangle PAB, are given PA, PB, the polar distances, and AB the true distance. Sought the angle PAB.

Sine AB. sine AP : R^2 :: v-sine BP — v-sine $\overline{AB - AP}$: v-sine PAB, per spherics.

Or, by using natural sines,

R^2 : co-sec. AB. co-sec. AP :: co-sine $\overline{AB - AP}$ $\mp$ co-sine BP : versed sine PAB.

R^2 : co-sec. dist. sec. dec. ob. farthest from mer. :: sine $\overline{\text{dist.} + \text{dec.}}$ $\mp$ sine other objects dec. : v-sine PAB.

Log. co-sec dist. + log. sec. dec. ob. farthest from mer. + log. sine $\overline{\text{dist.} + \text{dec.}}$ $\mp$ sine other objects dec. — 2 log. R. = log. rising of arch second.

In the spherical triangle PZA, given AZ, AP, the zenith and polar distances of the object farthest from the meridian, and the angle ZAP = BAP—BAZ, to find ZP the co-latitude, and the angle ZPA, the horary distance of the object farthest from the meridian.

Sine AZ. sine AP : R^2 :: v-sine ZP.—v-sine $\overline{AZ - AP}$: v-sine ZAP.

Now, the radius being 1, the above analogy may be expressed in form of an equation, as follows.

Ver-sine ZP—ver-sine $\overline{AZ - AP}$ = ver.-sine ZAP. sine AZ. sine AP.

Hence, v-sine ZP = v-sine $\overline{AZ - AP}$ + v-sine ZAP. sine AZ. sine AP.

That is, the natural co-versed sine of the latitude is equal to the sum of the natural versed sine of the difference between the zenith and polar distances of the object farthest from the meridian, and the natural number answering to the sum of the log. sines of these quantities, and the logarithmic rising of arch third from Table L.

Or, by substituting natural sines,

R^2 : sine AZ. sine AP :: v-sine ZAP : co-s. $\overline{AZ - AP}$ — co-s. ZP.

$$\frac{\text{Co-s. alt.. co-s. dec. v-sine ZAP}}{R} = \text{sine } \overline{\text{alt.} \pm \text{dec.}} - \text{sine latitude.}$$

$$\text{Sine lat.} = \text{sine } \overline{\text{alt.} \mp \text{dec.}} - \frac{\text{co-s. alt. co-s. dec. v-sine ZAP}}{R}.$$

Let natural number of log. co-sine alt. + log. co-sine dec. + log. co-sine ZAP — 2 log. R = a;

Then, nat. sine latitude = nat. sine $\overline{\text{alt} \mp \text{dec.}}$ — a.

Again, Sine ZP : sine ZA :: sine ZAP : sine ZPA,
Co s. lat. : co-s. alt. :: sine ZAP : sine ZPA,
Sec. lat. : sec. alt. :: co-sec. ZAP : co-sec. ZPA;

Log. co-sec. ZPA = log. co-sine lat. + log. sec. alt. + log. co-secant ZAP — 2 log. R.

But column of half elapsed time is a table of log. co-secants — log. R. Whence the operation is manifest. *Q. E. D.*

DEMON-

Demonstration

Of the Rule for computing the Difference of Longitude between the Moon and a fixed Star, or Planet, Page 204.

Let P, fig. 37, represent the pole of the ecliptic, PM the complement of the Moon's latitude, PS the complement of the star's latitude, MS the true distance between the Moon and star, and the angle MPS their difference of longitude, which is required.

Now, the polar distances being of the same affection,

$$\text{Co-sine } \tfrac{1}{2} P = \sqrt{\frac{\text{sine } \frac{1}{2}(PM + PS + MS).\text{sine } \frac{1}{2}(PM + PS - MS)}{\text{sine } PM.\ \text{sine } PS}}.$$

Let the Moon's latitude $= m$, the latitude of a star $= s$, and the distance $= d$.

$$\text{Then, } \tfrac{1}{2}(PM + PS + MS) = \tfrac{1}{2}(90 - m + 90 - s + d) = 90 - \tfrac{1}{2}(m + s - d$$
$$= 90 - \tfrac{1}{2}(m + s + d) - d.$$
$$\text{And, } \tfrac{1}{2}(PM + PS - MS) = \tfrac{1}{2}(90 - m + 90 - s - d) = 90 - \tfrac{1}{2}(m + s + d).$$

Hence,

$$\text{Co-s. } \tfrac{1}{2} P = \sqrt{\text{co-s. } \tfrac{1}{2}(m + s + d).\ \text{co-s. } \tfrac{1}{2}(m + s + d) - d.\ \text{sec. } m \text{ sec. } s}.$$

Again, let the polar distances be of a different affection,

$$\text{Sine } \tfrac{1}{2} P = \sqrt{\frac{\text{sine } \frac{1}{2}(MS + MP - PS).\ \text{sine } \frac{1}{2}(MS + PS - MS}{\text{sine } MP.\ \text{sine } PS}}.$$

Now, the same notation being used, we have

$$\tfrac{1}{2}(MS + MP - PS) = \tfrac{1}{2}(d + 90 - m - 90 + s) = \tfrac{1}{2}(d + s - m) = \tfrac{1}{2}$$
$$(d + s + m) - m.$$
$$\text{And } \tfrac{1}{2}(MS + PS - PM) = \tfrac{1}{2}(d + 90 +)\, s - 90 + m) = \tfrac{1}{2}(d + s + m).$$
$$\text{Hence, sine } \tfrac{1}{2} P = \sqrt{\text{sine} \tfrac{1}{2}(d + m + s).\text{sine} \tfrac{1}{2}(d + m + s) - m.\ \text{sec. } m.\text{sec. } s}.$$

Q. E. D.

Demonstration

Of the Rule for computing the Longitude, from an Observation of the Moon's Altitude, Page 223.

Let the Sun's right ascension $= s$, the Moon's $- m$, the Moon's relative motion in right ascension in 12 hours $= d$, Moon's horary angle $= h$, and the apparent time $= x$.

$$\text{Now, } 12\text{h.} : x \text{ h} :: d : \frac{dx}{12}$$

$$\text{And, } \overline{x + s} - m + \overline{\frac{dx}{12}} = h$$

$$x+s=h+m+\frac{dx}{12}$$

$$x-\frac{dx}{12}=h+m-s$$

$$12x-dx=\overline{h+m-s}\,.\,12$$

$$x=\frac{h+m-s}{12-d}\,.\,12=\frac{h+m-s}{180-d}\,.\,180^{\circ}.$$

Q. E. D.

Demonstration

Of the Rule for computing the Altitude and Longitude of the Nonagesimal.

Let the circle PEBQ, fig. 38, represent the solstitial colure, EQ the equator, ♋♑, the ecliptic, P,p their respective poles, ♈A the right ascension of the meridian PZAB, which passes through Z, the zenith of the place, whose latitude is AZ; hence, PZ is the co-latitude, Pp the distance of the poles, or the obliquity of the ecliptic; ZPp the right ascension of the meridian from ♑, ZP♋ the longitude of the nonagesimal from ♋; N the nonagesimal; hence, ZN is the zenith distance, and Zp the altitude of the nonagesimal.

Method First, *Page* 233.

In the right-angled spherical triangle PMZ, are given PZ the co-latitude of the place of observation, and the angle ZPM, the difference between the right ascension of the meridian and ♋, or ♑, to find PM.

R : co-sine ZPM :: tan. ZP : tan. PM $=\dfrac{\text{sine ZP♈. co-tan. latitude}}{\text{R}}$.

And PM ± Pp = Mp. The uppermost sign is to be used when the right ascension of the meridian is between o and XII. hours, and the other sign when the right ascension is between XII. and XXIV. hours.

Again, in the right-angled spherical triangles MPS, MpZ, sine MP : sine Mp : : co-tan. MPZ : co-tan. MpZ;

Hence, tan long nonagesimal $=\dfrac{\text{tan.♈PZ. co-secant MP. sine Mp}}{\text{R}^2}$

R : secant MpZ :: tan. Mp : tan. pZ,

Whence, tan. alt. nonagesimal $=\dfrac{\text{co-sec. ♈ pZ. tan. Mp}}{\text{R}}$. *Q. E. D.*

Method

METHOD SECOND, *Page* 235.

In the spherical triangle ZpP are given, ZP the co-latitude, Pp the obliquity of the ecliptic, and the angle pPZ the right ascension of the meridian from ♑, to find Zp the altitude, and the angle ♈ pN the longitude of the nonagesimal.

R^2 : sine PZ. sine Pp :: v-sine ZPp : v-sine Zp — v-sine $\overline{PZ - Pp}$

v-sine ZPp : co-sine $\overline{PZ - Pp}$ — co-s. Zp.

Now, the complement of the difference of two arches is equal to the sum of the least arch, and complement of the greater; therefore, as the latitude is assumed less than 66½ degrees;

R^2 : sine PZ. sine Pp :: v-sine ZPp : sine $\overline{Pp + \text{comp. } PZ}$ — co-s. Zp

R^2. sine $\overline{Pp + \text{comp. } PZ}$ — co-sine Zp = sine PZ.sine Pp. v-sine Zpp

$$\text{Hence, sine } \overline{AZ + Pp} - \text{co-sine } Zp = \frac{\text{co-sine } AZ.\ \text{sine } Pp\ \text{v-sine } ZPp}{R^2}$$

$$\text{And, co-sine } Zp = \text{sine } \overline{AZ + Pp} - \frac{\text{co-sine } AZ.\ \text{sine } Pp.\ \text{v.sine } ZPp.}{R^2}$$

Now, column of rising is a table of logarithmic versed sines.

Let the nat. N°. to log. co-sine AZ + log. sine Pp + log. rising ZPp — 2 log. R = a,

Then, co-sine altitude of nonagesimal = sine $\overline{AZ + Pp}$ — a.

Again, sine Zp : sine ZP :: sine ZPp : sine ZpP,

Therefore, co-sec. Zp : co-sec. ZP :: co-sec. ZPp : sec. ♈ pZ,

Hence, sec. ♈ pZ = co-sec. ZPp. co-sec. ZP. sine Zp, to radius 1.

But column of half elapsed time is a table of log. co-secants.

Therefore, log. secant long. nonagesimal = log. H, E, T, ZPp + log. co-sec. ZP + log. sine ZP. to radius 1. *Q. E. D.*

DEMONSTRATION

Of the Rule for computing the Parallax in Latitude and Longitude, Page 236.

Let ZNHEP, fig. 39, represent the meridian passing through the nonagesimal N, —NE the ecliptic, P its pole, Z the zenith, L the place of an object unaffected by parallax, and M its apparent place, both being in the same vertical ZLMn. Through L, M, draw the circles of latitude PLp, PMp, and through M draw the parallel of latitude MA; then will the angle APM be the parallax in longitude, and AL the parallax in latitude. Also ZP = NH = altitude of the nonagesimal; MPZ the apparent distance of the planet from the nonagesimal, and MP its apparent polar distance.

Now,

Now, in the right-angled triangle ALM, which may be considered as rectilineal,

R : co-sine AML :: ML : AM = ML. co-sine AML.

But co-sine AML = sine ZMP = $\frac{\text{sine P. sine ZP}}{\text{sine ZM}}$.

And ML = hor. parallax. sine ZM.
Hence, AM = hor. par. sine P. sine ZP.

And angle APM = $\frac{\text{hor par. sine P. sine ZP}}{\text{sine MP}}$.

That is, par. in lon = hor. par. sine dist. from non. sine alt. non. sec. lat.

Again, R : co-tang. ZMP :: AM : AL = AM. co-tangent ZMP.
But AM = hor. par. sine P. sine ZP.
Hence, A = hor. par. sine P. sine ZP. co-tangent ZMP.

But co-tangent ZMP = $\frac{\text{co-tan. ZP. sine MP.} - \text{co-s. P. co-sine MP}}{\text{sine P.}}$.

Whence, by substitution, we obtain
AL = hor. par. sine ZP. co-tan. ZP. sine MP — hor. par. sine ZP. co.sine P. co-sine MP.
= hor. par co-sine ZP. sine MP – hor. par. sine ZP. co-sine P co-sine MP.

That is, par. in lat. = hor. par. co-sine alt. non. co-sine lat. — hor. par. sine alt. non. co-sine dist. of object from non. sine lat. object.

Q. E. D.

Demonstration

Of the Rule for computing the Longitude of a Place by an Observation of a Solar Eclipse, or an Occultation of a Fixed Star by the Moon, Pages 239, 250.

Let CE, fig. 40, represent a small portion of the ecliptic, S a star, or the Sun's center*, M the apparent place of the Moon at the immersion, and L its apparent place at the emersion. Now the parallaxes in longitude at the immersion and emersion being applied to the Moon's relative motion in longitude in the interval between the observations, will give the apparent difference of longitude EC, which multiplied by the co sine of the Moon's apparent latitude, gives MN. The apparent difference of latitude LN is found, by applying the parallaxes in latitude to the computed change of latitude in the above interval.

In the right-angled triangle LNM, which may be considered as rectilineal, are given the sides MN, NL, to find the apparent inclination NML, and the apparent motion of the Moon in its relative orbit ML.

MN : NL :: R : tangent LMN = $\frac{\text{NL}}{\text{MN}}$.

R : secant LMN :: MN : ML = MN. secant LMN.

* But in this case BD will be a portion of the ecliptic.

Again,

Again, in the triangle MSL, the augmented semidiameters of the Moon (or sum of semidiameters of the Sun and Moon), MS, LS, and the Moon's relative motion ML being given, to find the angles SML, SLM; to which, the angle PLM = LMN being applied, the angles DSM, BSL, will be obtained.

Lastly, in the right-angled triangle MDS are given SM, and the angle DSM, to find DS; and hence AC, which being reduced to time, will be the interval between the immersion and conjunction. The interval between the emersion and conjunction may, in like manner, be inferred from the triangle SBL; and if no error be committed, the times of conjunction will agree. Now, the time of conjunction, thus found, being compared with that deduced from the Nautical Almanac, will give the longitude of the place of observation.

Q. E. D.

Remark.

If the time of conjunction be inferred from observations made at another place, then the difference between the times of conjunction at the two places will be the difference of their meridians in time; and, hence, the longitude of the other place being known, that of the given place will be more accurately determined, than by a comparison with the Nautical Almanac.

BOOK VI.

CONTAINING

Various Methods of finding the Latitude of a Place, and the Variation of the Compass.

CHAP. I.

Of finding the Latitude of a Place.

INTRODUCTION.

THE situation of a place, with respect to the equator, was anciently determined, by ascertaining the length of the longest day, and by the comparative length of the shadow of a *Gnomon* at that time. This instrument was afterwards used in many astronomical observations, such as for determining the obliquity of the ecliptic, the times of the tropics and equinoxes, the length of the year and seasons, &c. The method of reckoning the latitude in *degrees* and *minutes* being introduced, instruments for observing altitudes were divided accordingly.—The *Astrolabe*, (a circular ring, having a moveable index and sights,) was applied to observe altitudes at sea. It was, however, supplanted by the *Cross Staff*, and that again by the *Quadrants* of Davis and Hadley, in succession.

The most simple, and at the same time the most accurate method of determining the latitude of a place, is by an observation of the meridian altitude of the Sun, or of any other of the celestial bodies; and, therefore, such observations should not be neglected, when possible to be observed. It, however, frequently happens, that the meridian altitude cannot be observed, by reason of clouds, fog, &c.; or

or that the declination of the object is unknown. In these cases, therefore, recourse must be had to other methods.

Problem I.

Given the Sun's Meridian Altitude, to find the Latitude of the Place of Observation.

Rule.

Find the true altitude of the Sun's center, by Prob. x. page 104. Call it S or N, according as the Sun is south or north at the time of observation, which, subtracted from 90°, will give the zenith distance of a contrary denomination.

Reduce the Sun's declination to the meridian of the place of observation, by Prob. v. page 99. Then, the sum or difference of the zenith distance and declination, according as they are of the same, or of a contrary denomination, will be the latitude of the place of observation, of the same name with the greater*.

Examples.

I.

October 17, 1810, in longitude 32° E. the meridian altitude of the Sun's lower limb was 48° 53′ S. height of the eye 18 feet. Required the latitude?

Obs. alt. ☉'s low. limb	48° 53′ S.		☉'s dec. Oct. 17, noon	9° 7′ S.
Semidiameter - -	+ 16		Equation, Table XIII. - -	2
Dip and refraction	— 5			
			Reduced declination -	9 5 S.
True alt. Sun's center	49 4 S.		Zenith distance - -	40 56 N.
			Latitude - - -	31 51 N.

II.

November 6, 1811, in longitude 158° W. the meridian altitude of the Sun's lower limb was 87° 37′ N. height of the eye 12 feet. Required the latitude?

* A method very often practised at sea, especially by coasters, is, to correct the observed altitude, by adding 12′; and from thence, and the declination, the latitude is to be found. Or, subtract the altitude from 89° 48′, and the declination applied to the remainder will be the latitude. These methods may no doubt give the latitude tolerably exact in some cases, but in others, the mariner will be deceived above half a degree; and in any case they are seldom free of error.

Mer. alt. Sun's l. limb	87° 37′ N.	⊙'s dec. p. N. Alm.	15° 48′.8S.
Semidiameter -	+ 16.2	Equat. Tab. XIII.	+ 7.6
Dip and refraction	— 3.3		
		Reduced declination	15 56 .4S.
True altitude - -	87 49.9 N.	Zenith distance -	2 10 .1S.
		Latitude - -	18 6.5 S.

Remark.

The dip of the horizon, in Table III. answers to a free or unobstructed horizon; but if the land intervenes, and the ship at no great distance therefrom, the dip will be considerably greater, and that in proportion to the nearness of the ship to the land. This dip may be found as follows:

Let two persons observe the Sun's altitude at the same instant, the one being as near the mast head as possible, and the other on deck immediately under. Then to the log. of the sum of the heights of the observer above the sea, add the ar: co. of the log. of their difference, and the log. sine of the difference of altitude; the sum will be the log. sine of an arch. Now, half the sum of this arch, and the difference of altitude, will be the dip answering to the greatest height; and half their difference will be that corresponding to the height of the lowest observer.

If the distance of the ship from the land is known, the dip may found in Table IV.

This remark, in a similar case, is to be applied to the following problems.

Example.

Being close in with the land, September 28, 1812, a person on deck, 16 feet above the water, observed the meridian altitude of the Sun's lower limb to be 29° 51′ S. and another observer on the cross trees 68 feet above the sea, found the altitude at the same instant to be 30° 8′ S. Required the latitude?

Height - -	68 feet		
- - - - - -	16		
Sum - - - -	84	- - - log. - -	1.92428
Difference -	52	- - ar. co. log. - -	8.28400
Diff. of alt.	0° 13′	- - - sine - -	7.57767
Arch - -	0 21	- - - sine - -	7.78595
Sum - -	0 34	Half = 17′ = dip to the greatest altitude.	
Difference	0 8	Half = 4 = dip to the least altitude.	

Altitude

Altitude - - -	30° 8′ S.
Sun's semidiameter -	+ 16
Dip - - - - -	— 17
Refraction - - -	— 2
True altitude - -	30 5 S.
Zenith distance - -	59 55 N.
Declination - -	2 14 S.
Latitude - - -	57 51 N.

Problem II.

Given the Meridian Altitude of a Fixed Star, to find the Latitude of the Place of Observation.

Rule.

Reduce the observed to the true altitude, by Prob. ix. page 103, and find the star's zenith distance. Take the declination of the star from Table lii, and reduce it to the time of observation. Now, the sum or difference of the zenith distance and declination, according as they are of the same, or of a contrary name, will be the latitude of the place of observation.

Example.

December 1, 1812, the meridian altitude of Sirius was 59° 50′ S. height of the eye 14 feet. Required the latitude?

Observed altitude of Sirius - -	59° 50′ S.
Dip and refraction - -	—′ 4
True altitude - - -	59 46 S.
Zenith distance - - -	30 14 N.
Declination of Sirius -	16 28 S.
Latitude - - - - -	13 46 N.

Problem III.

Given the Meridian Altitude of a Planet, to find the Latitude of the Place of Observation.

Rule.

Compute the true altitude of the planet, as directed in last problem*. Take its declination from the Nautical Almanac, and reduce it

* Being sufficiently accurate for correcting altitudes observed at sea.

it to the time of observation. Then the sum or difference of the zenith distance and declination of the planet will be the latitude as before.

Example.

April 25, 1812, the meridian altitude of Saturn was 68° 42′ N. and height of the eye 15 feet. Required the latitude?

Observed altitude of Saturn -	68° 42′ N.
Dip and refraction - -	— 4
True altitude - - -	68 38 N.
Zenith distance - - -	21 22 S.
Declination - - - -	22 21 S.
Latitude - - - -	43 43 S.

Problem IV.

Given the Meridian Altitude of the Moon, to find the Latitude of the Place of Observation.

Rule.

Take the number from Table xx. answering to the given longitude, and daily variation of the Moon's passing the meridian; which being applied to the time of transit at Greenwich, by addition or subtraction, according as the longitude is west or east, will give the time of passage over the meridian of the ship.

Reduce this time to the meridian of Greenwich, and find the Moon's declination answering thereto, by Prob. vii. page 102, and the horizontal parallax and semidiameter, by Prob. viii.

Correct the observed altitude of the Moon's limb, by Prob. xi. page 105. Hence, the Moon's zenith distance will be known; the sum or difference of which, and the declination, will be the latitude of the place, as before.

Example.

June 21, 1812, in longitude 30° W. the meridian altitude of the Moon's lower limb was 81° 15′ N. height of the eye 16 feet. Required the latitude?

T. pas. over mer. Greenw. =	9h 38′	☽'s dec. at midn. —	14° 34 S.
Equation, Table xx.	+ 4	Eq. to time from midn. —	2
T. pas. over mer. ship -	9 42	Reduced dec. - -	14 32 S.

Longitude

Time pass. over mer. ship	9h 42′		
Longitude in time -	2 0	Moon's hor. par. -	55° 26′
		Moon's semidiameter	15 6
Reduced time - -	11 42	Augmentation -	+ 14
		Augmented semid.	15 20

Observed alt. Moon's lower limb =	81° 15′ N.
Augmented semidiameter -	+ 15
Dip - - - - -	— 4
Apparent alt. Moon's center -	81 26 N.
Correction, Table IX. -	+ 8
True altitude - - -	81 34 N.
Zenith distance - - -	8 26 S.
Declination - - - -	14 32 S.
Latitude - - - -	22 58 S.

Remark.

If the object, at the time of observation, is in the opposite meridian, then the sum of the true altitude, and the complement of the declination, will be the latitude. If the altitude is negative, it must be subtracted from the polar distance, in order to obtain the latitude.

Examples.

I.

July 1, 1812, in longitude 15° W. the meridian altitude of the Sun's lower limb was 8° 58′, height of the eye 18 feet. Required the latitude?

Sun's dec. at noon =	23° 7′.6 N.	Ob. alt. ☉'s l. limb =	8° 58′.0
Eq. Table XIII. to 15°	— .2	Semidiameter -	+ 15.8
- - to 12h. -	— 2.2	Dip and refraction	— 9.8
Reduced dec. - -	23 5.2	True altitude - -	9 4.0
Polar distance - -	66 54.8	- - - - -	66 54.8
		Latitude - -	75 58.8 N.

II.

May 18, 1804, in longitude 20° E. at midnight, the Sun's lower limb was observed to be in contract with the horizon, height of the eye 12 feet. Required the latitude?

Sun's

Sun's dec. at noon =	19° 35′	Observed alt. -	0° 0′
Eq. Table XIII. to 20° E. —	1	Semidiameter - -	+ 16
- - to 12h. - .	+ 6	Dip and refraction -	— 36
Reduced declination -	19 40	Depression - -	— 20
Polar distance -	70 20	- - - - -	70 20
		Latitude - - .	70 0 N.

Problem V.

Given equal Altitude of the Sun observed the same Day, with the Interval of Time between the Observations, to find the Latitude.

Method First.

Without a supposed Latitude.

Rule.

To the co-tangent of the declination, add the co-sine of half the interval in degrees, the sum will be the co-tangent of *arch first.*

To the co-secant of the declination, add the sine of the altitude, and the sine of arch first, the sum will be the co-sine of *arch second.*

If the latitude and declination are of the same name, the sum of arches first and second will be the latitude; but if of contrary names, their difference will the latitude.

Example.

In north latitude, at 11h. 10′, and at 12h. 40′ per watch, the altitude of the Sun's limb was the same; which being corrected, was 26° 55′, and the Sun's declination 5° 17′ S. Required the latitude?

Declination -	5° 17′	co-tan.	1·03398	- co-secant	1.03583
Half inter. 45 =	11 15	co-sine	9.99157	alt. 26° 55′. sine	9.65580
Arch first -	5 23	co-tan.	1.02555	- - sine	8.97229
Arch second -	62 32	- -		co-sine - -	9.66392
Latitude -	57 9 N.				

Method

Method Second.

The Latitude by Account being given.

Rule.

Take the log. answering to half the observed interval from column of rising, to which add the log. co-sines of the declination, and latitude by account. Find the natural number answering to the above sum; which being subtracted from the natural co-ver. sine of the corrected altitude, will give the natural ver. sine of the meridian zenith distance, and hence the latitude is to be found as formerly.

Example.

June 12, 1810, in latitude 42° 50′ N. by account, at 11h. 30′ A. M. per watch, the altitude of the Sun's lower limb was 69° 3′, and at 12h. 31′. the Sun had the same altitude; height of the eye 24 feet. Required the latitude?

Time per watch -	11h	32′		Observed altitude - -	69° 3′
- - - - -	12	31		Semid.—dip and refr.	+ 11
Elapsed time -	1	1		True altitude - -	69 14
Half elapsed time -	0	30′	30″	- - rising - -	2.94656
Declination - -	23	8		- - co-sine - -	9.96360
Latitude by account	42	50		- - co-sine - -	9.86530
Altitude - -	69	14		Nat. co-ver. sine = 06497	
				Nat. num. - - 596 -	2.77546
Mer. zen. distance -	19	47		Nat. ver. sine - 05901	
Declination - -	23	8			
Latitude - - -	42	55 N.			

If the ship makes any considerable run in the interval between the observations, the change of altitude, in consequence of that run, is to be estimated, and the index of the quadrant altered accordingly. The observed interval is also to be corrected by the change of longitude. See Prob. VIII. The half interval should also be corrected by the equation of equal altitudes, page 109.

Problem VI.

Given two Altitudes of the Sun, and the Apparent Times of Observation, to find the Latitude and Declination.

Rule.

To the ar. co. of the log. of the difference of the natural versed sines of the times from noon reduced to degrees, add the log. of the versed sine

sine of the greatest interval, and the log. of the difference of the natural sines of the corrected altitudes, the sum will be the log. of arch first.

To the constant log. 5.30103, add the two last-mentioned logs. the sum will be the log. of arch second

The sum of arch first, and the natural sine of the least altitude, will be the natural co-sine of arch third; and this natural co-sine being subtracted from arch second, leaves the natural co-sine of arch fourth.

Now, half the sum and half the difference of arches third and fourth, will be the latitude and declination respectively.

Remark.

The apparent time of each observation may be found, by taking equal altitudes, and correcting the half interval, by the equation answering thereto.

Example.

At 9h. 23′ 20′, A. M. apparent time, the true altitude of the Sun's center was 34° 29′: and at 11h. 9′ 32″, the altitude was 42° 19′. Required the latitude and declination?

T. fr. n.	39° 10′	N. V. S.	22469	log. = 4.35158	con. log.	5.30103
-	12 37	- -	02415			
		Differ. -	20054	ar.co.log. 5.69780	- -	5.69780
Alt. -	42 19	N. sine	67323			
- -	34 29	- -	56617			
		Differ. -	10706	log. 4.02963	- -	4.02963
Arch 1st	- -	-	11995	- - 4.07901	ar. 2d. 106772	5.02846
Least alt.	34 29	N. sine	56617			
Arch 3d	46 40	N. co-s.	68612	- - -	68612	
Arch 4th	67 34	-	Natural co-sine	-	38160	

Sum - 114 14 half = 57° 7′ N. = latitude
Differ. 20 54 half = 10 27 N. = declination.

Suppose the above observation to have been made some time between June and December, in the year 1812; then the time, answering to this declination, is August 26th.

Problem

Problem VII.

Given the Latitude by Account, the Declination, and two observed Altitudes of the Sun, and the Time elapsed between them, to find the true Latitude *.

Rule.

To the log. secant of the latitude by account, add the log. secant of the declination; the sum, rejecting 20 from the index, is the *log. ratio.* To this add the log. of the difference of the natural sines† of the two altitudes, and the logarithm of the half elasped time from Table XLVIII.

Find this sum in table of middle time, and take out the corresponding time, the difference between which and the half elapsed time will be the time from noon, when the greatest altitude was observed ‡.

Take the log. answering to this time from table of rising, from which subtract the log. ratio; the remainder is the logarithm of a natural number, which being added to the natural sine of the greater altitude, the sum is the natural co-sine of the meridian zenith distance; from which, and the Sun's declination, the latitude is obtained as formerly ||.

If

* The method of finding the latitude by two altitudes of the Sun and elapsed time, was proposed by Mr. Robert Hues, in his Treatise on the Globes, which was first published in the year 1594, and solved by him upon the globe. This problem was resolved, by projection, by Mr. John Collins, in the third part of his book entitled *The Mariner's Scale new plain'd*, printed in 1659, page 35; and it is performed, in a direct manner, by calculation, in Leadbetter's *Astronomy of the Satellites of Jupiter*, printed in 1729, page 32; and since by many succeeding writers. And in the Treatise on the Use of the Sliding Gunter, by the author of this work, the method of resolving this problem by that instrument is given. The present solution is that which was given in the *Actes de l'Académie de Haarlem* for 1754, by M. Cornelius Douwes. The investigation of this method is given in the *Phil. Transactions* for 1760, by Dr. Pemberton; in the *Connoissance des Temps*, for 1793; and *Phil. Transactions* for 1797, by Don Joseph de Mendoza y Rios; and by several other writers upon navigation.

† The table of nat. versed sines may be used in place of that of nat. sines. In that case the log. of the difference of nat. co-versed sines of the altitudes is to be taken; and the natural number answering to the excess of the log. rising above the log. ratio, being subtracted from the nat. co-versed sine of the greatest altitude, will give the nat. versed of the meridian zenith distance.

‡ The sum of the half elapsed time and middle time may be taken. In this case, the natural number is to be added to the natural sine of the least altitude, to find the natural co-sine of the meridian zenith distance.

|| This method is only an approximation, and requires to be used under certain restrictions, namely:

The observations must be taken between nine o'clock in the forenoon, and three in the afternoon. If both observations be in the forenoon, or both in the afternoon, the interval must not be less than the distance of the observation of the greatest altitude from noon. If one observation be in the forenoon, and the other in the afternoon, the interval must not exceed four hours and a half; and in all cases, the nearer the greater altitude is to noon the better.

If the Sun's meridian zenith distance be less than the latitude, the limitations are still more

If the latitude thus found differs considerably from that by account, the operation is to be repeated, using the computed latitude in place of that by account, until the latitude last found agrees nearly with that used in the computation.

Examples.

I.

June 4, 1804, in latitude by account 37° N. at 10h. 29′ forenoon per watch, the corrected altitude of the Sun was 65° 24′, and at 12h. 31′, the altitude was 74° 8′. Required the true latitude?

T. p. watch.	Alt. ⊙'s cent.	Nat. sine.			
			Lat. by ac.	37° 0′	sec. 0.09765
10h 29′	65° 24′	90924	Declinat.	22 27	sec. 0.03423
12 31	74 8	96190			
			Logarithm ratio	-	0.13188
2 2	Difference	5266	- - log.	-	3.72148
1 1	- - -	Half elapsed time		-	0,57999
31 10	-	Middle time	- -	-	4.43335
29 50	- - -	Rising	- -	-	2.92740
		Log. ratio	-	-	0.13188
Natural number	- -	624	- -	-	2.79552

Greatest altitude	74 8	N. sine	96190
Mer. zenith dist.	14 30	N. co-s.	96814
Declination -	22 27		
Latitude -	36 57 N.		

Remarks,

I.

The difference between the half elapsed time and the middle time, is the time from noon when the greatest altitude was observed; if, therefore, that altitude had been observed in the afternoon, the difference between the time per watch of that observation, and the rising, would be the error of the watch; but if the greatest altitude had been observed in the forenoon, subtract the rising from twelve hours, then, the difference between this time, and that, when the greatest altitude was observed, will be the error of the watch.

more contracted. If the latitude be double the meridian zenith distance, the observations must be taken between half past nine in the forenoon, and half past two in the afternoon, and the interval must not exceed three hours and a half. The observations must be taken still nearer to noon, if the latitude exceeds the meridian zenith distance in a greater proportion. See Req. Tab. 2d ed. and Brit. Mar. Guide.

Thus,

Thus, in the preceding example, the greatest altitude was observed.

Per watch at	- -	31′ 0″
Rising	- - -	29 50
Watch fast	- -	1 10

II.

If the greatest altitude be observed near noon, the latitude to be used in the computation, instead of that by account, may be deduced nearly, according to circumstances, from the above altitude and declination, particularly if no observations have been made for some days. This will be exemplified in the following

EXAMPLE.

II.

October 20, 1810, at 0h 20′ per watch, the corrected altitude of the Sun's center was 36° 26′ towards the South, and at 2h. 50′ the true altitude was 24° 34′. Required the true latitude?

Greatest altitude	- - -	36° 26′ S.
Zenith distance	- - -	53 34 N.
Declination	- - - -	10 13 S.
Latitude to be assumed	- -	43 21 N.

Time per watch.	Alt.	Nat. sine.			
			Lat. assumed.	43° 21′ sec.	0.13836
0h 20′	36° 26′	59389	Declination	10 13 sec.	0.00694
2 50	24 34	41575	- Log. ratio	-	0.14530
1 15	- -	17841	- Log.	- -	4.25076
2 30	- -	-	Half elapsed time	-	0.49290
1 31 10	- -	-	Middle time	- -	4.88896
0 16 10	- -	-	Rising	- - -	2.39567
Natural number	-	178	- - -	-	2.25037
Nat. co-s. m. z. dist. -		59567	.	53° 26′ N.	
	Declination		-	10 13 S.	
	Latitude		- -	43 13 N.	

REMARK.

In this example the log. ratio is subtracted from the log. rising without placing it underneath; and the natural number answering thereto, is placed immediately under the natural sines, and added

 to

to the greatest, without taking it down; by this means the work is contracted.

Example.

III.

October 18, 1804, in latitude 49° 48′ N. by account, at 0h. 32′ per watch, the altitude of the Sun's lower limb was 28° 32′, and at 2h. 41′ it was 19° 25′, height of the eye 12 feet. Required the true latitude;

First observed altitude -	28° 32′	Second altitude -	19° 25′
Semidiameter - -	+ 16	Semidiameter -	+ 16
Dip and refraction -	— 5	Dip and refraction -	— 6
True altitude - -	28 43	True altitude -	19 35

T. p. watch.	Alt. ☉'s cent.	Nat. sine.	Lat. by ac. 49° 48′ sec.	0.19013
0h 32′ -	28° 43′ -	48048	Declination 9 39 sec.	0.00619
2 41 -	19 35 -	33518		
			Log. ratio - -	0.19632
2 9 -	Diff.	14530	Log. - - -	4.16227
1 4 30 -	-		Half elapsed time - -	0.55637
1 37 0 -	-		Middle time - - -	4.91496
32 30 -	-		Rising - - -	3.00164
			Log. ratio - - -	0.19632
Natural number - -	- -	639	- -	2.80532

Greatest altitude -	38 43	Nat. sine	48048
Mer. zenith dist. -	60 52	Nat. co-s.	48687
Declination -	9 39		
Latitude - -	51 13		

As the latitude, by computation, differs 1° 25′ from that by account, the operation must be repeated.

Computed latitude -	51° 13′	secant	0.20316
Declination - -	9 39	secant	0.00619
Logarithm ratio - - -	-	- -	0.20935
Difference nat. sines -	14530	- log. -	4.16227
Half elapsed time -	1h 4′ 30″	- log. -	0.55637
Middle time - -	1 40 10	- log. -	4.92799
Rising - - -	35 40	- log. -	3.08225
Logarithm ratio - -	- -	-	0 20935
Natural number - -	745	- -	2.87190

Natural

Natural number - - -				745
Greatest alt. -	28°	43′	N. sine	48048
Mer. zen. dist. -	60	48	N. co-s.	48793
Declination -	9	39	S.	
Latitude - -	51	9	N.	

As this latitude differs only 4′ from that used in the computation, it may, therefore, be depended on as the true latitude.

PROBLEM VIII.

Given the Latitude by Account, the Declination, and two observed Altitudes of the Sun; the elapsed Time, and the Course and Distance run between the Observations, to find the Ship's Latitude at the Time of Observation of the greatest Altitude.

RULE.

Find the angle contained between the ship's course and the Sun's bearing, at the time of observation of the least altitude, with which enter a traverse table; and the difference of latitude, answering to the distance made good, will be the *reduction* of altitude.

Now, if the least altitude be observed in the forenoon, the reduction of altitude is to be added thereto, if the angle between the ship's course and Sun's bearing is less then eight points; but if that angle is greater than eight points, the reduction is to be subtracted from the least altitude. If the least altitude be observed in the afternoon, the reduction is to be subtracted therefrom, if the angle between the ship's course and the Sun's bearing is less than eight points; but if greater, the reduction is to be added to the least altitude.

The difference of longitude in time between the observations is also to be applied to the elapsed time, by addition or subtraction, according as it is east or west. This is, however, in many cases so inconsiderable as to be neglected.

With the corrected altitudes, and the interval of time between the observations, the lat. by D. R. and declination at the time of observation of the greatest altitude, the computation is to be performed by the last problem.

EXAMPLES.

I.

June 6, 1804, in latitude 58° 14′ N. by D. R. at 10h. 54′ A. M. per watch, the altitude of the Sun's lower limb was 53° 17′; and at 1h.

1h. 17′, the altitude was 52° 51′, and bearing per compass S. W. by W. The ship's course during the elapsed time was S. by W. ½ W. and hourly rate of sailing 8 knots, the height of the eye 16 feet. Required the true latitude at the time of observation of the greater altitude?

☉'s bearing at 2d obs.	S. W. by W	Interval betw. observ.	2h 23′
Ship's course -	S. by W. ½ W.	Distance run = 2 23 = 8 = 19m.	
Contained angle - -	3½ points.		

Now, to course 3½ points and distance 19 miles, the difference of latitude is 14.7 or 15 miles.

First observed alt. =	53° 17′	Second obser. altitude	52° 51′
Semidiameter -	+ 16	Semidiameter -	+ 16
Dip and refraction -	— 4	Dip and refraction -	— 4
		Reduction - - -	— 15
True altitude - -	53 29		
		Reduced altitude -	52 48

Time per watch.	Altitude.	Nat. sines.	Lat. by ac. 58° 14′ sec.	0.27863
10h 54′ -	53° 29′ -	80368	Declinat. 22 40 sec.	0.03491
1 17 -	52 48 -	79653		
			Logarithm ratio -	0.31354
2 23*	Difference -	715	- - - log - -	2.85431
1 11 30 - - -			Half elapsed time - -	0.51294
5 30 - - - - -			Middle time - -	3.68079
1 6 0 - - - -			Rising - - -	3.61469
			Log. ratio - - -	0.31354
Natural number - - - -		2001	-	3.30115
Greatest altitude -	53 29 Nat. sine	80368		
Mer. zenith distance -	34 33 N. co-sine	82369		
Declination - - -	22 40			
Latitude - - -	57 13			

Since the computed latitude differs so much from that by account, it will be necessary to repeat the operation.

Computed latitude - -	57° 13′	- secant -	0.26643
Declination - - -	22 40	- secant -	0.03491
Logarithm ratio - - - - - -			0.30134

* The correction of the elapsed time is here neglected. It, however, may be found thus. The variation inferred from the observations is about 2¼ points W. Hence the true course is S¾E, to which and the distance 19m. the departure is 2′. 8, and to latitude 57°, and 2′,8 in a lat. column, the distance is 5′, which reduced to time is 20″ additive, because the difference of longitude is east.

Logarithm

Logarithm ratio	- - -		- - -		0.30134
Difference natural sines	- -	715	- -	log. - -	2.85431
Half elapsed time	- -	1 11 30	- -	log. - -	0.51294
Middle time	- - -	5 20	- -	log. - -	3.66859
Rising	- - -	1 6 10	- -	log. - -	3.61686
Logarithm ratio	- - -		- - -		0.30134
Natural number	- - -	2068		- -	3.31552
Greatest altitude	-	53 29 N. sine 80368			

Mer. zenith dist.	-	34 29 N. co-s. 82436
Declination	- -	22 40
Latitude	- - -	57 9

As this latitude differs only 4 miles from that used in the computation, it may, therefore, be depended on as the true latitude.

II.

September 14, 1814, in latitude 38° 12′ N. by account, and longitude 27° W. at 9h. 28′ A. M. per watch, the altitude of the Sun's lower limb, was 40° 42′, and azimuth per compass S. E $\frac{3}{4}$ S. at 11h. 16′ A. M. the altitude was 53° 11′. The ship's course during the elapsed time was W. $\frac{1}{2}$ N. at the rate of 9 knots per hour, and the height of the eye 12 feet. Required the ship's true latitude at the time of the second observation?

☉'s bearing at 1st obs.	S. E. $\frac{3}{4}$ S.	Reduced declination -	3° 34′ N.
Ship's course -	W. $\frac{1}{2}$ N.	Elapsed time - -	1h 48′
		Distance run = 1h. 48′ × 9 = 16m.	
Contained angle -	11$\frac{3}{4}$ points		
Supplement - -	4$\frac{1}{4}$ points.		

To course 4$\frac{1}{4}$ points, and distance run 16 miles, the difference of latitude is 10′.7, or 11 miles.

First observed altitude	40° 42′	Second observed altitude	53° 11′
Sun's semidiameter -	+ 16	Semidiameter - -	+ 16
Dip and refraction -	— 4	Dip and refraction - -	— 4
Reduction of altitude -	— 11		
		Corrected altitude -	53 23
Reduced altitude -	40 43		

Time

Time per watch.	Altitude.	Nat. sine.	Lat. by ac. 38° 12′ sec.	0.10466
9h 28′	- 40° 43′	65232	Declination 3 34 sec.	0.00084
11 16	- 53 23	80264		
			Logarithm ratio -	0.10550
1 48	Difference -	15032 - -	log. - -	4.17702
0 54	- -	Half elapsed time	- -	0.63181
1 37	- -	Middle time	- -	4.91433
43	- -	Rising	- - -	3.24427
Natural number	-	1376	- - - -	3.13877

Mer. zen. dist.	35 16	N. co-s. 81640		
Declination -	3 34	- -	secant - -	0.00084
Latitude -	38 50	- -	secant - -	0.10848

Logarithm ratio - - - -		- - -	0.10932
Difference natural sines - -	15032	- log. -	4.17702
Half elapsed time - -	0 54 0	- log. -	0.63181
Middle time - -	1 37 50	- log. -	4.91815
Rising - - -	0 43 50	- log. -	3.26089
Natural number - -	1418	- - -	3.15157

Mer. zen dis.	35 14	Nat. co-s. 81682
Declination -	3 34	
Latitude -	38 48 N.	

Problem IX.

To find the Latitude by double Altitudes, and the elapsed Time, by Means of a Table of Log. Sines and Secants only.

Rule.

To the log. co-sine of half the sum of the true altitudes, add the log. sine of half their difference, the log. secant of the declination, the log. secant of the latitude by account, and the log. co-secant of half the interval of time between the observations reduced to degrees; the sum will be the log. sine of *arch first*.

To the log. co-sine of the declination and latitude, add the constant log. 0.30103, twice the log. sine of half the difference between the half interval and arch first, and the log. secant of the estimate meridian altitude;

altitude; the sum will be the log. sine of *arch second*, which, being added to the greatest altitude, will give the meridian altitude, and, hence, the latitude is found as usual.

EXAMPLE.

In latitude 57° N. by account, at 10h. 51′ A. M. the corrected altitude of the Sun's center was 25° 18′; and at 1h. 0′ P. M. the altitude was 26° 55′, the Sun's declination being 5° 17′ S. Required the latitude?

Greatest alt. 26° 55′ Inter. = 2h 9′, & $\frac{2h\ 9'}{8}$ = 16° 7½′ = half interval in degrees.
Least alt. - 25 18

Sum - -	52 18 half = 26 6½	co-sine	9.95326		
Difference -	1 37 half = 48½	sine -	8.14945		
Declination -	5 17 - -	secant -	0.00185	co-sine	9.99815
Latitude -	57 0 - -	secant -	0.26389	co-sine	9.73611
Half interval	16 7½ - -	co-secant	0.55637	con. log.	0.30103
Arch first .	4 49½ - -	sine . -	8.92482		

Difference 11 18 - - Est. mer. alt. 27° 43′ secant . 0.05293
Half - - 5 39 - - Twice log. sine = 8.99322 × 2 = 8.98644

Arch second - -	0° 41′	- sine -	8.07466
Greatest altitude -	26 55		
Meridian altitude	27 36		
Zenith distance -	62 24 N.		
Declination - -	5 17 S.		
Latitude - -	57 7 N.		

PROBLEM X.

To find the Latitude by Observations taken near the Meridian and Prime Vertical.

RULE.

When the Sun is near the east or west points, let its altitude be observed, from which, the estimated latitude and declination, compute the Sun's distance from the meridian, by Prob. VI. page 125; which, being applied to the time of observation per watch, will give the time per watch when the Sun is on the meridian.

Now, observe the Sun's altitude when near the meridian; then the difference between the time per watch of observation, and the time of noon before found, will be the Sun's distance from the meridian at the time of that observation, with which, and the altitude and declination, compute the meridian altitude by Prob. V. and hence the true latitude will be known.

Allowance is to be made for the change of longitude in the interval between the observations.

EXAMPLE.

April 29, 1803, in latitude 68° N. by account, and longitude 15° W. at 6h. 45′ 34″ A. M. the correct altitude of the Sun's center was 17° 58′, and at 11h. 42′ 18″, the true altitude was 36° 2′. Required the true latitude?

Declination at time of 1st obs. = 14° 12′ N. at the time of 2d = 14° 16′ N.

Altitude - - -	17° 58′		
Polar distance - -	75 48	- - co-secant - -	0.01348
Latitude - - - -	68 0	- - secant - -	0.42642
Sum - -	161 46		
Half - - -	80 53	- - co-sine - -	9.20768
Difference - -	62 55	- - sine - - -	9.94956
			19.59714
	38 58	- - sine - - -	9.79857
	8		
Time from noon -	5 11 44		
Time p. watch of obser.	6 45 34		
Time p. watch of noon	11 57 18	Lat. 68° 0′ co-sine	9.57358
Time of second obser. -	11 42 18	Dec. 14 16 co-sine	9.98640
Difference - -	15 0	- rising -	2.33063
Natural number - - - - - -		78 -	1.89061
Greatest altitude -	36 3	Nat. sine 58826	
Mer. zenith distance -	53 55	Nat. co-sine 58904	
Declination - -	14 16		
Latitude - - - -	68 11 N.		

If the latitude by account be assumed 69°, the latitude by computation will be 68° 12′, differing only one mile from the former computation.

PROBLEM XI.

Given the Altitudes of two known Stars, observed at the same, or at different Times; and if at different Times, the Interval of Time between the Observations, to find the Latitude.

RULE.

If both altitudes be observed at the same time, call the difference between their right ascensions the *reduced interval*; but if the altitude be

be taken at different times, reduce the interval between the observations to sidereal time, by adding thereto the proportional part answering to the interval, and 3′ 56″, the daily acceleration of the fixed stars, No •, to the right ascension of the first observed star, add the interval in sidereal time, and the difference between this sum, and the right ascension of the other star, will be the reduced interval.

To the log. rising of the reduced interval, add the log. co sines of the declinations of the stars; add the natural number answering to the sum of these three logarithms, to the nat. versed sine of the difference or sum of the stars declinations, according as they are of the same, or of a contrary name, and the sum will be the nat. co-versed sine of *arch first*.

To the log. co-sine of arch first, add the log. secant of the declination of the star nearest to the elevated pole, and the log. half elapsed time of the reduced interval, the sum will be the log. half elapsed of time *arch second*.

From the nat. versed sine of the difference between arch first, and the altitude of the star farthest from the elevated pole, subtract the nat. co-versed sine of the altitude of the other star, and find the log. of the remainder; to which add the log. secant of arch first, and the log. secant of the altitude of the star farthest from the elevated pole, the sum will be the log. rising of *arch third*. The difference between arches second and third is arch *fourth*.

To the log. rising of arch fourth, add the log. co sines of the declination and altitude of the star farthest from the elevated pole; add the corresponding natural number to the nat. versed sine of the difference between the altitude and declination, the polar distance being less than 90°; otherwise, to that of their sum, and the sum will be the natural co-versed sine of the latitude.

Remark.

If two altitudes of the same star be observed, the operation becomes more simple.

Examples.

I.

January 1, 1805, in north latitude, the true altitude of Capella was 69° 28′; and, at the same instant, the true altitude of Sirius was 16° 19′. Required the true latitude?

Right ascen. Cap. -	5h	2′	18″
Right ascen. Sirius	6	36	33
Interval - -	1	34	15

Interval

Interval - -	1h	34′	15″	- - rising - -			3.92103
Capella's declin. -	45	47	N.	- co-sine -			9.84347
Sirius' declin. -	16	27	S.	- - co-sine - -			9.98185
Sum - -	62	14		nat. ver. sine	53413		
					5576	-	3.74635
Arch first -	24	13		nat. co-v. sine	58989	co-s.	9.96000
Capella's declin.	45	47		- - secant - -			0.15653
Interval - -	1	34	15	- H. E. T. - -			0.39821
Arch second -	1	11	12	- H. E. T. -			0.51474
Arch first -	24	13		- - secant - -			0.04000
Sirius' altitude	16	19		- - secant - -			0.01785
Difference -	7	54		nat ver. sine	00949		
Capella's altitude	69	23		nat. co-v. sine	06404		
				Difference -	5455		3.73679
Arch third -	1	21	30	- - rising -			3.79464
Arch second	1	11	12				
Arch fourth -		10	8	- - rising -			1.99202
Sirius' declin. -	16	27		- - co-sine -			9.98185
- altitude -	16	19		- - co-sine -			9.98215
Sum - -	32	46		nat. ver. sine	15912		
					90	-	1.95602
Latitude -	57	8 N.		nat. co-v. sine	16002		

II.

In north latitude, December 20, 1806, the true altitude of Menkar was 43° 38′, and 1h. 18′ after, the altitude of Rigel was 29° 51′. Required the latitude?

Observed interval	1h	18′	0″
Equation - -	+		13
Int. in sid. time -	1	18	13
Right asc. Menkar	2	52	11
Sum - -	4	10	24
Right ascen. Rigel	5	5	11
Reduced Interval	0	54	47

Reduced

Reduced interval	0h 54′ 47″	- - rising		- 3.45383
Declin. of Menkar	3 20 N.	- . co-sine		- 9.99926
Declin. of Rigel	8 26 S.	- - co-sine		- 9.99528
Sum - -	11 46	n. v. sine	02101	
			2808	- - 3.44837
Arch first -	71 58	n.co-v.sine	04909	- co-sine - 9.49076
Declin. Menkar	3 20	- -	secant	- - 0.00074
Reduced interval	54 47	- -	H. E. T.	- - 0.62568
Arch second -	3 19 6	-	H. E. T.	- - 0.11718
Arch first -	71 58	- -	secant	- - 0.50924
Alt. Rigel -	29 51	- -	secant	- - 0.06181
Difference -	42 7	n. ver. sine	25822	
Alt. Menkar	43 38	n. co-ver. sine	30996	
		Difference	5174	- - 3.71383
Arch third -	2 24 40	- -	rising	- 3.28488
Arch second -	3 19 6			
Arch fourth -	54 26	- -	rising	- 3.44829
Declin. Rigel	8 26	- - -	co-sine	- 9.99528
Alt. Rigel -	29 51	- - -	co-sine	- 9.93819
Sum -	38 17	n. ver. sine -	21504	
			2409	- - 3.38176
Latitude -	49 32½	n. co-v. s. -	23913	

Problem XII.

Given Two Altitudes of the Moon, with the Times per Watch, and Longitude by Account, to find the Latitude of the Place of Observation.

Rule.

Reduce the given times of observation to the meridian of Greenwich, by applying thereto the longitude by account in time, by addition or subtraction, according as it is W. or E.; and to these times let the Moon's declination be found by the Nautical Almanac, and correct the observed altitudes by Prob. xi. page 112.

To the interval of time between the observations, add the change of the Sun's right ascension in that interval; and from the sum, subtract the corresponding variation of the Moon's right ascension in time, and the remainder will be the *reduced interval*.

Now,

Now, with the reduced interval, the altitudes and declinations of the Moon, the latitude is to be found, as directed in last problem.

Examples.

I.

December 12th, 1804, being in north latitude, and in longitude 24° W. by account: at 5h. 24′ P. M. per watch, the altitude of the Moon's lower limb was 41° 33′; and at 7h. 12′, the altitude of the Moon's lower limb was 52° 56′, height of the eye 20 feet. Required the latitude?

Time p. watch of 1st obs. -	5h	24′	Time p. w. of 2d obs.	7h	12′
Longitude in time - -	1	36	- - - - -	1	36
Reduced time - - -	7	0	- - - - -	8	48
Moon's declin. at noon -	12	16 N.	- - -	12	16 N.
Equation to 7h. -	+1	36	Equation to 8h. 48′	+2	0
Reduced declination -	13	52 N.	- - -	14	16 N.
Ob. alt. ☽'s l. l. at 1st ob.	41	33	Ob. alt ☽'s l. l. at 2d ob.	52	56
Semidiameter - -	+	16	Semidiameter -	+	16
Dip - - -	—	4	Dip - - - -	—	4
Apparent alt. ☽'s center	41	45	Appar. alt. ☽'s center	53	8
Correction - - -	+	44	Correction -	+	35
True altitude - -	42	29	True altitude -	53	43

Time of 1st observ.	5h.	24′	0″
- 2d -	7	12	0
Interval - -	1	48	0
Var. ☉'s right asc.	+		20
- ☽'s -	—	4	5

Reduced interval	1	44	15	- rising -		4.00725
Moon's dec. at 1st ob.	13	52		- co-sine -		9.98715
- at 2d ob.	14	16		- co-sine -		9.98640
Difference -	0	24		n. v. s.	00002	
					9567 - -	3.98080
Arch first -	64	44		n. co-v. s.	09569 - co-sine -	9.63026
☽'s dec. at great. alt.	14	16		- - secant	- -	0.01360
Reduced interval -	1	44	15	- - H. E. T.	- -	0.35718
Arch second -	5	43	53	- H. E. T.	- -	0.00104

Arch

Arch first - -	64° 44′	- -	secant	- -	0.36974
Least alt. - -	42 29	- -	secant	- -	0.13225
Difference -	22 15	n. v. s.	07446		
Greatest altitude -	53 43	n. co. v. s.	19390		
		Differ.	11944	- -	4.07715
Arch third - -	3 26 34	-	rising	-	4.57915
Arch second - -	5 43 53				
Arch fourth - -	2 17 19	-	rising	-	4.24102
Dec. at least alt. -	13 52	- -	co-sine	-	9.98715
Least alt. - -	42 29	- -	co-sine	-	9.86775
Difference - -	28 37	n. v. s.	12216		
			12462	- -	4.09592
Latitude - -	48 52	n. co-v. s.	24678		

II.

October 16, 1804, in S. latitude, and longitude 65° E. by account, at 8h 8′ P. M. per watch, the altitude of the Moon's upper limb was 55° 24′; and at 1h. 40′ A. M. the altitude of the Moon's lower limb was 28° 54′, the height of the eye 10 feet. Required the latitude?

Time per wat. of 1st ob.	8h 8′ P. M.	Time p. w. of 2d ob.	1h 50′ A. M.
Longitude in time -	4 20 E.	- -	4 20 E.
Reduced time -	3 48 P.M.	- - -	9 30 P. M.
Moon's dec. at noon	1 20 S.	- - - -	1 20 S.
Eq. to time 3h. 48′	1 0	Eq. to time 9h. 30′	2 30
Reduced declination	0 20 S.	- - -	1 10 N.
Ob. alt. ☽'s u.l. 1st ob.	55° 24	Ob. alt. ☽'s l.l. 2d ob.	28° 54′
Semidiameter -	— 16	Semidiameter -	+ 16
Dip - -	— 3	Dip - - -	— 3
App. alt. ☽'s center	55 5	App. alt. ☽'s center	29 7
Correction -	+ 33	Correction -	+ 50
True altitude -	55 38	True altitude -	29 57

Time of 1st ob.	8h 8′ P. M.
—— of 2d ob.	1 50 A. M.
Interval - -	5 42
Var. ☉'s r. asc. +	53
Var. ☽'s r. asc. —	12 4
Reduced inter.	5 30 49

Reduced

Reduced inter.	5h 30′ 49″	rising		4.94101
☽'s dec. at 1st ob.	0 20 S.	co-sine		9.99999
- at 2d ob.	1 10 N.	co-sine		9.99991
Sum	1 30	nat. ver sine	00034	
			87280	4.94091
Arch first	7 17	nat. co-ver. sine	87314	co-sine 9.99648
☽'s dec. at gr. alt.	0 20 N.	secant		0.00001
Reduced inter.	5 30 49	H. E. T.		0.00353
Arch second	5 57 50	H. E. T.		0.00002
Arch first	7° 17′	secant		0.00352
Least altitude	29 57	secant		0.06225
Difference	22 40	nat. ver. sine	07724	
Greatest alt.	55 38	nat. co-ver. sine	17456	
		Difference	9732	3.98820
Arch third	1 50 8	rising		4.05397
Arch second	5 57 50			
Arch fourth	4 7 42	rising		4.72376
☽'s dec. least alt.	1 10 N.	co-sine		9.99991
Least alt.	29 57	co-sine		9.93775
Sum	31 7	nat. ver. sine.	14388	
			45861	4.66142
Latitude	23 25 S.	nat. co-ver. sine	60249	

The latitude may be found in the same manner, from two altitudes of the Sun, or of a planet and the elapsed time, without using a supposed latitude.

Problem XIII.

Given three Altitudes of the Sun, taken at equal Intervals of Time, near the Meridian, to find the Latitude of the Place of Observation.

Rule.

Let the difference between the first and second altitudes be called A, and that between the first and third B.; also, let the difference between A and half B be called C, and that between 2 A and half B be called D.

Now from the log. of D subtract the log. of 2 C; and call the remainder the log. of *arch first.* To twice the log. of arch first add the

the log. of C, the sum will be the log. of *arch second*; and to the log. of arch first, add the log. of D, the sum will be the log. of *arch third*.

If half B is less than A, or greater than 2 A, the difference between arches second and third will be the correction; but if half B is greater than A, and less than 2 A, the sum of these arches will be the correction, which being added to the first altitude, will give the meridian altitude, with which, and the declination, the latitude is to be found as formerly.

Remark.

If the log. of the interval of time between two successive observations, be added to the log. of arch first, the sum will be the log. of the interval of time between the time of observation of the first altitude and noon: hence the time per watch of apparent noon will be obtained, and, of course, the error of the watch will be known.

Example,

November 20th, 1804, the following observations were made. Required the latitude?

Times per watch 11h 33′. True alt. ☉'s center 13° 2′
- - 12 14 - - 12 59 - A = 3
- - 12 55 - - 12 0 - B = 62

Now A ∽ ½ B = 3 ∽ 31 = 28 = C.
And 2 A ∽ ½ B = 6 ∽ 31 = 25 = D.

D - 25 - log. - 1.39794
2 C - 59 - log. - 1.74819

Arch first . - 9.64975 - - - - 9.64975
D. - 25 - log. - 1.39794

9.29950
C - 28 - log. - 1.44716 Arch 3d 12.7 - 1.04769

Arch sec. - - 0.74666 - 5.6

Correction - - - 7
First observed altitude - 13 2

Meridian altitude - - 13 9
Zenith distance - - 76 51
Declination - - - 19 44

Latitude - - - 57 7 N.

Arch first - - - - - - 9.64975
Interval - 41′ - log. - - 1.61278

Int. bet. 1st obs. & noon - 18′.38 - - - 1.26253

Int. bet. 1st obs. & noon - 13′.38
Time of first observ. - 11 33.

Time p. watch of noon 11 46.38
Watch slow - 13.62 = 13′ 37″

Remarks.

I.

The three altitudes being observed at equal intervals of time, if then two of these are equal, the correction of the greatest altitude will be found by dividing the difference between the unequal altitudes by 8.

Example.

Let the following observation be supposed to be made December 13, 1812, to find the latitude.

Times per watch 11 46.	True alt. ☉'s cent.	34° 4′.
12 16	- -	34 4
12 46	- - -	33 9

Now 55′, the difference between the unequal altitudes being divided by 8, the quotient is 7′ nearly, which added to 34° 4′, gives 34° 11′, the meridian altitude. Hence the zenith distance is 55° 49′, from which the declination 23° 11′ being subtracted, the remainder 32° 38 N. is the latitude.

II.

If two equal altitudes be observed near noon, and another altitude not far distant therefrom at an unequal interval of time, the correction of the equal altitudes may be found as follows.

Method First.

To the interval of time between the unequal altitudes, add half of that between the equal altitudes, and divide the sum by the above half interval. Now increase and diminish the resulting quotient by 1, and find the product of this sum and difference, by which the difference between the unequal altitudes being divided, will give the correction, to be added to either of the equal altitudes, to obtain the meridian altitude.

Method Second.

To twice the log. of half the interval of time between the equal altitudes, add the log. of the difference of the unequal altitudes. From

From this sum, subtract the sum of the logs. of the extreme interval of time, and of the interval between the nearest unequal altitudes, the remainder will be the log. of the correction to be added to the greatest altitude, and hence the latitude will be obtained.

The unequal altitude is supposed to be less than either of the equal altitudes.

Example.

May 2, 1804, the following altitudes of the Sun were observed, from whence it is required to find the latitude?

By Method First.

Times per watch	11h	13′.	True alt. ⊙	52° 47′	
- -	11	40	- -	53 42	Diff. 55′.
- -	12	16	- -	53 42	

Now, $\frac{11h.\ 40' + 12h.\ 16'}{2} = 11h.\ 58'$ = time per watch of appar. noon.

Then, 11h. 58′ − 11h. 13′ 45′; and 11h. 58′ − 11h. 40′ = 18′.

Now, $\frac{45}{18} = 2'.5$; and $2'.5 \pm 1 = 3'.5$ and $1'.5$ respectively.

Then, $\frac{55'}{3.5. \times 1.5} = \frac{55'}{5.25} = 10'.5$ nearly, the correction.

Hence, 53° 42′ + 10′.5 = 53° 52′.5, the meridian altitude.

Zenith distance	-	36 7.5
Declination	-	15 21.
Latitude	- -	51 31.5

By Method Second.

Half inter. equal altitudes	18′	- log.	- - -	1.25527
				1.25527
Diff. unequal altitudes	55	- log.	- - -	1.74036
Extreme interv. of time	63′	log. 1.79934	- - -	4.25090
Int. betw. unequal alt.	27	log. 1.43136		
		3.23070	- - -	3.23070
Correction - -	10.5		- - - -	1.02020
Greatest altitude	53 42.			
Meridian altitude	53 52.5			

 Meridian

Meridian alt. - -	53 52 5
Zenith distance -	36 7.5
Declination -	15 21.
Latitude - -	51 31.5

Various other methods might be given for ascertaining the latitude from altitudes of an object observed near noon. These, however, it is proposed to insert in a work expressly upon that subject.

Problem XIV.

Given three Altitudes of the Sun, with the Intervals of Time between them, to find the Latitude of the Place of Observation, and the Sun's Declination.

Rule.

Find the apparent time of each observation, by Prob. xv. p. 146; then, with any two of the altitudes and corresponding times from noon, compute the latitude and declination, by Prob. vi. p. 319.

Example.

Let the three altitudes be 57° 23′, 55° 0′, and 49° 55′, the interval of time between the two first 40′, and that between the two last 50′. The latitude and declination are required; and suppose these observations to have been taken in 1811, some time between June and December, the day of observation is sought?

First interval 40′ = 10° half = 5° 0′ = arch 1st
Second interval 50 = 12 30 h. = 6 15 = arch 2d } Sum = 11° 15′ = arch 3d.

Altitude -	57° 23′	Nat. sine	84230		
	55 0	- - -	81915	diff.	23:5 = arch 5th
	49 55	- - -	76511	- -	7719 = arch 4th

Arch IV. = 7719	ar-co-log.	6.11244	Arch I. = 5° 0′	sine - 8.94030		sine - 8.94030
Arch V. = 23 15	- log. -	3.36455	Arch II. = 6 15	co-s. - 9.99741		sine - 9.03690
Arch III. = 11 15	- sine -	9.29024	Arch VII. .08664	- log. - 8.93771		
Arch VI. - - - - -		8.76723	- - -	.05851		
Difference - - - - - - - - -				.02813	- - ar-co-log. -	1.55083
Arch VIII. - - - - - - - - - -				18° 38′	- tangent - -	9.52803
Arch III. - - - - - - - - - -				11 15		
Time from noon when greatest altitude was observed - - - - - - - }				7 23		
Hence, time of observ. of middle alt. -				17 23		

Time

Time of observ. 17° 23′ N. V. S.	04567	log.	3.65966	const. log.	5.30103
7 23	00829				
Difference	03738	ar-co-log.	6.42736		6.42736
Diff. natural sines altitudes	02315	log.	3.36455		3.36455
	2829		3.45157	1.23863	5.09294
Mid. altitude 55 0 N. sine	81915				
32 4 N. co-sine	84744			84744	
66 58 N. co-sine				39119	

Sum - - - - 92 2 - half = 49 31 = latitude N.
Difference - - 34 54 - half = 17 27 = declination N. which answers to Aug. 4th.

Problem XV.

Given the Sun's Declination and Semidiameter, and the Interval of Time between the Instants when the upper and lower Limbs of the Sun were in the Horizon, to find the Latitude of the Place of Observation.

Rule.

To the log. of the Sun's semidiameter expressed in seconds, add the ar-co. log. of the interval of time in seconds, and the constant log. 9.12494; the sum will be the log. co-sine of an arch.

To the log. sine of the sum of this arch and the Sun's declination, add the log. sine of their difference; half the sum will be log. sine of the latitude.

Examples.

I.

Let the Sun's declination be 18° 24′, semidiameter 15′ 52″, and time required to rise 4′ 48″. Required the latitude?

Sun's semidiameter = 15′ 52″ = 952″		log.	2.97864
Interval of time - - 4 48 = 288		ar-co-log.	7.54061
Constant log.			9.12494
Arch	63° 51′	co-sine	9.64419
Sun's declination	18 24		
Sum	82 15	sine	9.99601
Difference	45 27	sine	9.85287
			19.84888
Latitude	57 10	sine	9.92444

II.

June 21, 1810, the interval of time between the setting of the lower and upper limbs of the Sun was 3′ 56″. Required the latitude?

Sun's

Sun's semidiameter	= 15′ 47″	= 947″	log.	-	2.97635
Interval of time	- 3 56	= 236	ar-co-log.		7.62709
Constant log.	-	-	-	-	9.12494
Arch - -	57° 39′	-	co-sine	-	9.72838
Sun's declination	23 28				
Sum - -	81 7	-	sine	-	9.99476
Difference -	34 11	-	sine	-	9.74961
					19.74437
Latitude -	48 10	-	sine	-	9.87218

PROBLEM XVI.

Given the difference of Altitude between the Pole Star and the Pole, at different distances of the Star from the Meridian.

RULE.

Find the interval between the time of observation of the altitude of the pole star, and that of its passing the meridian,* and take out the corresponding equation from the table; which added to, or subtracted from the true altitude of the pole star, will give the latitude of the place of observation.

EXAMPLES.

I.

Let the corrected altitude of the pole star be 48° 12′ N. observed 9h. 20′ before its passage over the meridian. Required the latitude?

True altitude of the pole star - - -	48° 12′ N.
Equation from table XLVI. to 8h. 30′ -	+ 1 22
Latitude - - -	49 34 N.

II.

At 1h. 10′ after the passage of the pole star over the meridian, its altitude corrected was 58° 51′N. Required the latitude?

True altitude of the pole star - -	58° 51′ N.
Equation from Table XLVI. to 1h. 10′ -	— 1 42
Latitude - - -	57 9 N.

* If this interval is expressed in mean solar time, it ought to be reduced to sidereal time. This may be done with sufficient accuracy for most purposes, by adding the proportional part of the acceleration of the fixed stars, answering to the interval between the times of observation, and the transit of the star; but, at sea, this is absolutely unnecessary.

TABLE

TABLE I.

OF THE APPARENT TIME OF TRANSIT OF THE POLE STAR.

Days.	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
	P. M.	P. M.	P. M.	P. M.	A. M.	A. M.	A. M.	A. M.	A. M.	A. M.	P. M.	P. M.
1	6h 9′	3h 56′	2h 4′	0h 10′	10h 19′	8h 17′	6h 13′	4h 9′	2h 13′	0h 25′	10h 25′	8h 22′
2	6 4	3 52	2 0	0 7	10 15	8 13	6 9	4 5	2 10	0 21	10 21	8 18
3	6 0	3 48	1 57	0 3	10 12	8 9	6 5	4 1	2 6	0 18	10 17	8 13
4	5 55	3 44	1 53	0 0	10 8	8 5	6 1	3 57	2 3	0 14	10 13	8 9
				A. M.								
5	5 51	3 40	1 49	11 56	10 4	8 1	5 57	3 53	1 59	0 10	10 9	8 5
6	5 47	3 36	1 45	11 52	10 0	7 57	5 53	3 49	1 55	0 7	10 5	8 0
7	5 42	3 32	1 42	11 49	9 56	7 53	5 49	3 45	1 52	0 3	10 1	7 56
										P. M. 12 0		
8	5 38	3 28	1 38	11 45	9 52	7 49	5 44	3 42	1 48	11 56	9 57	7 52
9	5 33	3 24	1 34	11 41	9 48	7 45	5 40	3 38	1 45	11 52	9 53	7 47
10	5 29	3 20	1 31	11 38	9 45	7 41	5 36	3 34	1 41	11 48	9 49	7 43
11	5 25	3 16	1 27	11 34	9 41	7 36	5 32	3 30	1 37	11 45	9 45	7 38
12	5 20	3 12	1 23	11 30	9 37	7 32	5 28	3 26	1 34	11 41	9 41	7 34
13	5 16	3 8	1 20	11 27	9 33	7 28	5 24	3 23	1 30	11 37	9 37	7 30
14	5 12	3 4	1 16	11 23	9 29	7 24	5 20	3 19	1 27	11 34	9 33	7 25
15	5 7	3 0	1 12	11 19	9 25	7 20	5 16	3 15	1 23	11 30	9 29	7 21
16	5 3	2 57	1 9	11 16	9 21	7 16	5 12	3 11	1 19	11 26	9 25	7 16
17	4 59	2 53	1 5	11 12	9 17	7 11	5 8	3 8	1 16	11 22	9 20	7 12
18	4 55	2 50	1 1	11 8	9 13	7 7	5 4	3 4	1 12	11 19	9 16	7 7
19	4 50	2 46	0 58	11 4	9 9	7 3	5 0	3 0	1 9	11 15	9 12	7 3
20	4 46	2 42	0 54	11 1	9 5	6 59	4 56	2 57	1 5	11 11	9 8	6 59
21	4 42	2 38	0 50	10 57	9 1	6 55	4 52	2 54	1 1	11 7	9 4	6 54
22	4 38	2 34	0 47	10 53	8 58	6 51	4 48	2 50	0 58	11 4	9 0	6 50
23	4 33	2 30	0 43	10 50	8 54	6 47	4 44	2 46	0 54	11 0	8 56	6 45
24	4 29	2 27	0 40	10 46	8 50	6 42	4 40	2 43	0 51	10 56	8 52	6 41
25	4 25	2 23	0 36	10 42	8 46	6 38	4 36	2 39	0 47	10 52	8 48	6 36
26	4 21	2 19	0 32	10 38	8 42	6 34	4 32	2 35	0 43	10 48	8 44	6 32
27	4 17	2 15	0 29	10 34	8 38	6 30	4 28	2 32	0 40	10 44	8 39	6 27
28	4 13	2 11	0 25	10 31	8 34	6 26	4 24	2 28	0 36	10 41	8 35	6 23
29	4 8	2 8	0 21	10 27	8 30	6 22	4 20	2 24	0 33	10 37	8 31	6 19
30	4 4		0 18	10 23	8 26	6 17	4 16	2 21	0 29	10 33	8 26	6 14
31	4 0		0 14		8 22		4 12	2 17		10 29		6 10

TABLE II.

DIFFERENCE OF ALTITUDE OF THE POLE STAR AND POLE.

Argument. Distance of the Star from the Meridian, in Sidereal Time.

SUBTRACT.

Min.	0 Hour.	1 Hour.	2 Hours.	3 Hours.	4 Hours.	5 Hours.	
0	1° 46′.9	1° 43′.3	1° 32′.6	1° 15′.6	0° 53′.4	0° 27′.7	60
5	1 46.9	1 42.7	1 31.4	1 13.9	0 51.4	0 25.4	55
10	1 46.8	1 42.0	1 30.2	1 12.2	0 49.4	0 23.2	50
15	1 46.7	1 41.2	1 28.9	1 10.5	0 47.3	0 20.9	45
20	1 46.5	1 40.4	1 27.6	1 8.7	0 45.2	0 18.6	40
25	1 46.3	1 39.6	1 26.2	1 6.9	0 43.1	0 16.3	35
30	1 46.0	1 38.8	1 24.8	1 5.1	0 40.9	0 14.0	30
35	1 45.7	1 37.9	1 23.4	1 3.2	0 38.8	0 11.6	25
40	1 45.3	1 36.9	1 21.9	1 1.3	0 36.6	0 9.3	20
45	1 44.9	1 35.9	1 20.4	0 59.4	0 34.4	0 7.0	15
50	1 44.4	1 34.8	1 18.8	0 57.4	0 32.2	0 4.7	10
55	1 43.9	1 33.7	1 17.2	0 55.4	0 29.9	0 2.3	5
60	1 43.3	1 32.6	1 15.6	0 53.4	0 27.7	0 0.0	0
	11 Hours.	10 Hours.	9 Hours.	8 Hours.	7 Hours.	6 Hours.	Min.

ADD.

CHAP. II.

Of finding the Variation of the Compass.

INTRODUCTION.

THE attractive power of the magnet was known in Europe about 600 years before the Christian æra; and by the Chinese records it is said, that its directive property was known in that country at least 1000 years earlier. The invention of the compass is by some ascribed to John or Flavia Goya of Amalphi in Naples, about the year 1302. It, however, appears from some verses,* in a French tract entitled *La Bible de Guyot de Provins**, to have been known in that country previous to the year 1180. It is also shown in the *Journal des Scavans* for October 1782, by M. Leprince le jeune, that Jacques de Vitri, who lived towards 1200, speaks of the magnetic needle as being in common use, and of great service in the practice of navigation.

* Icelle estoile ne se muet,
Un arts font qui mentir ne puet,
Par la vertue de la manete,
Un pierre laide et brunette,
Ou l fers volontiers se jointe,
Ont, regardant l'or droit pointe,
Puez l'une aguile l'ont touchie,
Et en un festu l'ont fichie
En longue la mette sens plus,
Et il festue la tient desus;
Puis se torne la pointe toute,
Contre l'estoile, sans doute;
Quant li nuis est tenebre et brune,
C'on ne voit estoile ne lune,
L'or font a l'aguille alumer,
Puez ne puent ills assarrer,
Contre l'estoile, vers le pointe;
Per ce sont la marinier cointe
De la droite voir tenir;
C'est un art qui ne peut mentir.

TRANSLATION.

There is a star that never moves,
And an art that never deceives,
By virtue of the magnet,
An ugly brownish stone,
Which always attracts iron.
And which always points straight, (or right)
With this a needle they touched,
And on a bit of straw they set it,
Along the middle they put it,
And the straw supports it.
Then its point it turns,
Towards this star no doubt,
When the night is so extremely dark,
That neither stars nor moon can be seen,
Then looking at the needle with a light,
Are they not certain,
To see it pointing towards that star?
Upon this the mariners depend
For the right, or proper, course to keep,
This is an art that never deceives.

* Catalogue du Duc de Valliere, No. 2.707.

Until

Until the time of Columbus, the direction of the needle was supposed to be exactly in the plane of the meridian; and, therefore, the other points were supposed to agree with the correspondent points of the horizon. However, in the month of September 1492, Columbus first discovered the variation of the needle. This discovery is also said to have been made by Sebastian Cabot in the year 1497.

Soon after, the variation was found to be different at different places. It was, however, affirmed to be constant at the same place; but observation also overturned this hypothesis. In 1635 Mr. Henry Gellibrand published his discovery of the change of the variation, from a comparison of his own observations with those of his predecessors.

The magnetic needle is subject both to an annual and to a diurnal vibratory motion. In the first of these, the motion of the north end of the needle is in general towards the east, from the time of vernal equinox to the summer solstice; and, during the other nine months, its motion is in general towards the west. In the second, the needle is stationary from noon till about 3h. P.M. and from thence till about 8 in the evening it slowly approaches the east. It again continues stationary till about 8 in the morning; and, from that time till noon, it gradually approaches the west. The mean quantity of the diurnal variation at the Observatory at Paris, in each month of the year 1791, according to the observations of M. J. D. Cassini, is as follows.

January -	11′.3	May -	11′.3	September	11′.3
February	8.5	June -	11.3	October -	11.3
March -	12.7	July -	14.1	November	10.2
April -	15.5	August -	11.3	December -	9.8

The magnetic needle has another property, called the Inclination or Dip. Thus, let the needle, before it is rendered magnetical, be well balanced; then give it magnetism; and the needle, being suspended on the same point as before, will now be found to have lost its equilibrium; and the angle contained between the direction of the needle and an horizontal line, is called the Dip. This discovery was made by chance, in the year 1576, by Mr. Robert Norman, a compass-maker at Wapping, London †.

Defi-

† It is presumed the following account of Dip of the needle, as given by Norman in the third chap. of his *Newe Attractive*, will be acceptable to some of our readers.

"Hauing made many and diuers compasses, and using alwaies to finish and end them before I touched the needle, I found continually, that after I had touched the yrons with the Stone, that presently the north point thereof would bend or *Decline* downwards under the Horizon in some quantitie: insomuch that to the Flie of the Compasse, which before was

Definitions.

The Variation of the compass is the deviation of the points of the mariner's compass from the corresponding points of the horizon, and is denominated East or West Variation.

East Variation is when the north point of the compass is to the east of the true north point of the horizon.

West Variation is when the north point of the compass is to the west of the true north point of the horizon.

The Variation of the compass may be found by various methods, as *Amplitudes*, *Azimuths*, *Equal Altitudes*, &c.

The True Amplitude* of any celestial object, is an arch of the horizon, contained between the north or south points thereof, and the object's center at the time of its rising or setting.

The Magnetic Amplitude, is the arch contained between the object's center, when in the horizon, and the magnetic meridian— or, in other words, it is the bearing of the object per compass, when in the horizon.

The True Azimuth of an object, is the angle contained between the true meridian, and the vertical passing through the object's center.

The Magnetic Azimuth is the angle contained between the magnetic meridian, and the azimuth circle passing through the center of the object.

The true amplitude or azimuth is found by calculation; and the magnetic amplitude or azimuth of the Sun, or any celestial object, may be very accurately observed by Mr. M'Culloch's patent compass, of which the following is a description.

was made equall, I was still constrained to put some small peece of waxe in the South part thereof, to counterpoise this *Declining*, and to make it equall againe.

Which effect having many times passed my hands without any great regard thereunto, as ignorant of any such propertie in the Stone, and not before hauing heard nor read of any such matter: It chaunced at length that there came to my hands an Instrument to bee made with a Needle of sixe inches long, which needle after I had pollished, cut off at Just length, and made it to stand levell upon the pinne, so that nothing rested but onely the touching of it with the stone: when I had touched the same, presently the north part thereof *Declined* downe in such sort, that beeing constrayned to cut away some of that part, to make it equall againe, in the end I cut it too short, and so spoyled the needle wherein I had taken so much paynes.

Hereby beeing stroken in some choller, I applyed my self to seeke further into this effect, and making certayne learned and expert men (my friends) acquainted in this matter, they advised me to frame some Instrument, to make some exact tryal, how much the needle touched with the stone would *Decline*, or what greatest Angle it would make with the plaine of the Horizon. Whereupon I made diligent proofs."

* The amplitude is commonly reckoned from the east or west points. It, however, eems more proper to reckon it from the meridian.

Descrip-

Description *of the* Azimuth Compass.

Plate VIII. contains a perspective view of the azimuth compass ready for observation. The needle and card of this compass are similar to those of the steering compass, with this difference only, that a circular ring of silvered brass, divided into 360°, or rather four times 90°, circumscribes the card; *b* represents the compass box, which is of brass, and has a hollow conical bottom, *e* is the prop, or support of the compass box, which stands in a brass socket screwed to the bottom of the wooden box, and may be turned round at pleasure; *h* is one of the guards, the other, being directly opposite, is hid by the box. Each guard has a slit, in which a pin, projecting from the side of the box, may move freely in a vertical direction. 1 is a brass bar, upon which, at right angles, the side vanes are fixed; a line is drawn along the middle of this bar; which line, the lines in the vanes, and the thread joining their tops, are in the same plane. 2 is a coloured glass moveable in the vane 3; 4 is a magnifying glass moveable in the other vane, whose focal distance is nearly equal to the distance between the vanes; 5 is the vernier, which contains six divisions, and as the limb of the card is divided into half degrees, each division of the vernier is, therefore, five minutes. The interior surface of the vernier is ground to a sphere, whose radius is equal to that of the card. 6 is a slide or stopper, connected with the vernier; which serves to push the vernier close to the card, and thereby prevent it from vibrating, as soon as the observation of the amplitude or azimuth is completed; and hence the degrees and parts of a degree may be read off at leisure, with certainty. 7 is a convex glass to assist the eye in reading off the observed amplitude or azimuth.

Use *of the* Azimuth Compass.

I.

To observe the Sun's Amplitude.

Direct the vane, containing the magnifying glass, to the Sun; move the compass-box by means of the stop, and raise or depress the magnifying glass until the bright speck fall upon the slit in the other vane, then stop the card, turn round the box, and read off the amplitude.

Without using the magnifying glass, the sight may be directed through the dark glass, and the card is to be stopped when the Sun is bisected by the thread in the other vane, or when the thread is a tangent to either limb of the Sun; but, in this case, allowance must be made for the Sun's semidiameter. In this manner the amplitude of the Moon, a planet, or fixed star, may be observed.

II.

II.

To observe the Sun's Azimuth.

Raise the magnifying glass to the upper part of the vane, and move the box as before directed, until the bright speck falls on the slit in the other vane, or on the line in the horizontal bar, the card is to be stopped, and the divisions being read off, will be the Sun's magnetic azimuth.

If the card vibrates considerably at the time of observation, it will be better to observe the extreme vibrations, and their mean will be the magnetic azimuth.

PROBLEM I.

Given the Latitude of a Place, the Day of the Month, and the Sun's Magnetic Amplitude, to find the Variation of the Compass.

RULE.

To the log. secant of the latitude, add the log. sine of the Sun's declination; the sun will be the log. co sine of the true amplitude, to be reckoned from the north or south, according as the declination is north or south.

Let the true and observed amplitudes be both reckoned from the same point; that is, either both from the north, or both from the south. If, then, the amplitudes are either both in the eastern, or both in the western hemisphere, their difference is the variation: but if one is in the eastern, and the other in the western hemisphere, their sum is the variation.

If the observations be made in the eastern hemisphere, the variation will be east or west, according to the observed amplitude is nearer to, or more remote from the north than the true amplitude. The contrary rule holds good in observations taken in the western hemisphere. Or, let the amplitudes be reckoned from the north, if the observations were taken at Sun rising; but from the south, if at Sun-setting; then the variation will be *East*, if the observed amplitude is less than the true amplitude, but *West*, if greater.

Otherwise, let the observer be supposed to look directly towards the point representing the true amplitude; then, if the observed amplitude is to the left of the true amplitude, the variation is easterly; but if to the right, it is westerly.

EXAMPLES.

I.

May 15, 1804, in latitude 33° 10′ N. and longitude 18° W. about 5h. A. M. the Sun was observed to rise E. by N. Required the variation?

Sun's

Sun's decl. May 15, at noon -	18° 53′ N.			
Eq. from Tab. XIII. to 7h. from noon —	4			
- - - to 18° W. -	'+ 1			
Reduced declination - -	18 50	- sine	-	9.50896
Latitude - - - -	33 10	- secant	-	0.07723
True amplitude - -	N. 67 19 E.	co-sine	-	9.58619
Observed amplitude - -	N. 78 45 E.			
Variation - - - -	11 26			

Which is west, because the magnetic amplitude is more distant from the north than the true amplitude, the observation being made in the eastern hemisphere. Or, the observed amplitude being to the right hand of the true, the variation is, therefore, west.

II.

December 22, 1810, in latitude 31° 38′ S. and longitude 83° W. the Sun was observed to set S. W. Required the variation?

The Sun's declination at the time of observation is nearly the same as at Greenwich at the noon of the given day, namely, 23° 28′ S.

Latitude -	31° 38′	-	secant -	0.06985
Declination -	23 28	-	sine -	9.60012
True amplitude S.	62 7 W.	-	co-sine -	9.66997
Obs. amplitude S.	45 0 W.			
Variation -	17 7			

Which is east, because the observed amplitude is to the left of the true amplitude.

If the Sun's amplitude be observed at the instant its center is in the visible horizon, an allowance, depending on the horizontal refraction, the horizontal parallax*, the height of the eye, and the latitude of the place, is to be applied to the observed amplitude, in order to obtain the amplitude of the object, when in the true horizon. This, however, may be avoided, by observing the amplitude when the altitude of the Sun's lower limb is equal to the sum of 15′ and the dip of the horizon. Thus, if an observer be elevated 17 feet, the amplitude should be observed, when the altitude of the Sun's lower limb is 19′.

In favourable circumstances, and when great accuracy is required, the amplitude of the N. or S. limb of the Sun may be observed, to which the semidiameter being applied, will give the amplitude of the center.

* If the object is the Moon.

PROBLEM

Problem II.

Given the Magnetic Azimuth, the Altitude, and Declination of the Sun, together with the Latitude of the Place of Observation; to find the Variation of the Compass.

Rule.

Reduce the Sun's declination to the time and place of observation, and compute the true altitude of the Sun's center.

Find the sum of the polar distance and altitude of the Sun, and the latitude of a ship. Take the difference between half this sum and the polar distance.

To the log. secant of the altitude add the log. secant of the latitude, the log. co-sine of the half sum, and the log. co-sine of the difference; half the sum of these will be the log. sine of half the Sun's true azimuths, to be reckoned from the south in north latitude, but from the north in south latitude.

The difference between, or sum of, the true and observed azimuths, according as they are of the same, or of a contrary denomination, will be the variation as formerly.

Examples.

I.

November 19, 1835, in latitude 50° 22′ N. longitude 24° 30′ W. about three quarters past eight A. M. suppose the altitude of the Sun's lower limb to be 8° 10′, and bearing per compass S. 21° 18′ E. height of the eye 20 feet. Required the variation?

☉'s decl. Nov. 19, at noon	19° 23′ S.	Observed alt. ☉'s l. limb =	8° 10′
Eq. Tab. XIII. to 3 h.	— 2	Semidiameter - -	+ 16
- to 24° 30′ W.	+ 1	Dip and refraction -	— 10
Reduced declination -	19 22	True altitude .. -	8 16

Polar distance -	109 22		
Altitude - -	8 16	- secant -	0.00454
Latitude - -	50 22	- secant -	0.19527
Sum - - -	168 0		
Half - - -	84 0	- co-sine -	9.01923
Difference - -	25 22	- co-sine -	9.95597
			19.17501
Half true azimuth -	22 45	- sine - -	9.58750
	2		
True azimuth -	S. 45 30 E.		
Observed azimuth -	S. 21 18 E.		

Variation - - 24 12, which is westerly, because the observed azimuth is to the right of the true azimuth.

II.

II.

January 14, 1807, in latitude 33° 52′ S. longitude 53° 15′ E. about half past three P. M. the following altitudes and azimuths were observed, the height of the eye being 20 feet. Required the variation?

Ob. alt. ☉'s l.l.	41° 58′	Azim. N.	63° 24′ W.	☉'s dec. at noon	21° 26′ S.
- -	41 17	-	63 52	Eq. tot. fm. noon	− 2
- -	40 39	-	64 18	Eq. to longitude	+ 2
	54		154	Reduced decl.	21 26 S.
Mean -	41 18	-	63 51	Polar distance	68 34
Semidiameter	+ 16				
Dip and refr.	− 6				
True altitude	41 28				

Polar distance - - -	68° 34′		
Altitude - - -	41 28	- secant -	0.12532
Latitude - - -	33 52	- secant -	0.08075
Sum - - - -	143 54		
Half - - - -	71 57	- co-sine -	9.49115
Difference - - -	3 23	- co-sine -	9.99924
			19.69646
Half true azimuth - -	44 50	- sine -	9.84823
	2		
True azimuth - -	N. 89 40 W.		
Magnetic azimuth -	N. 63 51 W.		
Variation - - -	25 49		

Being west, because the observed azimuth is to the right of the true azimuth.

It is common for one person to observe the Sun's altitude at the same instant that another is taking the azimuth. These observations may, however, be made by the same person, by taking several azimuths and altitudes alternately, at short, and as nearly equal intervals of time as possible, remembering to end with an observation of the same kind that was first taken.

III.

III.

August 13, 1788, in latitude 57° 7′ N. longitude 2° 6′ W. about half past eight in the morning, the following observations were taken, height of the eye being 14 feet. Each quadrant of the card of the compass with which the azimuths were observed, was divided into 96 equal parts. Required the variation?

Azimuth Sun's center	S. 43° .5 E.	Alt. Sun's lower limb	31° 38′
	43 .3	- - - -	31 47
	43 .0	- - - -	31 55
	42 .5	- - - -	32 3
	42 .3		
	14 .6		203
Mean - - -	42.92	Mean - -	31 50¾
		Semidiameter -	+ 16
Now, $\frac{42.92 \times 90}{96} = 40°.237' = 40° 14'$		Dip and refraction	— 5¾
		True altitude - -	32 1

Sun's decl. 13th August -	14° 26′ N.		
Eq. to reduced time -	+ 3		
Reduced declination -	14 29 N.		
Polar distance - -	75 31		
Altitude - - -	32 1	- secant -	0.07166
Latitude - - -	57 7	- secant -	0.26526
Sum - - -	164 39		
Half - - -	82 19½	- co-sine -	9.12566
Difference - -	6 48½	- co-sine -	9.99693
			19.45951
Half true azimuth -	32 28	- sine -	9.72975
	2		
True azimuth - -	S. 64 56 E.		
Observed Azimuth -	S. 40 14 E.		

Variation - - - 24 42 W. because the observed azimuth is to the right of the true azimuth.

METHOD

Method Second.

For finding the true Azimuth.

Rule.

Take the difference between the natural sine of the declination, and the natural number answering to the sum of the log. sines of the latitude and altitude, if they are of the same name, otherwise take their sum, to the log. of which, its index being 9, if it consists of five figures, 8, if four figures, &c. add the log. secants of the latitude and altitude, and the sum will be the log. co-sine of the true azimuth.

Example.

Let the latitude be 43° 39′ N. the Sun's altitude 39° 28′ and declination 16° 37 N. Required the true azimuth?

Latitude -	43° 39′ N. Sine 9.83901	-	secant -		0.14052
Altitude -	39 28 N. Sine 9.80320	-	secant -		0.11239
	9.64221	nat. numb.	43875		
Declination	16 37 - - -	nat. sine -	28597		
			15278	-	9.18407
True azimuth S. 74° 8′ W.	- -	co-sine		-	9.43698

Problem III.

To find the Variation of the Compass by Equal Altitudes of the Sun.

Rule.

Let the Sun's altitude be observed in the eastern hemsphere, when its meridian distance exceeds two hours, and at the same instant let another observer take the azimuth. Then, when the Sun comes to the same altitude in the afternoon, let the bearing of its center be again observed. Now, half the difference between the eastern and western azimuths will be the variation—which, when the observations are made in the southern hemisphere, will be east or west, according as the eastern or western azimuth is greatest. The contrary rule is to be applied, when the azimuths are observed in the northern semicircle.

Remark.

The western azimuth will be affected with the change of declination in the interval between the observations. This correction may be found thus:

To the log. co-sine of the latitude, add the log. sine of the horary angle, and the log. of the change of declination in the elapsed time; the sum will be the log. of the correction of azimuth—which is additive to the western azimuth, when the object is approaching towards the elevated pole, but subtractive, if receding therefrom.

Examples.

I.

In the forenoon, the bearing of the Sun's center per compass was S. S. E. $\frac{3}{4}$ E. the altitude of its lower limb being 40° 18′; and in the afternoon, when the Sun had attained the same altitude, the azimuth was S. W. by W. $\frac{1}{4}$ W. Required the variation?

Eastern azimuth	=	S. S. E. $\frac{3}{4}$ E.	=	S. 2 $\frac{3}{4}$ E.
Western azimuth	=	S. W. by W. $\frac{1}{4}$ W.	=	S. 5 $\frac{1}{4}$ W.
Difference		- - - - -		2 $\frac{1}{2}$
Variation		- - - - -		1 $\frac{1}{4}$ points, which

is *West*, both observations being reckoned from the south, and the western azimuth the greater.

II.

In the forenoon the Sun's azimuth was N. 51° 28′ E. and in the afternoon, when the Sun had come to the same altitude, the azimuth was N. 67° 18′ W. Required the variation?

Eastern azimuth	- -	N. 51° 28′ E.
Western azimuth	- -	N. 67 18 W.
Difference	- - - -	15 50
Variation	- - - -	7 55, East, the ob-

servations being reckoned from the north, and the western azimuth the greater.

III.

February 22, 1804, the following equal altitudes, and corresponding azimuths, were observed; to find the variation of the compass?

Alt.	T. p. watch.	Azim.	T. p. watch.	Azim.
21° 23′	11h 25′ 30″	S. 15° 50′ W.	1 h 4′ 13″	S. 41° 0′ W.*
21 25	28 40	16 40	1 3 30	40 30
21 28	31 20	17 20	1 2 0	40 0
21 30	33 0	17 40	1 1 0	40 0
21 36	34 30	18 30	12 58 50	39 0
Sums	3 0	50	3 33	30
Means	11 30 36	S. 17 10 W.	1 1 55	40 6

Mean of morning set of observations -	S. 17° 10′ W.
Mean of afternoon set - - -	S. 40 6 W.
Sum - - - - - - - -	57 16
Variation - - - - - - - -	28 38,

being west, because the western azimuth is the greatest.

* The Azimuths were observed with a steering compass, having two sight vanes.

PROBLEM

Problem IV.

To find the Variation of the Compass, by Observations of equal Altitudes taken at different Places, the Course and hourly Rate of Sailing being given.

Rule.

Find the distance run in the elapsed time, and hence the difference of latitude. Correct the observed interval by the difference of longitude.

To the log. secant of the latitude at noon, add the log. co-tangent of the half elapsed time, and the log. of the difference of latitude; the sum will be the log. of the correction of azimuth—which is to be applied to the western azimuth, by addition or subtraction, according as the ship's latitude is decreasing or increasing. Apply also the change of azimuth arising from the change of declination; hence the reduced azimuth will be obtained. Then, half the difference between the eastern and western azimuths will be the variation, as formerly.

Example.

April 23, 1805, at 8h. 44′ A. M. per watch, the Sun's azimuth per compass was S. 59° 4′ E. and at 3h. 20′ P. M. the Sun's azimuth was S. 81° 17′ W. the altitude being the same as in the forenoon. The latitude of the ship at noon was 41° 48′ N. and course per compass W. S. W. at the rate of 7 knots an hour. Required the variation?

The interval of time is 6h. 36′, which multiplied by 7 knots, gives 46 miles, nearly, the distance run. By comparing the observed azimuths, it appears the variation is about one point West; hence, the corrected course is S. W. by W. to which, and distance 46 miles, the difference of latitude is 25.6, and departure 38.2. Now to latitude 41° 48′ or 42°, and 19′.1 half the departure, the difference of longitude is 26 miles; which subtracted from 49° 30′, half the elapsed time in degrees, the remainder 49° 4′ is the reduced half interval. The change of declination in the elapsed time is about 6 minutes.

Latitude 41° 48′ secant	- 0.12757	- - co sine	- -	9.87243
Half int. 49 4 cotang.	- 9.93814	- - sine	- -	9.87822
Diff. of latitude 25.6 - log.	1.40824	Change of dec. = 6′ log.		0.77815
First Correction 29.8 - log.	1.47395	Second Correct. 3.4 log.		0.52880

 Western

Western azimuth - - -	S. 81° 17′ W.
First correction - - -	+ 29.8
Second correction - - -	+ 3.4
Western azimuth corrected -	S. 81 50 W.
Eastern azimuth - - -	S. 59 4 E.
Difference - - - - -	22 46
Variation - - - - -	11 23W.

Remark.

If, in place of taking equal altitudes of the Sun or a Star, the points of the compass, upon which the object rises and sets, be observed, then half the difference will be the variation as before; and if the ship is under way during the interval between the observations, the western azimuth is to be corrected, as directed in this Problem.

Problem V.

Given the Latitude of a Place, the Sun's Declination and Magnetic Azimuth, and the Apparent Time of Observation; to find the Variation of the Compass.

Rule.

To the log. co-tangent of the Sun's declination, add the log. co-sine of its distance from the meridian in degrees; the sum will be the log. co-tangent of arch first; which being subtracted from, or added to the latitude of the place of observation, according as the latitude and declination are of the same, or of a contrary denomination, the difference, or sum, will be arch second.

Now, to the log. co-tangent of the Sun's distance from the meridian in degrees, add the log. secant of arch first, and the log. sine of arch second; the sum will be the log. co-tangent of the true azimuth, from the south in north latitude, otherwise from the north. The difference between the true and observed azimuths will be the variation.

Example.

August 19, 1813, in latitude 41° 46′ N. longitude 144° 20′ W. at 9h. 13′ 44″ A. M. apparent time, the magnetic azimuth of the Sun's center was S. 80° 20′ E. Required the variation?

App time obs. 9h 13′ 44″ A. M.	☉'s dec. at noon at Gr. 12° 51′ N.
	Equation to 2h. 46′ - + 2
Time fm. noon 2 46 16 = 41° 34′ - -	to 144° 20′ W. — 8
	Reduced declination - 12 45 N.

Declina-

Declination -	12° 45′ - co-tang. -	0.64536		
Sun's mer. dist.	41 34 - co-sine -	9.87401	co-tangent	0.05217
Arch first - -	16 50 - co-tang. -	0.51937	secant -	0.01902
Latitude - -	41 46 N.			
Difference -	24 56 - - - -		sine - -	9.62486
True azimuth - - -		S. 63° 35′ E.	co-tangent	9.69605
Observed azimuth - -		S. 80 20 E.		
Variation - - - -		16 45 E.		

The observed azimuth being to the left of the true.

Problem VI.

Given the Latitude of a Place, the Sun's Declination and Magnetic Azimuth, and the Interval between that Observation and the Time of Sun Rising or Setting; to find the Variation of the Compass.

Rule.

To the log. tangent of the latitude, add the log. tangent of the declination; the sum will be the log. sine of an arch; which, when the latitude and declination are of the same name, is to be added to 90°, but when of different names, to be subtracted therefrom.

From this arch take the observed interval of time in degrees; the remainder will be the Sun's distance from the meridian; with which, the declination and latitude, the true azimuth is to be found by last problem; and hence the variation of the Compass will be known.

Example.

February 17, 1822, in latitude 36° 18′ N. long. 33° 24′ W. the Sun's magnetic azimuth was S. 59° 40′ W. at 3h. 12′ P. M. and at 5h. 26′ the Sun was observed to be in the true horizon. Required the variation?

Time of obs. 3h 12′		Sun's dec. Feb. 17, at noon at Green.	12° 4′
5 26		Equation to middle interval - -	— 3
		- - to long. 33° 24′ W. - -	— 2
Interval - 2 14 = 33° 30′			
		Reduced declination - -	11 59
			Latitude

Latitude - -	36° 18′	- -	tangent - -	9.86603
Declination -	11 59	- -	tangent - -	9.32685
Arch - -	8 56	- -	sine - - -	9.19288
Subtract from	90			
Remainder -	81 2			
Interval -	33 30			

Dist. fm. mer.	47 32	- co-sine -	9.82941	- co-tang. -	9.96155
Declination -	11 59	- co-tang.	0.67315		
Arch first -	17 27	- co-tang.	0.50256	- secant -	0.02046
Latitude - -	36 18 N.				
Arch second	53 45	- - -		sine - -	9.90657

True azimuth -	S. 52° 16′ W.	- co-tangent -	9.88858
Magnetic azimuth -	S. 59 40 W.		

Variation - - - - 7 24 W. The observed azimuth being to the right of the true azimuth.

Problem VII.

Given the Declination and Altitude of the Sun, its Magnetic Amplitude, and the Interval of Time between the Observations; to find the Latitude and Variation of the Compass.

Rule.

To the log. sine of half the elapsed time, add the log. co-sine of the declination; the sum will be the log. sine of the half of arch first; and the sum of the log. sines of half the elapsed time and declination will be the log. co-tangent of arch second.

To the log. co-secant of arch first, add the log. sine of the altitude; the sum will be the log. sine of arch third. Subtract the sum of arches second and third from 180°, the remainder will be arch fourth; to the log. sine of which, add the log. co-sine of the declination; the sum will be the log. sine of the latitude. And to the log. co-sine of arch fourth, add the log. co-tangent of the declination; the sum will be the log. tangent of the true amplitude, to be reckoned from the north, when the declination is north, but from the south, when the declination is south; and hence, the variation will be obtained.

Example.

May 12, 1810, the Sun was oberved to rise at 3h. 58′ per watch, and amplitude then was N. 85° 6′ E. and at 8h. 58′, the true altitude of

of the Sun's center was 38° 34′. Required the latitude of the place of observation, and the variation of the compass?

Inter. 5h 0′								
Half 2 30 = 37° 30′		sine	- 9.78445	- -	tangent	-	9.88498
Declination	18 0	co-sine	9.97821	- -	sine	- -	9.48998
	35 23	sine	- 9.76266	Arch 2d 76° 40′	co-t.		9.37496
Arch first	- 70 46	co-sec.	0.02494				
Altitude	- 38 34	sine	- 9.79478				
Arch third	- 41 19	sine	- 9.81972				
Arch second	76 40						
Sum	- - 117 59						
Subtract from	180 0						
Arch fourth	62 1	sine	- - 9.94600	-	co-sine	- -	9.67137
Declination	18 0	co-sine	9.97821	-	co-tang.	-	0.48822
Latitude	- 57 7′	sine	- 9.92421	Tr.am.N.55 18E.	tan.		0.15959
				Observed amplitude	N.85 6E.		
				Variation - -	29 48 W.		

The observed azimuth being to the right of the true.

Problem VIII.

Given the Declination, and two observed Altitudes and Azimuths of the Sun; to find the Latitude of the Place of Observation, and the Variation of the Compass. *

Rule.

To the log. versed sine of the difference of the observed azimuths, add the log. co-sines of the corrected altitudes; find the natural number answering thereto, which being added to the natural versed sine of the difference of altitude, the sum will be the natural versed sine of arch first.

To the log. sine of arch first, add the log. sine of the difference of azimuth, and the log. co-sine of the greatest altitude; the sum will be the log. sine of arch second.

To the log. tangent of the Sun's declination, add the log. tangent of the

* If the true azimuths be given, the latitude may be found independent of the Sun's declination by the following formula, in which a represents the least altitude, b the greatest altitude, c the least azimuth, and d the greatest azimuth.

$$\text{Then, Tangent latitude} = \frac{\text{co-s.}\,a.\,\text{co-s.}\,d - \text{co-s.}\,b.\,\text{co-s.}\,c}{\text{sine}\,b - \text{sine}\,a}$$

the

the half of arch first; the sum will be the log. co-sine of arch third. The difference between arches second and third will be arch fourth, if the latitude and declination are of the same name; but if of a contrary name, subtract arch second from the supplement of arch third, and the remainder will be arch fourth.

Now, to the log. versed sine of arch fourth, add the log. co-sine of the least altitude, and the log. co-sine of the declination; the nat. number answering to this sum, being added to the nat. versed sine of the difference or sum of the least altitude and declination, according as the latitude and declination are of the same, or of contrary names; the sum will be the nat. co-versed sine of the latitude.

Then, to the log. sine of arch fourth, add the log. co-sine of the declination, and the log. secant of latitude; the sum will be the log. sine of the true azimuth, when the greatest altitude was observed. Hence, the variation will be known.

Example.

August 26, 1809, in the forenoon, the Sun's magnetic azimuth was S. 22° 41′ E. and corrected central altitude 33° 14′. Afterwards the magnetic azimuth was S. 14° 53′ W. and the true altitude 42° 36′. Required the latitude and variation?

First obs. azim.	S. 22° 41′ E.					
Second obs. az.	S. 14 53 W.					
Difference	37 34	Nat. ver. sine	20736		log.	4.31672
First altitude	33 14				co-sine	9.92244
Second altitude	42 36				co-sine	9.86694
Difference	9 22	Nat. ver. sine	01333			
			12767			4.10601
Arch first	30 48	Nat. ver. sine	14100		co-secant	0.29069
Half Arch first	15 24	tangent	9.44004	Diff. azim. 37° 34′	sine	9.78510
Declination	10 30	tangent	9.26796	Greatest alt. 42 36	co-sine	9.86694
Arch third	87 4	co-sine	8.70801	Arch second 61 13	sine	9.94273
Arch second	61 13					
Arch fourth	25 51	log. versed sine	4.00027		sine	9.63950
Least altitude	33 14	co sine	9.92244			
Declination	10 30	co-sine	9.99267		co-sine	9.99267
Difference	22 44	N. V. S.	07769			
			08230	3.91538		
Latitude	57 8½	N.co-V.S.	15999		secant	0.26555
		True azimuth	S. 52° 12′ E.		sine	9.89772
		Observed azimuth	S. 22 41 E.			
		Variation	29 31 W.			

Because the observed azimuth is to the right of the true azimuth.

Problem

Problem IX.

To find the Variation of the Compass by Circumpolar Stars *.

Rule.

The greatest azimuth of the star, and its altitude, when in that position, are to be computed as follows:

To the log. secant of the latitude, add the log. co-sine of the declination of the star, the sum, rejecting radius, will be the log. sine of the greatest azimuth of the star, from the north in N. latitude, but from the south in S. latitude.

To the log. sine of the latitude, add the log. co-secant of the declination, the sum, rejecting radius, will be the log. sine of the altitude of the star, when at its greatest azimuth.

Now, when the star has attained this altitude, its azimuth is to be observed: then, the difference between the observed and computed azimuths, if of the same name, or sum, if of a contrary denomination, will be the variation; which will be east or west, according as the observed azimuth is to the left or right of the greatest azimuth by computation.

Remarks.

I.

The time when a star is at its greatest azimuth may be found as follows:

To the tangent of the latitude, add the co-tangent of the declination the sum will be the co-sine of the distance of the star from the meridian. Reduce this to time; and if great accuracy is required which, however, is not necessary, subtract therefrom the equation from Table XVIII. answering to this time and $3\frac{1}{2}$ hours, and the remainder will be the meridian distance of the star in time.

From the right ascension of the star, subtract that of the sun at the noon of the given day; to this time apply the equation from Table XVIII. answering to the Sun's right ascension, and the longitude of the place of observation, by addition or subtraction, according as the longitude is E. or W.; and the sum, or remainder, will be the approximate time of transit of the star. From this time subtract the meridian distance of the star, and the remainder will be the approximate time of the greatest eastern azimuth of the star: from which subtract the equation from Table XVIII. answering to the time from

* If the declination of a star exceeds the complement of the latitude of a place, and of the same name, the star will not set at that place; hence, it is called a *Circumpolar Star.*

noon and the Sun's right ascension, the time being after the noon of the given day; but if the time is before noon, add the equation, and the remainder, or sum, will be the apparent time of the greatest eastern azimuth. Again, to the approximate time of transit add the meridian distance of the star, and from this sum subtract the equation from Table XVIII. answering thereto, and the Sun's right ascension, and the remainder will be the apparent time of the greatest western azimuth of the star.

II.

If the greatest eastern and western azimuths of the star be observed, then half the sum, or half the difference of these azimuths, according as the observations are on the same, or on opposite sides of the meridian, will be the variation.

Examples.

I.

Anno 1820, in latitude 33° 28′ N. the greatest eastern azimuth of Dubhe was observed to be N. E. ½ E. Required the variation?

Latitude - - -	33° 28′	- secant - -	0.07873
Declination - -	62 43	- co-sine - -	9.66124
Greatest azimuth - -	N. 33 20 E.	- sine - -	9.73997
Obs. azim N. E. ½ E. or	N. 50 37 E.		
Variation - - -	17 17,		

which is west, because the observed azimuth is to the right of the true.

II.

Anno 1824, in latitude 21° 40′ N. the greatest azimuth of Kochab, when in the western part of its diurnal path, was N. by E. ¾ E. Required the variation?

Latitude - -	21° 40′	- - secant - -	0.03182
Declination - -	75 53	- - co-sine - -	9.38721
Greatest azimuth -	N. 15 13 W.	- sine - -	9.41903
Observed azimuth -	N. 14 4 E.		
Variation - -	29 17W.		

III.

III.

The two extreme azimuths of the pole star were N. 6½° W. and N. 3° W. Required the variation?

Extreme azimuths - -	N. 6°	30′ W.
- - - - -	N. 3	0 W.
Sum - - - -	9	30
Variation - - -	4	45, which is east,

because the greatest of the observed azimuths is to the left of the meridian.

Problem X.

To find the Variation by the Transit of a fixed Star or Planet.

Rule.

Find the apparent time of the transit of the star, by prob. II. vol II. page 368; but if the object is a planet, the time of its passage over the meridian will be found in the Nautical Almanac. Then, the watch being regulated to apparent time, or its error found from observations of the sun or stars, at the instant of the computed time of transit of the star, or planet, observe its bearing per compass; the difference between which, and the N. or S. points, will be the variation, which will be east, if the observed azimuth is to the left of the meridian, but west, if it is to the right of the meridian.

Remarks.

I.

The less the altitude, and the greater the declination of the observed object, the more accurately will the variation be obtained.

II.

When the pole star is in the same vertical with Aliath, or Cor. Caroli, it is nearly in the meridian. If, therefore, its azimuth be observed when it is in this position, the variation will be obtained as before.

III.

If two stars be selected, whose right ascensions are either nearly equal, or differ 180°, then, when they are vertical, they will be upon the meridian; and the bearing of that star which is nearest the pole should be observed.

Examples.

I.

November 15, 1812, in latitude 34° S. and longitude 50° E. the bearing of Achenar, at the apparent time of its transit under the pole, was S. $25\frac{1}{2}$° W. Required the variation?

The variation is $25\frac{1}{2}$° W. the observed azimuth being to the right of the meridian.

II.

When the pole star and Cor. Caroli were in the same vertical, the azimuth per compass, of the pole star, was N. 21° W. Required the variation?

The variation is 21° E.; the observed azimuth being to the left of the meridian.

Problem XI.

Given the Latitude and Longitude of the Ship, to find the Variation, of the compass by the Variation Chart.

Rule.

Lay a scale over the given latitude, then take the difference between the given longitude and the nearest meridian, which being laid off the same way from that meridian, by the edge of the scale, will show, among the variation lines, the quantity and quality of the variation.

Example.

Let the latitude of a ship be 43° 20′ S. and longitude 38° 40′ E. Required the variation?

By proceeding as above directed on the variation chart, the variation will be found to be about 25° W.

Remark.

The true amplitude and azimuth, and hence the variation of the compass may be found by many different methods, such as by the globe, various modes of projection, Gunter's scale, particularly the sliding Gunter, of which see the author's treatise on the description and use of that instrument in navigation, &c.

Upon account of the variation of the compass, the true course between any two places is different from the course per compass. The true course is found by the common rules of navigation; and if

th

the variation of the compass be known, the magnetic course may be obtained. Again, in order to determine a ship's place from the course steered and distance run, the course must be previously corrected by variation. The method of applying the variation in these cases is as follows:

Problem XII.

Given the true Course between any two Places, and the Variation of the Compass; to find the Course per Compass.

Rule.

The variation, if westerly, being allowed to the right hand of the true course, or to the left hand, if easterly, will give the course per compass.

Example.

Required the course per compass from the Lizard to St. Mary's, the true course being S. W. $\frac{1}{4}$ W. and the variation at the Lizard 2$\frac{1}{2}$ points west?

Two and a half points being allowed to the right of S. W. $\frac{1}{4}$ W. gives W. S. W. $\frac{3}{4}$ W. the magnetic course; or course the ship must steer from the Lizard for St. Mary's; and the course must be altered accordingly as the variation alters.

Problem XIII.

Given the Course per Compass, and the Variation; to find the true Course.

Rule.

The variation, if westerly, being allowed to the left of the course steered, or to the right, if easterly, will give the true course.

Example.

Let the course steered be S. E. $\frac{1}{2}$ E. and the variation 1$\frac{1}{2}$ points W. Required the true course?

One and a half points allowed to the left of S. E. $\frac{1}{2}$ E. gives E. S. E the true course.

Remark.

The rectifier will be found by many persons to be a convenient instrument for performing the two last problems.

END OF THE FIRST VOLUME.

ADDITIONS AND CORRECTIONS.

VOLUME I.

In the Dedication, for "Warden," read "Wardens."

Page 20, line 38, after La Lande, add, "to complete which, only one star, was taken from the ancient constellation of Hydra, viz. the 44th. About the beginning of August, 1807, the University of Leipsig, named those in the belt and sword of Orion, and the stars adjacent thereto, the constellation *Napoleon*.

Page 28, for Chap. IV, read Chap. V.

—— 35, Chap. V. Chap. VI.

—— 46, Chap. VI. Chap. VII.

—— 150, line 4 from the bottom, after "parallax," read, "corrected by the number from Table XLIII. or XLIV. according as the distance between the Moon and the Sun, or a fixed Star, is observed."

—— 156, line 3d. from the bottom, being the log. 9,747049, one line down so as to be opposite to its corresponding natural number 257270.

—— 200, line 7, for "7h. 27′ 4″," read "7h. 37′ 4″;" and the operation being performed agreeable thereto, and the other date, the Latitude will be found to be 57° 37′ 10″ N, and Longitude 21° 23′ E.

—— 293, line 20, for "rep. of alt." read "rep. at alt."

—— 328, after the Example, add the following

Remark.

In observations for finding the latitude by double altitudes, it is often found that an observation of the sun, may be made when that object is in a proper position for observation; but that by reason of clouds or fog, no other altitude could be observed that day, yet the latitude may be found by comparing the above altitude, with an observation of the sun, taken the following day; but in this case, an equation depending on the change of declination, becomes necessary to be applied to one of the altitudes, which may be to the first, if the latitude of the second place of observation be required. This equation may be easily deduced from a fluxionary Spherical analogy, which will be obvious to those who are qualified to teach navigation. Upon the subject of these extensions, and other improvements in finding the latitude, it is probable the author may be induced to publish a treatise which he hopes will be found useful.

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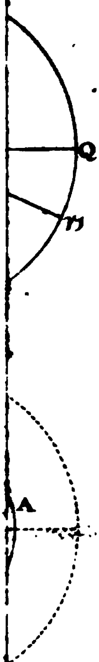
Fig. 5

Plate 1



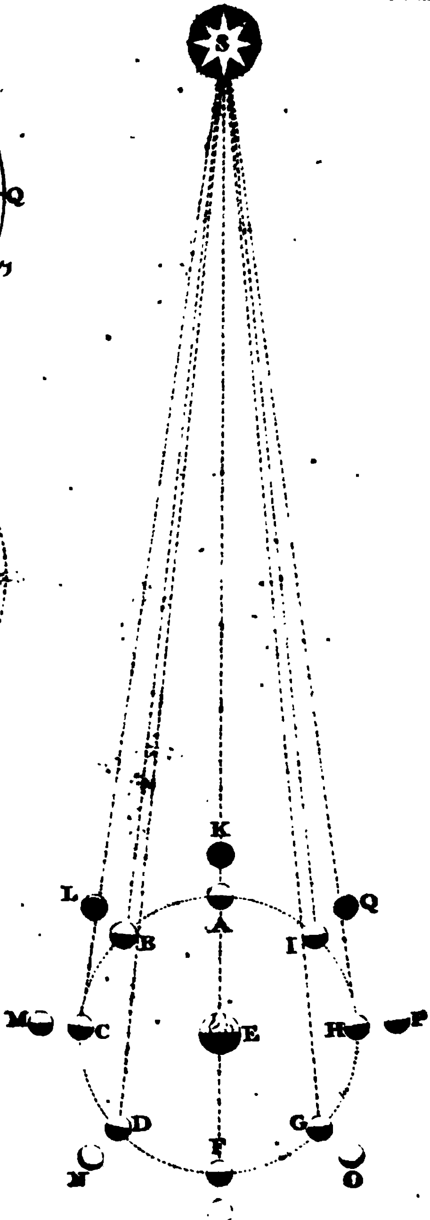


PLATE. II.

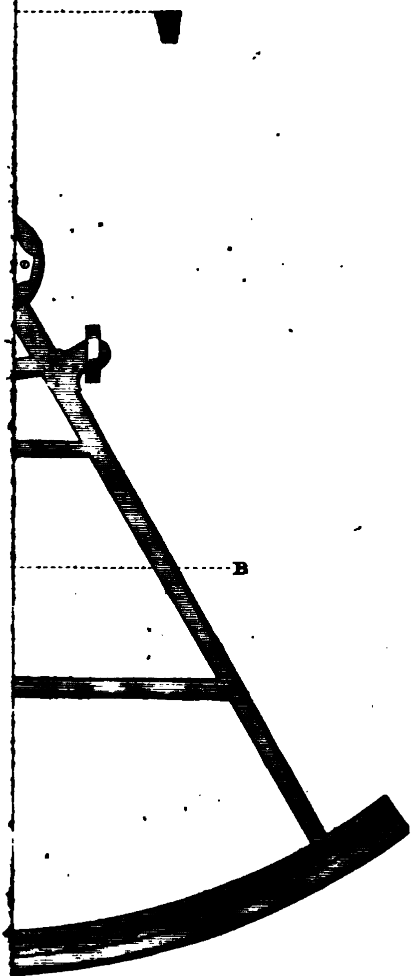


Fig. 10.

Carpenter sculpt London.

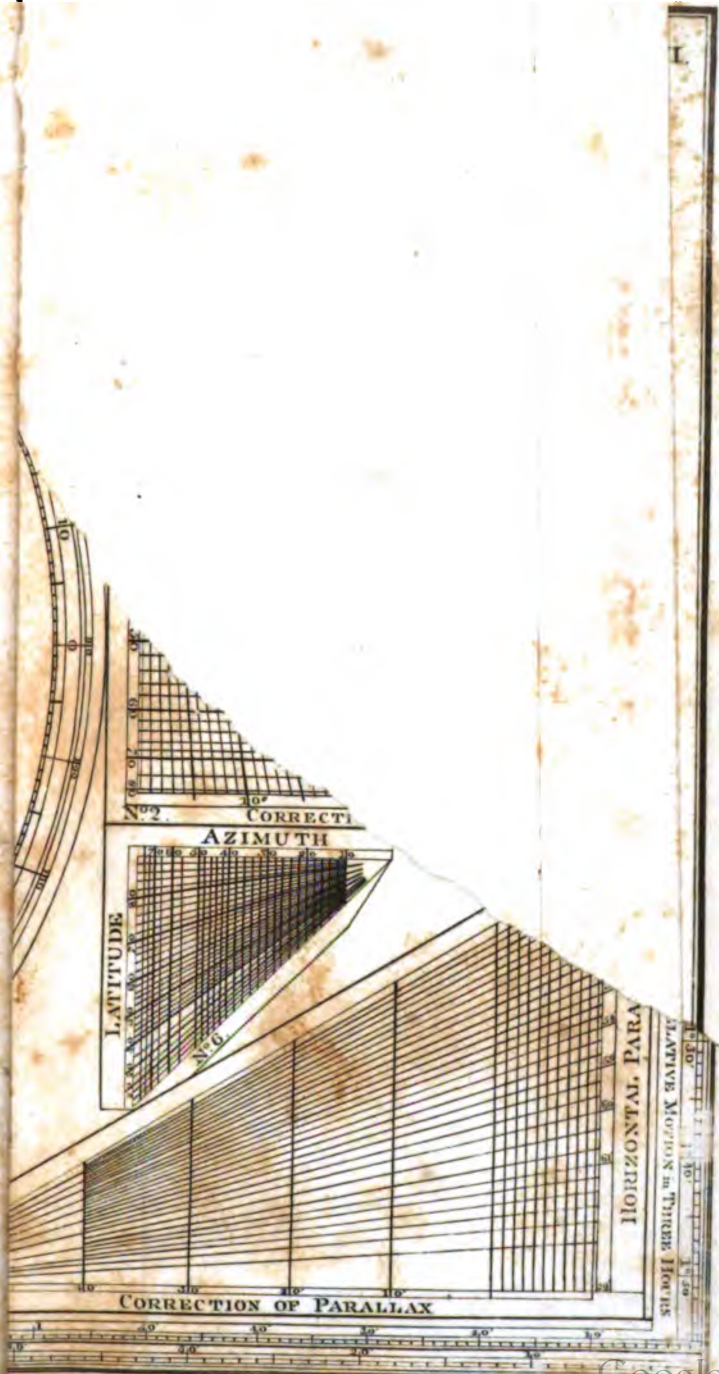

I.
N°2.
CORRECTI
AZIMUTH
LATITUDE
N°6.
HORIZONTAL PARA
LATIVE MOTION in THREE HOURS
CORRECTION OF PARALLAX

PLATE. VI.

Fig. 28.

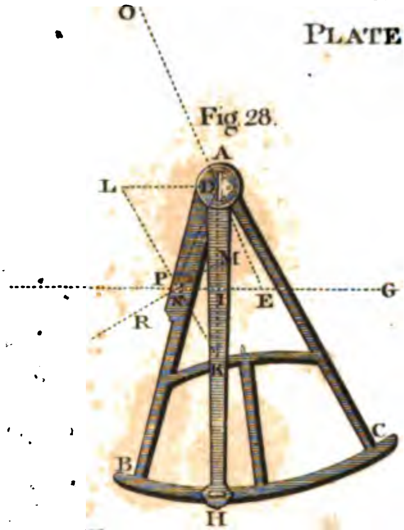
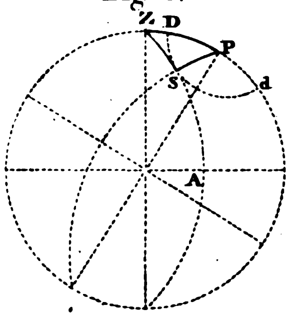


Fig. 30.



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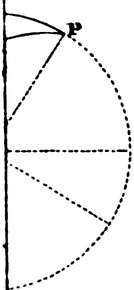


Fig. 32.

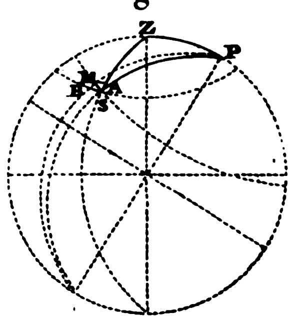


PLATE. VII.

Fig. 34.

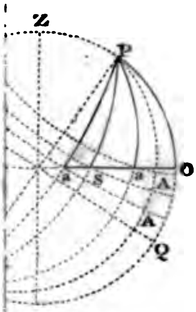
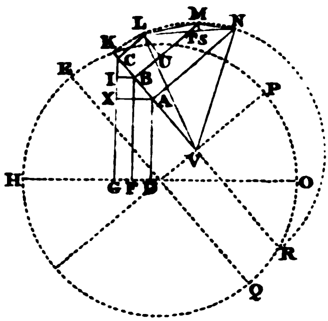


Fig. 35.



g 37.

Fig. 38.

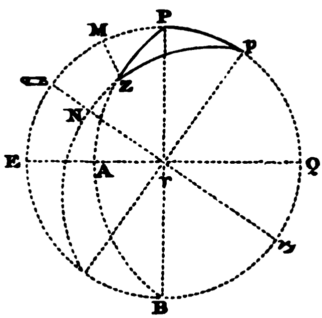


Fig. 40.

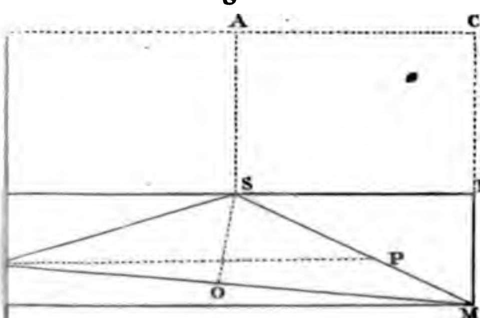


PLATE VIII.

AZIMUTH COMPASS.